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On Complex Fuzzy Matrix with Algebraic Operations, Similarity Measure and its Application in Identification of Reference Signal

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In the literature of application fields of science and technology, various matrix systems involving Lyapunov, Sylvester, & Stein matrix equations have been investigated and paid attention by different researchers in the last decades. In view of this, various algebraic operations and properties like addition, multiplication, associative, distributive, and commutative have been studied in the case of the complex fuzzy matrix (CFM). In addition to this, a new similarity measure has also been proposed for CFMs along with a detailed explanatory numerical example. The proposed modification and extension significantly contribute in order to model the uncertainty & inexactness a little more accurately. Further, an application in the area of identification of reference signals has been described with the help of the CFM and operations.

Keywords: complex fuzzy matrix, matrix binary operations, uncertainty, similarity measure, reference signal

1. INTRODUCTION

Various areas such as patternistic medical diagnosis [1], environment studies [2], multi-criteria decision-making problems [3], econometrics [4] *etc*. are full of complexities due to the uncertainties and incompleteness in the available information. In view of the available literature, it may be noted that the classical methodologies based on the crisp set [5] is not very efficient in solving such complex problems related to uncertain events. Different paradigm shifts and methodologies based on the variety of sets (Fuzzy Set (FS), Intuitionistic Fuzzy Set (IFS), Neutrosophic Set (NS) so on) have been put forward to deal with inherent fuzziness among the unsolved problems. In the very initial stage of development, Zadeh [6] extended the crisp set theory to fuzzy set which proves to be very efficient in dealing with the uncertainty issues among the various problems. Later in literature, various researchers like Atanassov (intuitionistic fuzzy set [7]), Samarandache (neutrosophic set [8]), Molodtsov (soft set [9]) and many more have contributed several

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novel concepts related to the extensional features of fuzzy set on the real plane in the interval [0,1].

The similarity and entropy measures play a critical role in the study of measuring uncertain information related to the data available for fuzzy sets and their hybrid structures. The necessary axioms for fuzzy entropy were first introduced and explained by De Luca and Termini [10]. On the other hand, the similarity measure is considered an important tool in comparison with the entropy measure due to its ability to calculate the similarity between the sets according to the data present in the literature. Majumdar and Samanta [11] presented the similarity and entropy measure between the two single-valued neutrosophic sets. Later, Pappis *et al*. [12, 13] presented an axiomatic view of similarity measure for a better understanding of the similarity measure concept. Various researchers have published many research articles on similarity and entropy measures and utilized it in many applications in the case of fuzzy soft sets [14], intuitionistic soft sets [15] and interval-valued fuzzy sets [16] so on.

Later in 2002, Ramot *et al*. [17] extended the concept of uncertainties from the interval [0,1] on the real plane to the unit circle on the complex plane. The complex fuzzy set (CFS) added the phase term to the amplitude term of the fuzzy set which helps to calculate the amount and periodicity of the considered event. This ability of the proposed concept encouraged various authors to work on the complex plane.

Further, it has been widely observed that the matrix form of fuzzy set plays a vital role in scientific and technical areas as the classical set matrix is not able to solve various real-life examples. This form was initially presented by Thomson [18], who also explained the convergence of the matrix. Further, the concept of convergence was discussed by many researchers [19] for a better understanding of the concept. This also proves the advantage of matrix form over the set form for the uncertainty parameter where one event can be taken at an interval of time instead of one problem. A better understanding of the range of giving solutions to the problems by respective proposed fuzzy theories is tabulated below.

Sets	Year	Advantage
Fuzzy set [6]	1965	Deals with membership function
Intuitionistic fuzzy set [7]	1983	Added on-membership function
Neutrosophic set [8]	1995	Added indeterminate function
Fuzzy matrix [18]	1994	Added tabular form to fuzzy set to increase clarity
Complex fuzzy set [17]	2008	increased range to unit circle in complex plane
Complex fuzzy matrix [23]	2016	Added tabular format to complex fuzzy set

Due to the extensional developments of various types of fuzzy sets, one of the major applications in the field of the detection of the reference signal has been carried out utilizing complex fuzzy set theories. This concept was first discussed by Ramot [17] with the application of signal detection and further Zhang *et al*. [16] demonstrated the expansivity of the application by taking the case of δ-equalities of complex fuzzy sets. Later, Hu *et al*. [20] discussed the signal detection and orthogonality for complex fuzzy sets in detail. The new algorithm using the Fourier and inverse Fourier transformation was used to detect the exact values of two signals and this concept of identifying reference signals among the large number of signals received by the source is explained by Ma *et al*. [21]. Recently, Khan *et al*. [22] introduced the concept of complex fuzzy soft matrices with some basic operations (Union, intersection, complement) and validated this concept along with its application in the detection of signals.

