Spatio-Temporal UML Modeling and Verifying with Description Logic*

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Spatio-temporal data modeling is an important basis for spatio-temporal data management. Unified Modeling Language (UML) is a widely used modeling language. Therefore, how to model spatio-temporal data based on UML and then how to further verify the correctness of the spatio-temporal UML models have become important issues. In this paper we propose a spatio-temporal UML model and a Description Logic (DL) method for verifying the model. *First of all*, we present a UML-based spatio-temporal data model. *Also*, an abstract definition and semantic description of the spatio-temporal UML models are given, and a case of cadastral change process is provided. *Then*, by adding some special concepts, roles, and axioms into the DL *ALCIQ*, a method for mapping the spatio-temporal UML models to *ALCIQ* knowledge bases is proposed, and a mapping example is provided. *Further*, several verification tasks of the spatio-temporal UML models are equivalently converted to the inference problems of the mapped *ALCIQ* knowledge bases, and the inference results can be returned and the verification of spatio-temporal UML models are realized with the help of the DL inference abilities.

Keywords: spatio-temporal data modeling, UML, description logic, verification, transformation

1. INTRODUCTION

Many applications involve temporal and spatial information, especially in GIS (Geographic Information System), which is particularly important for the management of spatio-temporal data [31]. Spatio-temporal data modeling is a key basis for spatio-temporal data management. Spatio-temporal data models, which can effectively express, organize and model entities and their interactions with time and space, play a key role in several subsequent spatio-temporal data management tasks such as storage, query, analysis, and reasoning [4, 7, 26]. For modeling spatio-temporal data, some spatio-temporal data models have emerged, including snapshot model [15], event-based model [18], three-domain model [35], spatio-temporal ER (Entity-Relationship) model [27], and spatio-temporal object-oriented model [21] (please refer to the survey [19] for details). In particular, UML (Unified Modeling Language) [30], standardized by OMG (Object Ma nagement Group), can model rich semantics at a high abstract level, and thus it has been widely used for conceptual modeling in many applications, *e.g.*, databases and software engineering [22]. Naturally, UML was used to model spatio-temporal data, and several spatio-temporal UML models are accordingly developed [16, 20, 24].

With the development and application of spatio-temporal data models, it is expected to check and amend some inconsistencies as early as possible in the modeling process by verifying the correctness of the models (*e.g.*, checking whether the constraints of a model

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are not in conflict) [2]. Therefore, how to verify and infer spatio-temporal data models has become a key issue. Over the years, researchers proposed kinds of qualitative and quantitative spatio-temporal inference methods, *e.g.*, rule-based [13] and point graphsbased [36] (see [14, 33] for surveys). Description Logics (DLs) [5], as a family of knowledge formalization and inference languages, are employed to verify and infer various data models, *e.g.*, ER model [8] and UML model [6, 11]. Also, for inferring on spatiotemporal knowledge, several spatio-temporal DLs are proposed, such as temporal DLs [17], spatial DLs [9, 32], and spatio-temporal DLs [28]. And the temporal DLs are used to infer temporal EER (Extended Entity-Relationship) conceptual models [3].

It can be observed from the existing works that there exist several key issues still need to be settled for the spatio-temporal UML modeling and verifying with DLs. From the point of spatio-temporal UML modeling, several existing spatio-temporal UML models [16, 20, 24] still cannot meet various application requirements, *e.g.*, they cannot represent and deal with some complex semantic relationships among spatio-temporal classes/objects (*e.g.*, RCC-8 spatial topological relations [23] and Allen-13 temporal topological relations [1]). Also, the existing works did not give abstract definition and semantic description methods of spatio-temporal UML models. From the point of verifying spatio-temporal UML models with DLs, as far as we know, less research on representation and verification of spatio-temporal UML models with DLs has been done.

In this paper we propose a spatio-temporal UML model and a Description Logic (DL) method for verifying the model, including the following main contributions:

- We present a UML-based spatio-temporal data model, and give its abstract definition and semantic description. The model uses UML's extended mechanism stereotypes to represent spatio-temporal classes, spatio-temporal associations, several complex spatio-temporal semantic relationships (*e.g.*, generalization and aggregation), and spatiotem poral objects and relations (including RCC-8 spatial topological relations [23] and Allen-13 temporal topological relations [1]).
- We propose a Description Logic (DL) method for verifying the spatio-temporal UML models, including: (i) we *first* present a method for mapping the spatio-temporal UML models to the DL *ALCIQ* knowledge bases; (ii) we *further* convert several verification tasks of the spatio-temporal UML models to the inference problems of the mapped *ALCIQ* knowledge bases, so that the verification of spatio-temporal UML models can be handled by DL inference abilities.

Note that, we choose the existing DL *ALCIQ* [5] because it has enough expressive power and several existing practical reasoners (*e.g.*, FaCT++ [29] and Pellet [25]). This implies that the spatio-temporal UML models can be verified with the help of the existing DL reasoners.

The remainder of this paper is organized as follows: Section 2 recalls some preliminaries. Section 3 proposes a spatio-temporal UML model. Sections 4 and 5 propose a DL method for verifying the spatio-temporal UML models. Section 6 introduces the related work and makes a detailed comparison. Section 7 concludes the paper.

2. PRELIMINARIES ON SPATIO-TEMPORAL RELATIONS AND DLS

In this section, preliminaries on spatio-temporal relations and DLs are recalled.

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2.1 Spatio-Temporal Relations

There are some constraint calculi that are used to represent and reason about temporal or spatial relations (*e.g.*, Region Connection Calculus RCC and Allen's temporal Interval Calculus [12]). We will mainly focus on the RCC-8 spatial topological relations [23] and Allen-13 temporal topological relations [1]. RCC-8 can represent eight kinds of spatial topological relations: *DC*(DisConnected), *EC*(Externally Connected), *PO*(Partial Overlap), *EQ*(EQual), *TPP*(Tangential Proper Part), *NTPP*(Non-Tangential Proper Part), and their converse relations *TPPi* and *NTPPi*. Allen-13 can represent thirteen relations between temporal intervals: <(before), *m*(meets), *o*(overlaps), *d*(during), *s*(starts), *f*(finishes), their converse relations (>(after), *mi*(met-by)), *oi*(overlapped-by), *di*(includes), *si*(started-by), *fi*(finished-by)), and =(equal).

Further, composition tables are very important techniques for reasoning on temporal or spatial relations. As introduced in [1, 10], given two relations, a composition table can be used to answer an inference question. For example, given two spatial topological relations DC(x, y) and TPPi(y, z), we can infer that there is a relation DC(x, z) between x and z according to the spatial composition table [10]. Similarly, given two temporal intervals $\prec(x, y)$ and m(y, z), a relation $\prec(x, z)$ can be inferred according to the temporal composition table [1]. Such kind of computation is frequently very useful. The details about the composition tables for RCC-8 and Allen-13 can be found in [1, 10].

2.2 Description Logic ALCIQ

Here we briefly recall the syntax and semantics of the DL ALCIQ [5]. The ALCIQ concepts and roles can be constructed inductively:

$$C_1, C_2 \rightarrow \mathsf{T} \mid \perp \mid A \mid \neg C_1 \mid C_1 \sqcap C_2 \mid C_1 \sqcup C_2 \mid \forall R.C_1 \mid \exists R.C_1 \mid \geq kR. C_1 \mid \leq kR.C_1$$
$$R \rightarrow P \mid R^{-}$$

where A, P are atomic concept and role, C_1 and C_2 are concepts, R is an inverse role of R.

The semantics of *ALCIQ* is provided by interpretations \mathcal{I} . A TBox and an ABox compose a DL knowledge base (KB). A TBox is a set of terminology axioms $C_1 \sqsubseteq C_2$ or $C_1 \equiv C_2$. An ABox is a set of assertions $C_1(a)$ or R(a, b). An interpretation \mathcal{I} satisfies a DL KB if it satisfies all axioms and assertions in the KB.

The basic inference tasks in an *ALCIQ* KB include concept satisfiability and subsumption, ABox consistency, knowledge base satisfiability, and retrieval, which can be checked through the inference techniques for *ALCIQ* [5] and the existing reasoners (*e.g.*, FaCT++ [29] and Pellet [25]). Please refer to [5] for more details about the DL *ALCIQ*.

3. SPATIO-TEMPORAL UML MODELING

In this section we propose a UML-based spatio-temporal data model. Further, an abstract definition and semantic description of the model are given, and a case of cadastral change process is modeled by the proposed model.