Later in 2016, Zhao & Ma [23] presented the novel concept of the complex fuzzy matrix (CFM) which was given in the form of $[a_{ij} + ib_{ij}]_{m\times n}$ with discussion on its convergence. In the current work, various algebraic operations for complex fuzzy matrices have been discussed in detail. A new similarity measure for the concept is also described with a numerical example. Further, the applicability of the presented concept has been explained by taking the problem of identification of the signals into consideration so as to have a better understanding of the concept. The motivation and advantages behind presenting the theory of complex fuzzy matrix are given below:

- The concept of complex fuzzy matrix plays a significant role in the complex plane and therefore in the current manuscript the concept is explained in detail for a better understanding of the concept.
- Various new operators for the proposed matrix have been presented for a detailed understanding of the concept.
- A similarity measure for the proposed matrices has also been designed and subsequently its applicability in the field of identification of reference signals has been presented.

All the above-mentioned points sufficiently establish the novelty of the concept in solving various problems which may be related to the field of medicine, signals, and multi-criteria decision-making problems. It may also be noted that the proposed concept sufficiently addresses the benefits of complex fuzzy sets and fuzzy matrices for solving such decision-making problems. This will be of great help to the researchers for further study in its extensional application in various other fields.

The current manuscript has been organized as follows. Section 2 includes the basic preliminaries and fundamental definitions. The description of the complex fuzzy matrix and its theoretic algebraic operations have been given in Section 3. In Section 4, the new similarity measure for the complex fuzzy matrices have been defined along with a numerical example. Further, in Section 5, the problem of identification of the reference signal has been considered for the sake of application of the concept. Finally, the work has been concluded in Section 6 with the outlines of the possible scope for future work.

2. PRELIMINARIES

In this section, we are presenting some of the fundamental and basic definitions which are popularly available in literature and have a preliminary connection with the extended theory of complex fuzzy matrix (CFM).

Definition 1 (Similarity Measure) *[24]"The grade of similarity $S(P,Q)$ of two fuzzy sets P and Q in X, satisfy the following properties:*

- $0 \leq S(P,Q) \leq 1$;
- $S(P,Q) = S(Q,P)$;
- $P = Q \implies S(P,Q) = 1$;
- $min(P,Q) = 0 \implies S(P,Q) = 0$;
- $S(P,P^c) = 0$;
- $S(P,Q) + S(Q,R) \leq S(P,R)$; $P,Q,R \in X$, *where P^c is the complement of P".*

Definition 2 (Complex fuzzy set) *[17] "A complex fuzzy set P, defined on a universe of discourse X, is characterized by a membership function $\mu_P(x)$ that assigns any element $x \in X$ to be a complex-valued grade of membership function in P. As per the definition, it may be noted that the value $\mu_P(x)$ may lie within the unit circle in the complex plane, and are thus of the form $a_p(x)e^{ib_p(x)}$, where $i = \sqrt{-1}$, $a_p(x)$ and $b_p(x)$ are both real-valued, and $a_p(x) \in [0,1]$.The complex fuzzy set P may be represented as the set of ordered pairs $P = \{(x, \mu_P(x)) | x \in X\}$."*

Definition 3 (Complex fuzzy matrix) *[23] "A complex fuzzy matrix, denoted by P, defined by $P = (a_{ij}(x) + ib_{ij}(x))_{m \times n}$; where $(1 \leq i \leq m,\ 1 \leq j \leq n)$."*

3. COMPLEX FUZZY MATRIX WITH ALGEBRAIC OPERATIONS

In this section, we have extended the concept of the complex fuzzy matrix with its examples. In addition to this, various set-theoretic operations viz. addition, multiplication, union, and intersection on the CFM have been described to increase the understanding of the basics.