3.1 Spatio-Temporal UML Models

We use UML's extended mechanism stereotypes to represent spatio-temporal clas-

ses, spatio-temporal associations, several complex spatio-temporal semantic relationships, and spatio-temporal objects and their relations (including RCC-8 spatial topological relations [23] and Allen-13 temporal topological relations [1]).

3.1.1 Spatio-temporal UML classes

In a spatio-temporal UML model, spatio-temporal objects with the same characteristics constitute a *spatio-temporal class*. We use the UML stereotypes <<spatial cla>>, <<temporal cla>>, and <<spatio-temporal cla>> to indicate that a class is a spatial class, temporal class, and spatio-temporal class, respectively. For simplicity, throughout this paper we take the spatio-temporal class as example to introduce the model. In a spatiotemporal class, at least one of attributes is a spatio-temporal attribute, and its type is related to a spatial range and a timestamp. Similarly, we use the UML stereotypes <<spatio-temporal att>> to indicate that an attribute is a spatio-temporal attribute in Fig. 1.

3.1.2 Spatio-temporal UML associations

A spatio-temporal association, which models a relation between spatio-temporal classes, declares that there are links between objects of the associated spatio-temporal classes. We use the UML stereotypes <<spatial ass>>, <<temporal ass>>, and <<spatio-temporal ass>> to indicate that an association is a spatial association, temporal association, and spatio-temporal association, respectively. The participation of a spatio-temporal class in an association is called a role (*e.g.*, r_1 and r_2) and has a unique name. A multiplicity $m_i...n_i$ specifies that each instance object of a spatio-temporal class can participate at least m_i times and at most n_i times to a spatio-temporal association in Fig. 2.

	< <spatio-temporal cla="">> Class</spatio-temporal>		< <spatio-temporal cla="">> Class1</spatio-temporal>	<< sp	atio-temporal a Association	ss>> r2	<pre><<spatio-temporal cla="">> Class2</spatio-temporal></pre>
	- attribute < <spatio-temporal att="">> : type</spatio-temporal>			$m_{1}n_{1}$		m_2n_2	
_		-					

Fig. 1. Spatio-temporal UML classes/attributes.

Fig. 2. Spatio-temporal UML associations.

3.1.3 Spatio-temporal UML relationships

Several complex spatio-temporal semantic relationships are useful in spatio-temporal data modeling. In the following we give their UML representation forms.

• *Spatio-temporal aggregation*: it is a special situation of association. If two spatio-temral classes have a "whole and part" relationship, then such an association is called spatio-emporal aggregation as in Fig. 3.

• Spatio-temporal generalization: it can describe the relationship between general and specific spatio-temporal classes. The spatio-temporal class with general description is called "superclass" and with specific description is called "subclass" as in Fig. 4. In a spatio-temporal generalization, any spatio-temporal object belonging to the subclass also belongs to the superclass.

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3.1.4 Spatio-temporal objects

An instance of a spatio-temporal class is called a *spatio-temporal object*. The associations among spatio-temporal objects are called *links*. Fig. 5 shows the graphical form of spatio-temporal objects in a spatio-temporal UML model.

< <spatio-temporal obj="">> Object₁: Class₁</spatio-temporal>	link	<

Fig. 5. Spatio-temporal UML objects.

Moreover, we use eight specific spatial links <<DC ass>>, <<EC ass>>, <<PO ass>>, <<EQ ass>>, <<TPP ass>>, <<TPPi ass>>, <<NTPP ass>>, and <<NTPPi ass>> to model the RCC-8 spatial topological relations between spatial objects as mentioned in Section 2.1. Similarly, we use thirteen specific temporal links <>before ass>>, <<after ass>>, <<equal ass>>, <<meets ass>>, <<metBy ass>>, <<overlaps ass>>, <<overlaps ass>>, <<overlappedBy ass>>, <<during ass>>, <<includes ass>>, <<starts ass>>, <<startedBy ass>>, <<finishes ass>>, <<finishedBy ass>> to model the Allen-13 temporal topological relations between temporal objects as mentioned in Section 2.1.

3.2 Abstract Definition and Semantic Description of Spatio-Temporal UML Models

Based on Section 3.1, we further give an abstract definition and semantic description of spatio-temporal UML models based on [6, 8, 34]. Here, we further add spatio-temporal objects into the definition and consider the spatio-temporal features of UML models. As usual we suppose that a spatio-temporal UML model has an alphabet of distinct spatio-temporal class names C_{ST} , object names O_{ST} , attribute names A_{ST} , association name S_{ST} , role names R_{ST} , domain names D_{ST} . The domains of attributes may be the basic data types (*e.g., string* and *integer*), the temporal data types (*e.g., time point* and *time interval*), and the spatial data types (*e.g., points, lines, and regions*).

Definition 1 (formalization): A spatio-temporal UML model $ST_{UML} = (cfa, sfc, wfp, sfs, ofc)$, where:

- *cfa*: $C_{ST} \rightarrow A_{ST}$ is a mapping function from spatio-temporal classes C_{ST} to attributes A_{ST} with domains D_{ST} .
- *sfc*: $S_{ST} \rightarrow C_{ST}$ is a mapping function from spatio-temporal associations S_{ST} to classes C_{ST} labeled by roles R_{ST} and restricted by multiplicity (m_i, n_i) .
- wfp: $C_{ST} \rightarrow C_{ST}$, which is a mapping function from spatio-temporal whole classes C_{ST} to part classes C_{ST} , is used to model the aggregation relationships.
- sfs: $C_{ST} \rightarrow C_{ST}$, which is a mapping function from spatio-temporal subclasses C_{ST} to

superclasses C_{ST} , is used to model the generalization relationships.

• of $c: O_{ST} \rightarrow C_{ST}$ is a mapping function from spatio-temporal objects O_{ST} to spatio-temporal classes C_{ST} .

Being similar to UML models [6, 8, 34], the semantics of a spatio-temporal UML model can be described by spatio-temporal object states (*STOS*) in Definition 2.

Definition 2 (semantics): A spatio-temporal object state *STOS* w.r.t. a spatio-temporal UML model ST_{UML} is a tuple $STOS = (\Delta^{\text{STOS}}, \bullet^{\text{STOS}})$, where Δ^{STOS} is a set of spatio-temporal objects and \bullet^{STOS} is a function that maps:

- Each spatio-temporal class $C \in C_{ST}$ to a subset of Δ^{STOS} , *i.e.*, $C^{STOS} \subseteq \Delta^{STOS}$.
- Each spatio-temporal object $O \in O_{ST}$ to an element of Δ^{STOS} , *i.e.*, $O^{STOS} \in \Delta^{STOS}$.
- Each spatio-temporal domain $D \in D_{ST}$ to a set D^{STOS} .
- Each spatio-temporal attribute $A \in A_{ST}$ to a subset of $\Delta^{STOS} \times D^{STOS}$.
- Each spatio-temporal association (including aggregation) $S \in S_{ST}$ to a set S^{STOS} of tuples over Δ^{STOS} labeled by roles R_{ST} . That is, with any $S \in S_{ST}$ with $sfc(S) = [R_1:C_1, ..., R_n:C_n]$, where $R_i \in R_{ST}$, $C_i \in C_{ST}$, $i \in \{1...n\}$, each element in S^{STOS} is a tuple of $[R_1:O_1, ..., R_n:O_n]$, where $O_i \in C_i^{STOS}$.
- With any $C \in C_{ST}$ with $cfa(C) = [A_1:D_1, ..., A_k:D_k]$, where $A_i \in A_{ST}$, $D_i \in D_{ST}$, $i \in \{1, ..., k\}$, then for any spatio-temporal object $O \in O_{ST} \in C^{STOS}$ and $i \in \{1...,k\}$, there exists exactly an element $a_i = \langle O, d_i \rangle \in A_i^{STOS}$, where $d_i \in D_i^{STOS}$.
- With any multiplicity (m_i, n_i) in a spatio-temporal association S, *i.e.*, $sfc(S) = [R_i (m_i, n_i): C_i]$, for each spatio-temporal object $O \in O_{ST} \in C_i^{STOS}$, it follows $m_i \le \#\{s \in S^{STOS} \mid s[R_i] = O]\} \le n_i$, where $\#\{\}$ denotes the cardinal number of set $\{\}$.
- With any generalization such that $sfs(C_{sub}) = C_{super}$, where C_{sub} , $C_{super} \in C_{ST}$, then $C_{sub-STOS} \subseteq C_{super}$.
- With any object $O \in O_{ST}$ such that ofc(O) = C, and $C \in C_{ST}$, then $O^{STOS} \in C^{STOS}$.

The elements of C^{STOS} , O^{STOS} , D^{STOS} , A^{STOS} , and S^{STOS} are called instances of *C*, *O*, *D*, *A*, and *S* respectively.

3.3 Examples of Cadastral Changes in Spatio-Temporal UML Models

Cadastral management information system is an important part of geographic information system. The cadastral objects include land parcels, and the basic forms of land parcel change are segmentation and mergence. A cadastral change process is shown in Fig. 6. As time goes on, land parcels are continuously segmented and merged. In the following we use the spatio-temporal UML model in Section 3.1 to model the cadastral change process and the spatial and temporal topological relations among the land parcels.



Fig. 6. A cadastral change process.

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Fig. 7 gives a spatio-temporal UML class diagram for the cadastral change in Fig. 6. In the class diagram, a generic *Parcel* class is defined to specify some common attributes for all parcel classes. According to the cadastral change process, several different land parcel classes, such as *Industry*, *Resident*, *Park*, *Commerce*, and *University*, need to be created. All of them inherit the *Parcel* class and have their own attributes. Further, Fig. 8 gives a spatio-temporal UML object diagram for modeling the temporal and spatial relationships among the land parcel objects. Here we consider that several associations (*e.g.*, *Industry* and *Commerce*, *Commerce* and *Resident*, and *Park* and *Resident*) are similar with the associations have been provided in Figs. 7 and 8. For example, the association between *Industry* and *Commerce* is similar with the association between *Industry* and



Fig. 7. A spatio-temporal UML class diagram for cadastral change in Fig. 6.



Fig. 8. A spatio-temporal UML object diagram w.r.t the class diagram in Fig. 7.

Park in Fig 7, and the association between *Park* and *Resident* is similar with the association between *University* and *Resident* in Fig. 7. Therefore, we omit the several associations for simplification.

Further, according to the Definition 1, Fig. 9 gives the abstract form of the spatialtemporal UML model in Figs. 7 and 8. After constructing the spatio-temporal UML model of the above-mentioned cadastral change process, it is necessary to further verify the correctness of the model, for example, to determine whether the model has inconsistencies or not. In addition, users often want to know what topological relationship exists between any two land parcels in the process of cadastral change. Thus, Sections 4 and 5 will investigate the verification of spatio-temporal UML models based on DLs.

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The spatio-temporal UML model in Figs. 7 and 8 can be formalized as ST_{UML} = (cfa, sfc, wfp, sfs, ofc):

cfa(Parcel) = [area: float, ownership: string, validBeginTime: date, ...];

cfa(Industry) = [industrialType: enum{heavy industry, light industry}];

cfa(Park) = [environmentalProtectionLevel: string];

cfa(Resident) = [residenceType: string, serviceLife: date];

sfc(st-ass_1) = [ind_1: Industry, par_1: Park];

sfs(Resident) = Parcel; sfs(Industry) = Parcel; sfs(Park) = Parcel;

sfs(Commerce) = Parcel; sfs(University) = Parcel;

ofc(residentB) = Resident; ofc(industryA) = Industry; ofc(parkC) = Park;

ofc(commerceD) = Commerce; ofc(universityE) = University;

... //The other spatio-temporal constructors can be given similarly.
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Fig. 9. The abstract form of the spatial-temporal UML model in Figs. 7 and 8.

4. MAPPING OF SPATIO-TEMPORAL UML MODELS TO DL ALCIQ

Based on the spatio-temporal UML models in Section 3, we will further propose a method for verifying the spatio-temporal UML models based on Description Logics (DLs). In Section 4 we map a spatio-temporal UML model to a DL knowledge base (KB). *Firstly*, we add several syntactic sugar concepts, roles, and axioms into DL *ALCIQ*. *Then*, we map a spatio-temporal UML model to a DL *ALCIQ* KB, and prove the correctness. *Finally*, we give a mapping example. In Section 5 we further convert the verification tasks of the spatio-temporal UML model to the inference problems of the DL KB.

4.1 Syntactic Sugar Extensions of the DL ALCIQ

In order to verify the spatio-temporal UML model, our work borrows the idea of [32] for ensuring the decidability of the DL. We also treat spatial/temporal relations as special concepts, and express RCC-8 and Allen-13 topological relations in the form of DL axioms. For this purpose, we need to add several syntactic sugar concepts, roles, and axioms into the DL *ALCIQ* [5].

Firstly, in Allen's algebraic theory, a temporal interval $[e_1, e_r]$ is represented by two endpoints (*i.e.*, the left endpoint e_1 and the right endpoint e_r). To express the Allen-13 topological relations between two temporal intervals TI_1 , TI_2 , we need to add the following twelve atomic concepts into *ALCIQ*:

- ξ (*ss-before*) denotes the e_1 of TI_1 precedes the e_1 of TI_2 , e.g., $\frac{e_1 \quad TI_1 \quad e_1}{e_1 \quad TI_2}$
- ξ (*se-before*) denotes the e_1 of TI_1 precedes the e_r of TI_2 ;
- ξ (*es-before*) denotes the e_r of TI_1 precedes the e_1 of TI_2 ;
- ξ (*ee-before*) denotes the e_r of TI_1 precedes the e_r of TI_2 ;
- $\xi(ss-after)$ denotes the e_1 of TI_1 is behind the e_1 of TI_2 , e.g., e_1
- ξ (*se-after*) denotes the e_1 of TI_1 is behind the e_r of TI_2 ;
- ξ (*es-after*) denotes the e_r of TI_1 is behind the e_1 of TI_2 ;
- $\xi(ee-after)$ denotes the e_r of TI_1 is behind the e_r of TI_2 ;
- ξ (*ss-meet*) denotes the e_1 of TI_1 meets the e_1 of TI_2 ;
- ξ (*se-meet*) denotes the e_1 of TI_1 meets the e_r of TI_2 ;
- ξ (*es-meet*) denotes the e_r of TI_1 meets the e_1 of TI_2 ;
- ξ (*ee-meet*) denotes the e_r of TI_1 meets the e_r of TI_2 .

Based on the above concepts, the complex Allen-13 temporal relations can be expressed by using these concepts combined with atomic negation (\neg) and intersection (\sqcap) operators. For example, *before* can be defined as *ALCIQ* axiom: $\xi(before) \equiv \xi(ss\text{-before}) \sqcap \xi(se\text{-before}) \sqcap \neg \xi(se\text{-before}) \sqcap \neg \xi(se\text{-after}) \sqcap \neg \xi(se\text{-after}) \sqcap \neg \xi(se\text{-after}) \sqcap \neg \xi(se\text{-meet}) \sqcap \neg \xi(se\text{-meet}) \sqcap \neg \xi(se\text{-meet})$.

Moreover, for any pair of temporal objects xy (x and y are two different temporal objects) such that $\xi(ss-before)(xy)$ in the ALCIQ KB, we need to add its inverse $\xi(ss-after)(yx)$ into the ALCIQ KB, and similarly for the other pairs of basic inverse temporal relations $\langle \xi(se-before), \xi(es-after) \rangle$, $\langle \xi(es-after) \rangle$, $\langle \xi(ee-before), \xi(ee-after) \rangle$, $\langle \xi(ee-before) \rangle$, $\langle \xi(se-after), \xi(ee-before) \rangle$, $\langle \xi(se-after), \xi(es-after) \rangle$, $\langle \xi(es-after), \xi(es-after) \rangle$, $\langle \xi(es-meet) \rangle$, $\langle \xi(es-meet) \rangle$, $\langle \xi(ee-meet) \rangle$.

Secondly, in order to express RCC-8 topological relations between spatial objects SO_1 , SO_2 , following the idea of [32], the four atomic concepts are added into *ALCIQ*:

- ξ (*ii-ints*) denotes the interior of SO_1 intersects the interior of SO_2 ;
- ξ (*ib-ints*) denotes the interior of SO_1 intersects the border of SO_2 ;
- ξ (*bi-ints*) denotes the border of SO_1 intersects the interior of SO_2 ;
- ξ (*bb-ints*) denotes the border of SO_1 intersects the border of SO_2 .