Definition 4 *A complex fuzzy matrix $C_{m \times n}$, defined on a universe of discourse U, is characterized by a membership function $\mu_C(x_{ij})$ that assigns any element $x_{ij} \in U$. All the values of function $\mu_C(x_{ij})$ will lie in the unit disk of complex plane and will be of the form $r_C(x_{ij})e^{j\omega_C(x_{ij})}$, where $j = \sqrt{-1}$, $r_C(x_{ij})$ & $\omega_C(x_{ij})$ are both real-valued functions subject to $r_C(x_{ij}) \in [0,1]$. Then, CFM $(C_{m \times n})$ can be represented as $C_{m \times n} = \left\{(x_{ij},\ \mu_C(x_{ij}))_{m \times n} | x_{ij} \in U\right\}$.*

Example: Suppose that we have an example of a medical situation in which there is a set of three patients, say, $B = (b_1,\ b_2,\ b_3)$, who are suffering from diseases having similar symptoms. Then, the possibility of a patient suffering from the set of particular diseases $D = (d_1,\ d_2,\ d_3)$, can be represented through the following matrix, *i.e.*,

$$
\begin{array}{c c}
 & \begin{array}{ccc} d_1 & d_2 & d_3 \end{array} \\
\begin{array}{c} b_1 \\ b_2 \\ b_3 \end{array} & \begin{bmatrix} f_{11}e^{ig_{11}} & f_{12}e^{ig_{12}} & f_{13}e^{ig_{13}} \\ f_{21}e^{ig_{21}} & f_{22}e^{ig_{22}} & f_{23}e^{ig_{23}} \\ f_{31}e^{ig_{31}} & f_{32}e^{ig_{32}} & f_{33}e^{ig_{33}} \end{bmatrix}
\end{array}
$$

where $\left(f_{11}e^{ig_{11}},\ f_{12}e^{ig_{12}},\ \ldots,\ f_{33}e^{ig_{33}}\right)$ represents the degree of membership function for the patients in cases of a particular disease.

Theoretic Algebraic Operations on Complex Fuzzy Matrices

Let us consider two complex fuzzy matrices whose entries are of the form $r_C(x_{ij})$ $e^{j\omega_C(x_{ij})}$ and given by

$$C^1_{2\times 2} = \begin{bmatrix} h_{11}e^{i\theta_1} & h_{12}e^{i\theta_2} \\ h_{21}e^{i\theta_3} & h_{22}e^{i\theta_4} \end{bmatrix} \& \ C^2_{2\times 2} = \begin{bmatrix} J_{11}e^{i\alpha_1} & J_{12}e^{i\alpha_2} \\ J_{21}e^{i\alpha_3} & J_{22}e^{i\alpha_4} \end{bmatrix}. \tag{3.1}$$

- **Addition Operation of Two Complex Fuzzy Matrices**

The sum of $C^1_{2\times 2}$ & $C^2_{2\times 2}$ is defined and represented as follows:

$$C^1_{2\times 2} + C^2_{2\times 2} = \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix};$$

where $f_{11} = \max\{h_{11}, J_{11}\}e^{i\,max\{\theta_1,\alpha_1\}}$; $\quad f_{12} = \max\{h_{12}, J_{12}\}e^{i\,max\{\theta_2,\alpha_2\}}$;

$f_{21} = \max\{h_{21}, J_{21}\}e^{i\,max\{\theta_3,\alpha_3\}}$; $\quad f_{22} = \max\{h_{22}, J_{22}\}e^{i\,max\{\theta_4,\alpha_4\}}$.

Example: In view of the particular examples, the sum of the given two matrices is illustrated as follows:

$$C^1_{2\times 2} = \begin{bmatrix} 0.6e^{i0.3} & 0.1e^{i0.7} \\ 0.2e^{i0.1} & 0.5e^{i0.4} \end{bmatrix} \& \ C^2_{2\times 2} = \begin{bmatrix} 0.5e^{i0.1} & 0.4e^{i0.3} \\ 0.8e^{i0.6} & 0.7e^{i0.2} \end{bmatrix}.$$

Then, $C^1_{2\times 2} + C^2_{2\times 2} = \begin{bmatrix} 0.6e^{i0.3} & 0.4e^{i0.7} \\ 0.8e^{i0.6} & 0.7e^{i0.4} \end{bmatrix}$.