Based on the concepts above, the complex RCC-8 spatial relations can be expressed by using these concepts. For example, the RCC-8 disjoint relation *DC* can be defined as the *ALCIQ* axiom: $\xi(DC) \equiv \neg \xi(ii\text{-ints}) \sqcap \neg \xi(ib\text{-ints}) \sqcap \neg \xi(bb\text{-ints})$.

Moreover, in an ALCIQ KB, we use a form of $\xi(DC)(xy)$ or $\xi(ii-ints)(xy)$ to represent that two spatial objects x and y have a spatial relation such as DC or *ii-ints*. Further, for any pair of spatial objects xy (x and y are two different spatial objects) such that $\xi(ii-ints)(xy)$ in the ALCIQ KB, we need to add its inverse $\xi(ii-ints)(yx)$ into the ALCIQ KB. Similarly, if there is $\xi(ib-ints)(xy)$, $\xi(bi-ints)(xy)$, or $\xi(bb-ints)(xy)$ in the ALCIQ KB, we add their inverse $\xi(bi-ints)(yx)$, $\xi(ib-ints)(yx)$, or $\xi(bb-ints)(yx)$ into the ALCIQ KB.

Thirdly, for expressing combination tables as mentioned in Section 2.1, several special *ALCIQ* roles and assertions are introduced [32]:

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- $\xi(UTB)$: Mapping the first temporal (spatial) object to a pair of temporal (spatial) objects, *e.g.*, $\xi(UTB)(x, xy)$.
- $\xi(BTU)$: Mapping a pair of temporal (spatial) objects to the second temporal (spatial) object, *e.g.*, $\xi(BTU)(xy, y)$.
- ξ(BTT): Mapping a pair of the first and last temporal (spatial) objects to a ternary temporal (spatial) object, e.g., ξ(BTT)(xz, xyz).
- $\xi(TTB_1)$: Mapping a ternary temporal (spatial) object to a pair of the first two temporal (spatial) objects, *e.g.*, $\xi(TTB_1)(xyz, xy)$.
- ζ(TTB₂): Mapping a ternary temporal (spatial) object to a pair of the last two temporal (spatial) objects, *e.g.*, ζ(TTB₂)(xyz, yz).

Moreover, there exist several temporal (spatial) objects in an *ALCIQ* KB (*e.g.*, *x*, *y*, and *z*), in this case, we need to add their permutations (*e.g.*, *xy*, *xz*, *yx*, *zx*, *zy*, *xyz*, *xzy*, *yxz*, *zxy*, *zyx*, *zyy*, *zyx*, *zyx*, *zyy*, *zyx*, *zyx*, *zyy*, *zyy*,

4.2 Mapping of Spatio-Temporal UML Models to ALCIQ KBs

A spatio-temporal UML model $ST_{\text{UML}} = (cfa, sfc, wfp, sfs, ofc)$ in Definition 1 can be mapped to an *ALCIQ* KB $\xi(ST_{\text{UML}})$ according to the rules in Table 1.

A spatio-temporal UML model ST _{UML}	An ALCIQ KB $\xi(ST_{\text{UML}})$
A spatio-temporal class $C \in C_{ST}$	A concept $\xi(C)$
A spatio-temporal attribute $A \in A_{ST}$	A role $\xi(A)$
A spatio-temporal association $S \in S_{ST}$	A concept $\xi(S)$
A spatio-temporal role $R \in R_{ST}$	A role $\xi(R)$
A spatio-temporal domain $D \in D_{ST}$	A concept $\xi(D)$
A spatio-temporal object $O \in O_{ST}$	An individual $\xi(O)$
A spatio-temporal class $C \in C_{ST}$ with $cfa(C) = [A_1:D_1,$	$\xi(C) \sqsubseteq \forall \xi(A_1). \ \xi(D_1) \sqcap \le 1 \ \xi(A_1) \sqcap \dots \sqcap \forall \xi(A_k). \ \xi(D_k) \sqcap \le 1 \ \xi(A_k)$
$\dots, A_k: D_k]$, where $A_i \in A_{ST}, D_i \in D_{ST}$	$\xi(S) \sqsubseteq \exists \xi(R_1). \ \xi(C_1) \sqcap \le l \ \xi(R_1) \sqcap \ldots \sqcap \exists \xi(R_n). \ \xi(C_n) \sqcap \le l \ \xi(R_n)$
A spatio-temporal association $S \in S_{ST}$ with $sfc(S) =$	$\xi(C_1) \sqsubseteq \ge m_1 \xi(R_1)^- \xi(S) \sqcap \le n_1 \xi(R_1)^- \xi(S)$
$[R_1:C_1(m_1, n_1),, R_n:C_n(m_n, n_n)]$, where $R_i \in R_{ST}, C_i \in$	$\xi(C_n) \sqsubseteq \ge m_n \ \xi(R_n)^{-}. \ \xi(S) \sqcap \le n_n \ \xi(R_n)^{-}. \ \xi(S)$
$C_{\text{ST}}, i \in \{1n\}$	
A matrix terms of C $\rightarrow C$ with the	Generating a role $\xi(stagg)$
A spatio-temporal aggregation $w_{fp}(C_{whole}) = C_{part}$, where	$\top \sqsubseteq \forall \xi(stagg). \ \xi(C_{part}) \sqcap \forall \xi(stagg) . \ \xi(C_{whole})$
$C_{\text{whole}}, C_{\text{part}} \in C_{\text{ST}}$	$\xi(C_{\text{sub}}) \sqsubseteq \xi(C_{\text{super}})$
A spatio-temporal generalization $sfs(C_{sub}) = C_{super}$,	
where C_{sub} , $C_{\text{super}} \in C_{\text{ST}}$	
n object $Q \in Q_{\text{st}}$ such that $ofc(Q) = C$	$\zeta(C)(\zeta(D))$
	$\xi(DC) \equiv \neg \xi(ii\text{-}ints) \sqcap \neg \xi(ib\text{-}ints) \sqcap \neg \xi(bi\text{-}ints) \sqcap \neg \xi(bb\text{-}ints)$
A spatio-temporal link with $sfc(DC) = [R_1:O_1, R_2:O_2]$	$\xi(DC)(\xi(O_1)\xi(O_2))$
A spatio-temporal link with $sfc(EC) = [R_1:O_1, R_2:O_2]$	$\xi(EC) \equiv \neg \xi(ii\text{-}ints) \sqcap \neg \xi(ib\text{-}ints) \sqcap \neg \xi(bi\text{-}ints) \sqcap \xi(bb\text{-}ints)$

Table 1. Mapping rules from a spatio-temporal UML model to an ALCIQ KB.

A spatio-temporal link with $sfc(PO) = [R_1:O_1, R_2:O_2]$	$\xi(EC)(\xi(O_1)\xi(O_2))$		
A spatia temporal link with $cfc(TPD) = [P \cdot O P \cdot O]$	$\xi(PO) \equiv \xi(ii\text{-ints}) \sqcap \xi(ib\text{-ints}) \sqcap \xi(bi\text{-ints}) \sqcap \xi(bb\text{-ints})$		
A spano-temporal link with $Sc(IFF) = [K_1, O_1, K_2, O_2]$	$\zeta(FO)(\zeta(O_1)\zeta(O_2))$ $\exists TPP = \exists ii_inte) \Box = \exists ib_inte) \Box \exists bb_inte)$		
A spatio-temporal link with $sfc(TPPi) = [R_1:O_1, R_2:O_2]$	$\xi(TPP)(\xi(O_1)\xi(O_2))$		
A spatio-temporal link with $sfc(NIPP) = [R_1:O_1, R_2:O_2]$	$\mathcal{H}TPPi$) = $\mathcal{H}ii_{i}ints$) $\Box \mathcal{H}ih_{i}ints$) $\Box = \mathcal{H}hi_{i}ints$) $\Box \mathcal{H}hh_{i}ints$)		
A spatio-temporal link with $sfc(NTPPi) = [R_1:O_1, R_2:O_2]$ A spatio-temporal link with $sfc(FQ) = [R_2:Q_2, R_2:Q_2]$	$\begin{aligned} \zeta(TPPi)(\zeta(0_1)\zeta(0_2)) \\ \zeta(NTPP) &= \zeta(ii-ints) \sqcap \neg \zeta(ib-ints) \sqcap \zeta(bi-ints) \sqcap \neg \zeta(bb-ints) \\ \zeta(NTPP) &= \zeta(ii-ints) \sqcap \neg \zeta(ib-ints) \sqcap \zeta(bi-ints) \sqcap \neg \zeta(bb-ints) \\ \zeta(NTPP)(\zeta(0_1)\zeta(0_2)) \end{aligned}$		
A spatie temperation with $sfe(L_2) = [R_1, O_1, R_2, O_2]$	$\xi(NTPPi) \equiv \xi(ii-ints) \sqcap \xi(ib-ints) \sqcap \neg \xi(bi-ints) \sqcap \neg \xi(bb-ints)$ $\xi(NTPPi)(\xi(O_1)\xi(O_2))$		
A spatio-temporal link with $sfc(before) = [K_1:O_1, K_2:O_2]$	$\xi(EQ) \equiv \xi(iii-ints) \sqcap \neg \xi(ib-ints) \sqcap \neg \xi(bi-ints) \sqcap \xi(bb-ints)$ $\xi(EQ)(\xi(O_1)\xi(O_2))$		
A spatio-temporal link with $sfc(after) = [R_1:O_1, R_2:O_2]$	$ \begin{array}{l} \xi(before) \equiv \xi(ss-before) \sqcap \xi(se-before) \sqcap \xi(es-before) \sqcap \\ \xi(ee-before) \sqcap \neg \xi(ss-after) \sqcap \neg \xi(se-after) \sqcap \neg \xi(es-after) \sqcap \\ \neg \xi(ee-after) \sqcap \neg \xi(ss-meet) \sqcap \neg \xi(se-meet) \sqcap \neg \xi(es-meet) \sqcap \\ \neg \xi(ee-meet); \ \xi(before)(\xi(O_1)\xi(O_2)) \end{array} $		
A spatio-temporal link with $sfc(equal) = [R_1:O_1, R_2:O_2]$	$\begin{aligned} \zeta(after) &\equiv \neg \zeta(ss\text{-}before) \sqcap \neg \zeta(se\text{-}before) \sqcap \neg \zeta(es\text{-}before) \sqcap \\ \neg \zeta(ee\text{-}before) \sqcap \zeta(ss\text{-}after) \sqcap \zeta(se\text{-}after) \sqcap \zeta(es\text{-}after) \sqcap \zeta(ee\text{-}after) \\ \sqcap \neg \zeta(ss\text{-}meet) \sqcap \neg \zeta(se\text{-}meet) \sqcap \neg \zeta(es\text{-}meet) \sqcap \neg \zeta(ee\text{-}meet); \zeta(after) \\ ter)(\zeta(O_1)\zeta(O_2)) \end{aligned}$		
A spatio-temporal link with $sfc(meets) = [R_1:O_1, R_2:O_2]$	$\begin{aligned} \xi(equal) &\equiv \neg \xi(ss-before) \sqcap \neg \xi(se-before) \sqcap \neg \xi(es-before) \sqcap \\ \neg \xi(ee-before) \sqcap \neg \xi(ss-after) \sqcap \neg \xi(se-after) \sqcap \neg \xi(es-after) \sqcap \\ \neg \xi(ee-after) \sqcap \xi(ss-meet) \sqcap \neg \xi(se-meet) \sqcap \neg \xi(es-meet) \sqcap \\ \xi(ee-meet); \xi(equal)(\xi(O_1)\xi(O_2)) \end{aligned}$		
A spatio-temporal link with $sfc(metBy) = [R_1:O_1, R_2:O_2]$	$ \begin{aligned} &\xi(meets) \equiv \xi(ss\text{-}before) \sqcap \xi(se\text{-}before) \sqcap \neg \xi(es\text{-}before) \sqcap \\ &\xi(ee\text{-}before) \sqcap \neg \xi(ss\text{-}after) \sqcap \neg \xi(se\text{-}after) \sqcap \neg \xi(es\text{-}after) \sqcap \\ &\neg \xi(ee\text{-}after) \sqcap \neg \xi(ss\text{-}meet) \sqcap \neg \xi(se\text{-}meet) \sqcap \xi(es\text{-}meet) \sqcap \\ &\neg \xi(ee\text{-}meet); \xi(meets)(\xi(O_1)\xi(O_2)) \end{aligned} $		
$R_2:O_2]$	$\begin{aligned} &\xi(metBy) \equiv \neg \xi(ss\text{-}before) \sqcap \neg \xi(se\text{-}before) \sqcap \neg \xi(es\text{-}before) \sqcap \\ \neg \xi(ee\text{-}before) \sqcap \xi(ss\text{-}after) \sqcap \neg \xi(se\text{-}after) \sqcap \\ \neg \xi(ee\text{-}after) \sqcap \neg \xi(ss\text{-}meet) \sqcap \neg \xi(se\text{-}meet) \sqcap \\ \neg \xi(ee\text{-}meet); \xi(metBy)(\xi(O_1)\xi(O_2)) \end{aligned}$		
A spatio-temporal link with $sfc(overlappedBy) = [R_1:O_1, R_2:O_2]$	$\begin{aligned} \xi(overlaps) &\equiv \xi(ss\text{-}before) \sqcap \ \xi(se\text{-}before) \sqcap \neg \xi(es\text{-}before) \sqcap \\ \xi(ee\text{-}before) \sqcap \neg \xi(ss\text{-}after) \sqcap \neg \xi(se\text{-}after) \sqcap \\ \neg \xi(ee\text{-}after) \sqcap \neg \xi(ss\text{-}meet) \sqcap \neg \xi(se\text{-}meet) \sqcap \neg \xi(es\text{-}meet) \sqcap \\ \neg \xi(ee\text{-}meet); \ \xi(overlaps)(\xi(O_1)\xi(O_2)) \end{aligned}$		
A spatio-temporal link with $sfc(during) = [R_1:O_1, R_2:O_2]$	$\begin{aligned} &\xi(overlappedBy) \equiv \neg \xi(ss-before) \sqcap \xi(se-before) \sqcap \neg \xi(es-before) \\ &\sqcap \neg \xi(ee-before) \sqcap \xi(ss-after) \sqcap \neg \xi(se-after) \sqcap \xi(es-after) \sqcap \\ &\xi(ee-after) \sqcap \neg \xi(ss-meet) \sqcap \neg \xi(se-meet) \sqcap \neg \xi(es-meet) \sqcap \\ &\neg \xi(ee-meet); \xi(overlappedBy)(\xi(O_1)\xi(O_2)) \end{aligned}$		
A spatio-temporal link with $sfc(includes) = [R_1:O_1, R_2:O_2]$	$\begin{aligned} \zeta(during) &\equiv \neg \zeta(ss\text{-before}) \sqcap \zeta(se\text{-before}) \sqcap \zeta(es\text{-before}) \sqcap \\ \neg \zeta(ee\text{-before}) \sqcap \zeta(ss\text{-after}) \sqcap \neg \zeta(se\text{-after}) \sqcap \\ \neg \zeta(ee\text{-after}) \sqcap \neg \zeta(ss\text{-meet}) \sqcap \neg \zeta(se\text{-meet}) \sqcap \neg \zeta(es\text{-meet}) \sqcap \\ \neg \zeta(ee\text{-meet}); \zeta(during)(\zeta(O_1)\zeta(O_2)) \end{aligned}$		
A spatio-temporal link with $sfc(starts) = [R_1:O_1, R_2:O_2]$	$ \begin{array}{l} \xi(includes) \equiv \xi(ss\mbox{-}before) \sqcap \ \xi(se\mbox{-}before) \sqcap \neg \xi(es\mbox{-}before) \sqcap \\ \neg \xi(ee\mbox{-}before) \sqcap \neg \xi(ss\mbox{-}after) \sqcap \neg \xi(se\mbox{-}after) \sqcap \\ \xi(ee\mbox{-}after) \sqcap \neg \xi(ss\mbox{-}meet) \sqcap \neg \xi(es\mbox{-}meet) \sqcap \\ \end{array} $		
A spatio-temporal link with $sfc(startedBy) = [R_1:O_1,$	$\neg \xi(ee\text{-meet}); \ \xi(\text{includes})(\xi(O_1)\xi(O_2))$ $\xi(\text{starts}) \equiv \neg \xi(\text{ss-before}) \sqcap \xi(\text{se-before}) \sqcap \neg \xi(es\text{-before}) \sqcap$		