Commutativity and Associativity of Addition for CFMS:

Theorem 1 *Suppose that there are three CFMSs, say, P, Q and R, then the addition operation is commutative and associative.*

(i) $P+Q = Q+P$. $\qquad$ *(ii)* $(P+Q)+R = P+(Q+R)$.

Proof: Consider the following three complex fuzzy matrices:

$$P = \begin{bmatrix} a_{11}e^{i\theta_1} & a_{12}e^{i\theta_2} \\ a_{21}e^{i\theta_3} & a_{22}e^{i\theta_4} \end{bmatrix}, Q = \begin{bmatrix} b_{11}e^{i\alpha_1} & b_{12}e^{i\alpha_2} \\ b_{21}e^{i\alpha_3} & b_{22}e^{i\alpha_4} \end{bmatrix} \& \ R = \begin{bmatrix} g_{11}e^{i\gamma_1} & g_{12}e^{i\gamma_2} \\ g_{21}e^{i\gamma_3} & g_{22}e^{i\gamma_4} \end{bmatrix}.$$

Suppose that $P+Q = \begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{bmatrix} = Y$;

where $x_{11} = \max\{a_{11}, b_{11}\}e^{i\,max\{\theta_1,\alpha_1\}}$; $\quad x_{12} = \max\{a_{12}, b_{12}\}e^{i\,max\{\theta_2,\alpha_2\}}$;

$x_{21} = \max\{a_{21}, b_{21}\}e^{i\,max\{\theta_3,\alpha_3\}}$; $\quad x_{22} = \max\{a_{22}, b_{22}\}e^{i\,max\{\theta_4,\alpha_4\}}$.

Similarly, $Q+P = Y$. Hence, $P+Q = Q+P$. Also, in the case of associativity,

$$(P+Q)+R = Y+R = \begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{bmatrix} + \begin{bmatrix} g_{11}e^{i\gamma_1} & g_{12}e^{i\gamma_2} \\ g_{21}e^{i\gamma_3} & g_{22}e^{i\gamma_4} \end{bmatrix} = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} = K;$$

where $k_{11} = \max\{a_{11},\ b_{11},\ g_{11}\}e^{i\ max\{\theta_1,\alpha_1,\gamma_1\}}$; $k_{12} = \max\{a_{12},\ b_{12}, g_{12}\}e^{i\ max\{\theta_2,\alpha_2,\gamma_2\}}$;
$k_{21} = \max\{a_{21},\ b_{21}, g_{21}\}e^{i\ max\{\theta_3,\alpha_3,\gamma_3\}}$; $k_{22} = \max\{a_{22},\ b_{22}, g_{22}\}e^{i\ max\{\theta_4,\alpha_4,\gamma_4\}}$.

Next, $Q+R = \begin{bmatrix} b_{11}e^{i\alpha_1} & b_{12}e^{i\alpha_2} \\ b_{21}e^{i\alpha_3} & b_{22}e^{i\alpha_4} \end{bmatrix} + \begin{bmatrix} g_{11}e^{i\gamma_1} & g_{12}e^{i\gamma_2} \\ g_{21}e^{i\gamma_3} & g_{22}e^{i\gamma_4} \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} = Y'$;

where $y_{11} = \max\{b_{11},\ g_{11}\}e^{i\ max\{\alpha_1,\gamma_1\}}$; $y_{12} = \max\{b_{12},\ g_{12}\}e^{i\ max\{\alpha_2,\gamma_2\}}$;

$y_{21} = \max\{b_{21},\ g_{21}\}e^{i\ max\{\alpha_3,\gamma_3\}}$; $y_{22} = \max\{b_{22},\ g_{22}\}e^{i\ max\{\alpha_4,\gamma_4\}}$.