$R_2:O_2$]	ξ (ee-before) $\sqcap \neg \zeta$ (ss-after) $\sqcap \neg \zeta$ (se-after) $\sqcap \zeta$ (es-after) $\sqcap \neg \zeta$ (ee-after) $\sqcap \zeta$ (ss-meet) $\sqcap \neg \zeta$ (ee-meet); ζ (starts)(ζ (O_1) ζ (O_2))
A spatio-temporal link with $sfc(finishes) = [R_1:O_1, R_2:O_2]$	$\begin{aligned} &\zeta(startedBy) \equiv \neg \zeta(ss\text{-}before) \sqcap \neg \zeta(se\text{-}before) \sqcap \neg \zeta(es\text{-}before) \sqcap \\ \neg \zeta(ee\text{-}before) \sqcap \neg \zeta(ss\text{-}after) \sqcap \neg \zeta(se\text{-}after) \sqcap \neg \zeta(es\text{-}after) \sqcap \neg \zeta(ee\text{-}after) \sqcap \neg \zeta(ss\text{-}meet) \sqcap \neg \zeta(se\text{-}meet) \sqcap \neg \zeta(ee\text{-}meet); \zeta(startedBy)(\zeta(O_1)\zeta(O_2)) \end{aligned}$
A spatio-temporal link with $sfc(finishedBy) = [R_1:O_1, R_2:O_2]$	$\begin{aligned} &\zeta(finishes) \equiv \neg \zeta(ss-before) \sqcap \zeta(se-before) \sqcap \neg \zeta(es-before) \sqcap \\ \neg \zeta(ee-before) \sqcap \zeta(ss-after) \sqcap \neg \zeta(se-after) \sqcap \zeta(es-after) \sqcap \\ \neg \zeta(ee-after) \sqcap \zeta(ss-meet) \sqcap \neg \zeta(se-meet) \sqcap \neg \zeta(es-meet) \sqcap \\ \zeta(ee-meet); \zeta(finishes)(\zeta(O_1)\zeta(O_2)) \end{aligned}$
	$\begin{aligned} &\xi(finishedBy) \equiv \neg \xi(ss-before) \sqcap \neg \xi(se-before) \sqcap \neg \xi(es-before) \sqcap \\ \neg \xi(ee-before) \sqcap \neg \xi(ss-after) \sqcap \neg \xi(se-after) \sqcap \\ \neg \xi(ee-after) \sqcap \neg \xi(ss-meet) \sqcap \neg \xi(se-meet) \sqcap \neg \xi(es-meet) \sqcap \\ \xi(ee-meet); \ \xi(finishedBy)(\xi(O_1)\xi(O_2)) \end{aligned}$

As stated in Section 3.2, the semantics of a spatio-temporal UML model is describeed by the spatio-temporal object states (*STOS*). The semantics of a DL *ALCIQ* KB is defined by the interpretations \mathcal{I} . Following the idea of [6, 8], the correctness of mapping can be ensured by creating the correspondences between *STOS* and \mathcal{I} as shown in Theorem 1.

Theorem 1: With a spatio-temporal UML model $ST_{UML} = (cfa, sfc, wfp, sfs, ofc)$, STOS is its spatio-temporal object state, $\xi(ST_{UML})$ is its mapped ALCIQ KB, and \mathcal{I} is an interpretation of $\xi(ST_{UML})$. Then there are mappings: (i) σ : $STOS \rightarrow \mathcal{I}$, such that $\sigma(STOS)$ is an interpretation of $\xi(ST_{UML})$, and (ii) $\rho: \mathcal{I} \rightarrow STOS$, such that $\rho(\mathcal{I})$ is spatio-temporal object state of ST_{UML} .

Proof: Given a spatio-temporal object state STOS of the spatio-temporal UML model ST_{UML} , an interpretation \mathcal{I} of the ALCIQ KB $\xi(ST_{\text{UML}})$ is defined in the following:

- Interpretation domain $\Delta^{\mathcal{I}} = \Delta^{\sigma(STOS)} = \Delta^{STOS} \cup \bigcup_{S \in S} S^{STOS}$.
- The atomic concepts and atomic roles in the *ALCIQ* KB $\xi(ST_{UML})$ are consist of a set of labels $L = C_{ST} \cup A_{ST} \cup S_{ST} \cup R_{ST} \cup D_{ST}$ in the spatio-temporal UML model ST_{UML} , and $(\xi(X))^{\mathcal{I}} = (\xi(X))^{\sigma(STOS)} = \{X^{STOS} \mid X \in L\}$.
- For association $S \in S_{\text{ST}}$, $sfc(S) = [R_1:C_1, ..., R_n:C_n], n \ge 2, (\xi(R_i))^{\mathcal{I}} = (\xi(R_i))^{\sigma(\text{STOS})} = \{<s, O_i \ge \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}} \mid s \in S^{\text{STOS}} \land O_i \in C_i^{\text{STOS}} \land s[R_i] = O_i\}, i = 1...n.$

The following proves that $\mathcal{I} = \sigma(STOS)$ is an interpretation of $\xi(ST_{\text{UML}})$.

(i) Assuming that C ∈ C_{ST} with cfa(C) = [A₁:D₁, ..., A_K:D_K], and considering an instance O ∈ (ζ(C))^T. Firstly, according to the interpretation I above, it follows O ∈ C^{STOS}, and further according to Definition 2, there is at least one element a ∈ A^{STOS} = (ζ(A))^T for any C, where the first element is O, and the second element is d ∈ D^{STOS} = (ζ(A))^T, *i.e.*, a = <O, d> ∈ A^{STOS}. Similarly, there may be a unique element a_i = (ζ(A_i))^T ∈ A_i^{STOS}, the first part of which is O and the second part of which is d_i ∈ D_i^{STOS}, *i.e.*, a_i = <O, d≥ ∈ A_i^{STOS}. That is to say, I satisfies the axiom transformed by the constraint cfa.

- (ii) Assuming that S ∈ S_{ST} is an association satisfies sfc(S) = [R₁:C₁, ..., R_n:C_n], n ≥ 2, and considering an instance s ∈ (ξ(S))^T. Firstly, according to the definition of spatiotemporal object state, s is a labeled tuple like [R₁:O₁, ..., R_n:O_n], where O_i ∈ C_i^{STOS}, i ∈ {1, ..., n}, that is to say s is a function defined on {R₁, ..., R_n}, and we can found O_i ∈ (ζ(C_i))^T by the definition of the interpretation I above. Moreover, if there is a cardinality constraint (m_i, n_i) defined on S ∈ S_{ST}, by Definition 2, m_i ≤ #{s ∈ S^{STOS} | s[R_i]= O]} ≤ n_i. By the definition of the interpretation I above, (ζ(C_i))^T ⊆ {O_i | m_i ≤ #{s ∈ (ζ(S))^T | <s, O_i> ∈ (ζ(R_i))^T} ≤ n_i}, *i.e.*, I satisfies the axiom transformed by the constraint sfc.
- (iii) Assuming that there are two saptio-temporal objects $O_1 \in C_{ST}$ and $O_2 \in C_{ST}$. If there is one of the *Allen*-13 temporal relations between O_1 and O_2 , for example, there is a relation *before* between O_1 and O_2 . Then, according to the definition of the relation *before*, the relation of O_1 and O_2 can be defined by 12 basic temporal relations. As shown in Table 1, the relation *before* is represented by an axiom in $\xi(ST_{UML})$, which is an axiom constituted by the operators of 12 basic temporal relations. Therefore, the interpretation \mathcal{I} satisfies the axiom transformed by the temporal constraint. Similarly, the proof can also be extended to the axioms transformed by the *RCC*-8 spatial relations in ST_{UML} .
- (iv) Assuming that $sfs(C_{sub}) = C_{super}$ is a generalization relationship. Firstly, according to the definition of the spatio-temporal object state, there is $C_{sub}^{STOS} \subseteq C_{super}^{STOS}$. Then, we have $C_{sub}^{STOS} = (\xi(C_{sub}))^{\sigma(STOS)} = (\xi(C_{sub}))^{\mathcal{I}}$ and $C_{super}^{STOS} = (\xi(C_{super}))^{\sigma(STOS)} = (\xi(C_{super}))^{\mathcal{I}}$. Therefore, there is $(\xi(C_{sub}))^{\mathcal{I}} \subseteq (\xi(C_{super}))^{\mathcal{I}}$, *i.e.*, the interpretation \mathcal{I} satisfies the axiom transformed by the constraint *sfs*.

The two parts of the proof process above are a mutually inverse process, and thus the proof of the part two can be given analogously. According to the proof process above, it can be shown that the method proposed in Table 1 can map spatio-temporal UML models to *ALCIQ* KBs.

4.3 Mapping Example

In the following we provide a mapping example from the spatio-temporal UML model ST_{UML} in Section 3.3 to the DL *ALCIQ* KB $\xi(ST_{\text{UML}})$ as shown in Fig. 10.

 ξ (University) $\sqsubseteq \forall \xi$ (tem-foundingYear). ξ (date) $\sqcap \leq 1 \xi$ (tem-foundingYear);

 $\xi(st-ass_1) \sqsubseteq \exists \xi(ind_1). \ \xi(Industry) \sqcap \le 1 \xi(ind_1) \sqcap \exists \xi(par_1). \ \xi(Park) \sqcap \le 1 \xi(par_1);$

 ξ (Industry) $\equiv = 1 \xi$ (ind₁)⁻. ξ (st-ass₁); ξ (Park) $\equiv = 1 \xi$ (par₁)⁻. ξ (st-ass₁); ξ (Resident)(ξ (residentB)); ξ (Industry)(ξ (industryA)); ξ (Park)(ξ (parkC)); ξ (Commerce)(ξ (commerceD)); ξ (University) (ξ (universityE));

 $\xi(DC) \equiv \neg \xi(ii\text{-}ints) \sqcap \neg \xi(ib\text{-}ints) \sqcap \neg \xi(bb\text{-}ints) \sqcap \neg \xi(bb\text{-}ints);$

 $\xi(DC)(\xi(industryA) \xi(residentB)); \xi(PO)(\xi(universityE)\xi(commerceD));$

 $[\]xi(ST_{\text{UML}}) = \{\xi(Resident) \sqsubseteq \xi(Parcel); \ \xi(Industry) \sqsubseteq \xi(Parcel); \ \xi(Parcel)$

 $[\]begin{array}{l} \xi(Parcel) \sqsubseteq \forall \xi(spa-area). \ \xi(float) \sqcap \leq 1 \ \xi(spa-area) \sqcap \forall \ \xi(ownership). \ \xi(string) \sqcap \\ \leq 1 \ \xi(ownership) \sqcap \forall \ \xi(landCategory). \ \xi(string) \sqcap \leq 1 \ \xi(landCategory) \sqcap \forall \ \xi(tem-validBeginTime). \\ \xi(date) \sqcap \leq 1 \ \xi(tem-validBeginTime) \sqcap \forall \ \xi(tem-validEndTime). \ \xi(date) \sqcap \leq 1 \ \xi(tem-validEndTime); \\ \xi(Resident) \sqsubseteq \forall \ \xi(residenceType). \ \xi(string) \sqcap \leq 1 \ \xi(residenceType) \sqcap \forall \ \xi(tem-serviceLife). \ \xi(date) \\ \sqcap \leq 1 \ \xi(tem-serviceLife); \end{array}$