Further, $P+(Q+R) = P+Y' = \begin{bmatrix} a_{11}e^{i\theta_1} & a_{12}e^{i\theta_2} \\ a_{21}e^{i\theta_3} & a_{22}e^{i\theta_4} \end{bmatrix} + \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} = \begin{bmatrix} l_{11} & l_{12} \\ l_{21} & l_{22} \end{bmatrix} = L$;

where $l_{11} = \max\{a_{11},\ b_{11},\ g_{11}\}e^{i\ max\{\theta_1,\alpha_1,\gamma_1\}}$; $l_{12} = \max\{a_{12},\ b_{12}, g_{12}\}e^{i\ max\{\theta_2,\alpha_2,\gamma_2\}}$;
$l_{21} = \max\{a_{21},\ b_{21}, g_{21}\}e^{i\ max\{\theta_3,\alpha_3,\gamma_3\}}$; $l_{22} = \max\{a_{22},\ b_{22}, g_{22}\}e^{i\ max\{\theta_4,\alpha_4,\gamma_4\}}$.

Hence, $(P+Q)+R = P+(Q+R)$.

- **Multiplication Operation of Two Complex Fuzzy Matrices**
  Suppose $C^1_{2\times2}$ & $C^2_{2\times2}$ given by equation 3.1 are two CFMs, then their product is defined as follows:

$$C^1_{2\times2}C^2_{2\times2} = \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix};$$

where $d_{11} = \left\{\max\left\{\min\left\{h_{11}e^{i\theta_1}, J_{11}e^{i\alpha_1}\right\}, \min\left\{h_{12}e^{i\theta_2}, J_{21}e^{i\alpha_3}\right\}\right\}\right\}$;

$$d_{12} = \left\{\max\left\{\min\left\{h_{11}e^{i\theta_1}, J_{12}e^{i\alpha_2}\right\}, \min\left\{h_{12}e^{i\theta_2}, J_{22}e^{i\alpha_4}\right\}\right\}\right\};$$

Similarly, d_{21} and d_{22} can be expressed accordingly.
Example: The product of the given two matrices is obtained as follows:

$$C^1_{2\times2} = \begin{bmatrix} 0.6e^{i0.3} & 0.1e^{i0.7} \\ 0.2e^{i0.1} & 0.5e^{i0.4} \end{bmatrix} \ \& \ C^2_{2\times2} = \begin{bmatrix} 0.5e^{i0.1} & 0.4e^{i0.3} \\ 0.8e^{i0.6} & 0.7e^{i0.2} \end{bmatrix}.$$

Then, $C^1_{2\times2}C^2_{2\times2} = \begin{bmatrix} 0.5e^{i0.6} & 0.4e^{i0.3} \\ 0.5e^{i0.4} & 0.5e^{i0.2} \end{bmatrix}$.

- **Union of Two Complex Fuzzy Matrices**
  Now again, taking the value of $C^1_{2\times2}$ & $C^2_{2\times2}$ from Eq. 3.1 and then, the union of these matrices is given as

$$C^1_{2\times2} \cup C^2_{2\times2} = \begin{bmatrix} \check{d}_{11} & \check{d}_{12} \\ \check{d}_{21} & \check{d}_{22} \end{bmatrix};$$

where $\check{d}_{11} = \left\{\max\{h_{11},\ J_{11}\}e^{i\min\{\theta_1,\alpha_1\}}\right\}$; $\check{d}_{12} = \left\{\max\{h_{12},\ J_{12}\}e^{i\min\{\theta_2,\alpha_2\}}\right\}$;

$\check{d}_{21} = \left\{\max\{h_{21},\ J_{21}\}e^{i\min\{\theta_3,\alpha_3\}}\right\}$; $\check{d}_{22} = \left\{\max\{h_{11},\ J_{11}\}e^{i\min\{\theta_1,\alpha_1\}}\right\}$.

Example: The union of given two matrices is obtained as follows:

$$C^1_{2\times 2} = \begin{bmatrix} 0.6e^{i0.3} & 0.1e^{i0.7} \\ 0.2e^{i0.1} & 0.5e^{i0.4} \end{bmatrix} \;\&\; C^2_{2\times 2} = \begin{bmatrix} 0.5e^{i0.1} & 0.4e^{i0.3} \\ 0.8e^{i0.6} & 0.7e^{i0.2} \end{bmatrix}.$$

$$\text{Then, } C^1_{2\times 2} \cup C^2_{2\times 2} = \begin{bmatrix} 0.6e^{i0.1} & 0.4e^{i0.3} \\ 0.8e^{i0.1} & 0.7e^{i0.2} \end{bmatrix}.$$

- **Intersection of Two Complex Fuzzy Matrices**
  Similarly, suppose that $C^1_{2\times 2}$ & $C^2_{2\times 2}$ are two CFMs, then the intersection of these matrices is defined as follows:

$$C^1_{2\times 2} \cap C^2_{2\times 2} = \begin{bmatrix} \check{\check{d}}_{11} & \check{\check{d}}_{12} \\ \check{\check{d}}_{21} & \check{\check{d}}_{22} \end{bmatrix};$$

where $\check{\check{d}}_{11} = \left\{ \min\{h_{11},\, J_{11}\} e^{i\max\{\theta_1,\alpha_1\}} \right\}$; $\check{\check{d}}_{12} = \left\{ \min\{h_{12},\, J_{12}\} e^{i\max\{\theta_2,\alpha_2\}} \right\}$;

$$\check{\check{d}}_{21} = \left\{ \min\{h_{21},\, J_{21}\} e^{i\max\{\theta_3,\alpha_3\}} \right\}; \;\; \check{\check{d}}_{22} = \left\{ \min\{h_{11},\, J_{11}\} e^{i\max\{\theta_1,\alpha_1\}} \right\}.$$

Example: The intersection of the given two matrices is obtained as follows:

$$C^1_{2\times 2} = \begin{bmatrix} 0.6e^{i0.3} & 0.1e^{i0.7} \\ 0.2e^{i0.1} & 0.5e^{i0.4} \end{bmatrix} \;\&\; C^2_{2\times 2} = \begin{bmatrix} 0.5e^{i0.1} & 0.4e^{i0.3} \\ 0.8e^{i0.6} & 0.7e^{i0.2} \end{bmatrix}.$$

$$\text{Then, } C^1_{2\times 2} \cap C^2_{2\times 2} = \begin{bmatrix} 0.5e^{i0.3} & 0.1e^{i0.7} \\ 0.2e^{i0.6} & 0.5e^{i0.4} \end{bmatrix}.$$

- **Commutativity, Associativity and Distributivity Properties of CFMs:**

Theorem 2 *Suppose that there are three CFMSs, say, P, Q, and R, then the union operation of CFMs is commutative and associative and distributive over the intersection.*

(i) $P \cup Q = Q \cup P$.

(ii) $P \cup (Q \cup R) = (P \cup Q) \cup R$.

(iii) $P \cup (Q \cap R) = (P \cup Q) \cap (P \cup R)$.

Proof: Consider the following three complex fuzzy matrices:

$$P = \begin{bmatrix} a_{11}e^{i\theta_1} & a_{12}e^{i\theta_2} \\ a_{21}e^{i\theta_3} & a_{22}e^{i\theta_4} \end{bmatrix}, Q = \begin{bmatrix} b_{11}e^{i\alpha_1} & b_{12}e^{i\alpha_2} \\ b_{21}e^{i\alpha_3} & b_{22}e^{i\alpha_4} \end{bmatrix} \& R = \begin{bmatrix} g_{11}e^{i\gamma_1} & g_{12}e^{i\gamma_2} \\ g_{21}e^{i\gamma_3} & g_{22}e^{i\gamma_4} \end{bmatrix}.$$

Commutative Property:

$$P \cup Q = \begin{bmatrix} \max(a_{11}, b_{11}) e^{i\min(\theta_1,\alpha_1)} & \max(a_{12}, b_{12}) e^{i\min(\theta_2,\alpha_2)} \\ \max(a_{21}, b_{21}) e^{i\min(\theta_3,\alpha_3)} & \max(a_{22}, b_{22}) e^{i\min(\theta_4,\alpha_4)} \end{bmatrix} = Q \cup P.$$