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\begin{aligned} \xi(PO) &= \xi(ii\text{-}ints) \sqcap \xi(ib\text{-}ints) \sqcap \xi(bi\text{-}ints) \sqcap \xi(bb\text{-}ints); \ \xi(meets) &= \xi(ss\text{-}before) \sqcap \ \xi(se\text{-}before) \sqcap \ \neg \xi(ss\text{-}after) \sqcap \ \neg \xi(se\text{-}after) \dashv \ \neg \xi(s
```

Fig. 10. The mapped ALCIQ KB E(STUML) from the spatio-temporal UML model STUML in Section 3.3.

After mapping the spatio-temporal UML model to the *ALCIQ* KB, it is possible to verify the correctness of the model by means of the inference ability in Section 5.

5. VERIFICATION OF SPATIO-TEMPORAL UML MODELS BASED ON DL ALCIQ

In this section we employ the inference ability of the DL *ALCIQ* to assist in solving some verification tasks of the spatio-temporal UML models. First, we introduce several typical verification tasks of the spatio-temporal UML models. Then we further convert these verification tasks into the inference problems of the mapped *ALCIQ* KBs. That is, the verification of spatio-temporal UML models can be handled by DL inference abilities.

Being similar to the ER and UML models [5, 6, 8], several typical verification tasks of the spatio-temporal UML models mainly include satisfiability, subsumption, and qualitative spatio-temporal query problems:

- Satisfiability: it is to determine if a spatio-temporal UML model ST_{UML} is satisfiable. That is, whether there exists at least one object state *STOS* of ST_{UML} .
- Subsumption: it is to determine if one spatio-temporal class is the subclass of another spatio-temporal class in ST_{UML} . That is, with any two spatio-temporal classes C_{sub} , $C_{\text{super}} \in C_{\text{ST}}$ in Definition 2, whether $sfs(C_{\text{sub}}) = C_{\text{super}}$.
- Qualitative spatio-temporal query problems: In spatio-temporal applications, verification tasks that are more complex than the above ones are usually required, including qualitative spatio-temporal query problems, *e.g.*, retrieving which area has an inscribed spatial relation with industrial area in Section 3.3.

Theorem 2: With any spatio-temporal UML model ST_{UML} , $\xi(ST_{\text{UML}})$ is the mapped AL-CIQ KB in Section 4, then ST_{UML} is satisfiable iff $\xi(ST_{\text{UML}})$ is satisfiable.

Proof: " \Rightarrow ": If a spatio-temporal class *C* is satisfiable, then there is a spatio-temporal object state *STOS* of *ST*_{UML} such that $C^{STOS} \neq \emptyset$. From the Theorem 1, $\sigma(STOS)$ is an interpretation of $\xi(ST_{UML})$ so that $C^{STOS} = (\xi(C))^{\sigma(STOS)}$. Therefore, it can be inferred that $(\xi(C))^{\sigma(STOS)} \neq \emptyset$, *i.e.*, $\xi(C)$ is satisfiable. That is to say, if all classes of *ST*_{UML} are satisfiable (*i.e.*, *ST*_{UML} is satisfiable), then all of $\xi(ST_{UML})$ are satisfiable (*i.e.*, $\xi(ST_{UML})$) is satisfiable).

" \Leftarrow ": If a concept $\xi(C)$ in $\xi(ST_{\text{UML}})$ is satisfiable, *i.e.*, there is an interpretation \mathcal{I} of $\xi(ST_{\text{UML}})$ such that $(\xi(C))^{\mathcal{I}} \neq \emptyset$. From the Theorem 1, $\rho(\mathcal{I})$ is a spatio-temporal object state of ST_{UML} such that $(\xi(C))^{\mathcal{I}} = C^{\rho(\mathcal{I})}$. Therefore, it can be inferred that $C^{\rho(\mathcal{I})} \neq \emptyset$, *i.e.*, C is satisfiable. That is to say, if all of $\xi(C)$ in $\xi(ST_{\text{UML}})$ are satisfiable (*i.e.*, $\xi(ST_{\text{UML}})$ is

satisfiable), then all classes of ST_{UML} are satisfiable (*i.e.*, ST_{UML} is satisfiable).

Theorem 3: With any spatio-temporal UML model ST_{UML} in Section 3, C_{sub} , $C_{\text{super}} \in C_{\text{ST}}$ are two spatio-temporal classes in ST_{UML} , $\xi(ST_{\text{UML}})$ is the mapped ALCIQ KB in Section 4, and $\xi(C_{\text{sub}})$ and $\xi(C_{\text{super}})$ are two concepts in $\xi(ST_{\text{UML}})$. Then C_{sub} is the subclass of C_{super} in ST_{UML} iff $\xi(ST_{\text{UML}}) \models \xi(C_{\text{super}})$.

Proof: " \Rightarrow ": Assuming $\xi(ST_{UML}) \nvDash \xi(C_{sub}) \sqsubseteq \xi(C_{super})$, then there exists an interpretation \mathcal{I} of $\xi(ST_{UML})$ so that $(\xi(C_{sub}) \sqcap \neg \xi(C_{super}))^{\mathcal{I}} \neq \emptyset$, *i.e.*, $\exists O. \ O \in (\xi(C_{sub}))^{\mathcal{I}}$ and $O \notin (\xi(C_{super}))^{\mathcal{I}}$. From Theorem 1 and the mapping rules in Section 4.2, $\rho(\mathcal{I})$ is a spatio-temporal object state of ST_{UML} so that $(\xi(C_{sub}))^{\mathcal{I}} = C_{sub}\rho^{(\mathcal{I})}$ and $(\xi(C_{super}))^{\mathcal{I}} = C_{super}\rho^{(\mathcal{I})}$. By combining these conditions, there exists O so that $O \in C_{sub}\rho^{(\mathcal{I})}$ and $O \notin C_{super}\rho^{(\mathcal{I})}$, *i.e.*, $\xi(C_{sub}) \not\subseteq \xi(C_{super})$. According to anti-evidence method, $\xi(ST_{UML}) \vDash \xi(C_{sub}) \sqsubseteq \xi(C_{super})$.

"⇐": Assuming $sfs(C_{sub}) \neq C_{super}$, then there exists a spatio-temporal object state STOS of ST_{UML} so that $O \in C_{sub}^{STOS}$ and $O \notin C_{super}^{STOS}$. From the Theorem 1 and the mapping rules in Section 4.2, $\sigma(STOS)$ is an interpretation of $\xi(ST_{UML})$ so that $C_{sub}^{STOS} = (\xi(C_{sub}))^{\sigma(STOS)}$ and $C_{super}^{STOS} = (\xi(C_{super}))^{\sigma(STOS)}$. By combining these conditions, $O \in (\xi(C_{sub}))^{\sigma(STOS)}$ and $O \notin (\xi(C_{super}))^{\sigma(STOS)}$, *i.e.*, $\xi(ST_{UML}) \nvDash \xi(C_{sub}) \sqsubseteq \xi(C_{super})$. According to the anti-evidence method, $sfs(C_{sub}) = C_{super}$.

Theorem 4: With any spatio-temporal UML model ST_{UML} , $\xi(ST_{UML})$ is the ALCIQ KB in Section 4. Then a qualitative spatio-temporal query problem in ST_{UML} as mentioned above can be equivalently reduced to the individual retrieval problem in $\xi(ST_{UML})$.

Proof: Given a spatio-temporal query problem Q in ST_{UML} , for simplification, if Q = DC(O_1 , ?y), denoting that which object has a DC spatial relation with the object O_1 . From the Theorem 1 and the mapping rules in Section 4.2, the spatial relation DC can be represented by a concept $\neg \xi(ii\text{-ints}) \sqcap \neg \xi(ib\text{-ints}) \sqcap \neg \xi(bb\text{-ints}) \sqcap \neg \xi(bb\text{-ints})$ in the mapped ALCIQ KB $\xi(ST_{UML})$, and also the query Q = DC (O_1 , ?y) can be represented by the assertion $\xi(DC)(\xi(O_1)$?y) in $\xi(ST_{UML})$. That is to say, the query problem Q in ST_{UML} can be equivalently reduced to the individual retrieval problem in $\xi(ST_{UML})$.

Until now, based on our mapping and verification methods, the verification of the spatio-temporal UML models can be handled by employing the inference ability of the DL *ALCIQ* as shown in Fig. 11. In general, when given the spatio-temporal UML model in Figs. 7 and 8, the graphical model can be formalized and interpreted according to the abstract definition and semantic description of the model proposed in Section 3. Then, the formal spatio-temporal UML model can be mapped into an *ALCIQ* knowledge base by our approach in Section 4. Subsequently, on the basis of the mapped *ALCIQ* knowledge base, the verification tasks of the spatio-temporal UML model can be further converted to the inference problems of *ALCIQ* by our approach in Section 5. Final, the inference of *ALCIQ* can be done by means of the existing reasoners (*e.g.*, FaCT++ [29] and Pellet [25]), and the reasoners can return the results. For example, retrieving which area has an inscribed spatial relation with the industrial area as mentioned above can be converted to find all individuals *O* such that $\zeta(ST_{UML}) \models \exists \zeta(UTB)$. ($\zeta(TPPi) \sqcap (\exists \zeta(BTU)$. (*Industry*))), and the reasoners return the results $\zeta(parkC)$ and $\zeta(commerceD)$.



Fig. 11. The overall description of the application of our proposed approach on the case study.

6. RELATED WORK

In this section we make a detailed comparison between our work and the existing work. There are three main categories of approaches related to our work, *i.e.*, *the spatio-temporal data modeling methods*, *the spatio-temporal description logics (DLs)*, and *the representation and verification of spatio-temporal data models based on DLs*.

Firstly, regarding to the methods of spatio-temporal data modeling, there were some spatio-temporal data models, including spatio-temporal snapshot model [15], event-bas ed model [18], and three-domain model [35]. Also, the traditional database models were extended to model the spatio-temporal data, such as the spatio-temporal ER (Entity-Relationship) model [27] and the spatio-temporal object-oriented data model [21]. Please refer to [19] for more details. In particular, several spatio-temporal UML models were accordingly developed [16, 20, 24]. In [16], the authors proposed an extended spatiotemporal UML state chart for cyber-physical systems. The state chart was based on the UML Profile for modeling and analysis of real-time and embedded systems. In the state charts, they unified the logical time and the chronometric time variables, and extended the traditional events to the cyber-physical system events. In [20], the authors presented a spatio-temporal modeling approach based on UML that uses graphical notations to replace the stereotypes in UML, added five new graphical labels to extend the spatio-temporal UML, and also used a specification box to describe the semantics of the spatio-temporal data represented by the five kinds of graphical labels. In [24], the authors developed a spatio-temporal UML model, using the UML extension mechanism to define some stereotypes such as spatio-temporal classes, attributes and relations, and then they used these stereotypes in the class diagram to represent the housing market system. In this paper we further consider the complex semantic relationships (*e.g.*, spatio-temporal associations with multiplicity and aggregations). Also, we represent and deal with some complex semantic relationships among spatio-temporal classes/objects (*e.g.*, RCC-8 spatial topological relations [23] and Allen-13 temporal topological relations [1]). Moreover, we give abstract definition and semantic description of spatio-temporal UML models.

Secondly, for reasoning on spatio-temporal knowledge, several spatio-temporal DLs were proposed, such as the temporal DLs [17], the spatial DLs [9, 32], and the spatio-temporal DL [28] (please refer to the survey [9, 17]). In our work, for representing the spatio-temporal UML models and ensuring the decidability of the DL, we add several syntactic sugar extensions to the existing DL *ALCIQ* [5] because it has enough expressive power and several existing practical reasoners (*e.g.*, FaCT++ [29] and Pellet [25]).

Finally, regarding to the representation and verification of spatio-temporal data models based on DLs, as far as we know, less research work has been done. In [3], the temporal DL was used to verify and infer the *temporal EER* (Extended Entity-Relation-ship) conceptual model. But the *spatio-temporal UML model in our paper is completely different from the temporal EER model in* [3]. The temporal EER model cannot represent most of the spatio-temporal UML features, including spatio-temporal classes, spatio-temporal associations, several complex spatio-temporal semantic relationships (*e.g.*, generalization and aggregation), and spatio-temporal objects and their relations (including RCC-8 spatial topological relations [23] and Allen-13 temporal UML model to DL and also cannot verify and infer the spatio-temporal UML model with the DL.

All of these works give beneficial inspirations for our study in this paper, but as mentioned and compared above, the existing work cannot represent and reason on the spatio-temporal UML model with the DL. To this end, in this paper we proposed a spatio-temporal UML model and a DL method for verifying spatio-temporal UML models.

7. CONCLUSIONS AND FUTURE WORK

We proposed a spatio-temporal UML model and a Description Logic (DL) method or verifying the spatio-temporal UML models. First of all, a UML-based spatio-temporal data model was proposed. The abstract definition and semantic description of the spatio-temporal UML models were given, and a case of cadastral change system was modeled. Then, by adding some special concepts, roles, and axioms into the DL *ALCIQ*, a method for mapping the spatio-temporal UML models to the DL *ALCIQ* KBs was developed, and a mapping example was provided. Finally, several typical verification tasks of the spatio-temporal UML models were reduced to the inference problems of the mapped *ALCIQ* KBs, and the inference results can be returned and the verification of the spatio-temporal UML models are realized with the help of the DL inference abilities.

In future works, we aim at implementing a mapping tool and testing the performance of the method and tool with more cases, and considering and investigating more spatio-temporal features based on DLs. Moreover, we will further make some more detailed and deeper comparisons between our work and the existing work.

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