Associative Property:$P\cup(Q\cup R)=\begin{bmatrix}\check{e}_{11} & \check{e}_{12}\\ \check{e}_{21} & \check{e}_{22}\end{bmatrix}=(P\cup Q)\cup R;$

where $\check{e}_{11}=\max(a_{11},\ b_{11},\ g_{11})e^{i\min(\theta_1,\alpha_1,\ \gamma_1)}$; $\check{e}_{12}=\max(a_{12},\ b_{12},\ g_{12})e^{i\min(\theta_2,\alpha_2,\ \gamma_2)}$;

$\check{e}_{21}=\max(a_{21},\ b_{21},\ g_{21})e^{i\min(\theta_3,\alpha_3,\ \gamma_3)}$; $\check{e}_{22}=\max(a_{22},\ b_{22},\ g_{22})e^{i\min(\theta_4,\alpha_4,\ \gamma_4)}$.

Distributive Property:

$P\cup(Q\cap R)=P\cup\check{Q}$; where $\check{Q}=\begin{bmatrix}\check{\check{e}}_{11} & \check{\check{e}}_{12}\\ \check{\check{e}}_{21} & \check{\check{e}}_{22}\end{bmatrix}$;

and $\check{\check{e}}_{11}=\min(\ b_{11},\ g_{11})e^{i\max(\alpha_1,\ \gamma_1)}$; $\check{\check{e}}_{12}=\min(b_{12},\ g_{12})e^{i\max(\alpha_2,\ \gamma_2)}$;

$\check{\check{e}}_{21}=\min(\ b_{21},\ g_{21})e^{i\max(\alpha_3,\ \gamma_3)}$; $\check{\check{e}}_{22}=\min(b_{22},\ g_{22})e^{i\max(\alpha_4,\ \gamma_4)}$.

Next, remaining part of the equation is calculated that is $P\cup\check{Q}=\begin{bmatrix}\check{\check{p}}_{11} & \check{\check{p}}_{12}\\ \check{\check{p}}_{21} & \check{\check{p}}_{22}\end{bmatrix}$;

where $\check{\check{p}}_{11}=a_{11}e^{i\theta_1}\cup\check{\check{e}}_{11}$; $\check{\check{p}}_{12}=a_{12}e^{i\theta_2}\cup\check{\check{e}}_{12}$; $\check{\check{p}}_{21}=a_{21}e^{i\theta_3}\cup\check{\check{e}}_{21}$; $\check{\check{p}}_{22}=a_{22}e^{i\theta_4}\cup\check{\check{e}}_{22}$.

Similarly, we will obtain the right-hand side of the identity and after calculation, it is observed that the desired values are obtained. In this manner, the distributive property is satisfied.
Now, the above three identities are being illustrated with the help of numerical examples for a better understanding of the concept.
Suppose the matrices are of the following form:

$$P=\begin{bmatrix} 0.5e^{i0.7} & 0.3e^{i0.1}\\ 06e^{i0.3} & 02e^{i0.5}\end{bmatrix},\ Q=\begin{bmatrix} 0.7e^{i0.1} & 0.5e^{i0.3}\\ 02e^{i0.4} & 09e^{i0.2}\end{bmatrix}\ \&\ R=\begin{bmatrix} 0.4e^{i0.3} & 0.3e^{i0.5}\\ 06e^{i0.1} & 07e^{i0.2}\end{bmatrix}.$$

Commutative law: $P\cup Q=\begin{bmatrix}0.7e^{i0.1} & 0.5e^{i0.1}\\ 0.6e^{i0.3} & 0.9e^{i0.2}\end{bmatrix}=Q\cup P.$

Associative law:

$$(P\cup Q)\cup R=\begin{bmatrix}0.7e^{i0.1} & 0.5e^{i0.1}\\ 0.6e^{i0.3} & 0.9e^{i0.2}\end{bmatrix}\cup\begin{bmatrix}0.4e^{i0.3} & 0.3e^{i0.5}\\ 0.6e^{i0.1} & 0.7e^{i0.2}\end{bmatrix}=\begin{bmatrix}0.7e^{i0.1} & 0.5e^{i0.1}\\ 0.6e^{i0.1} & 0.9e^{i0.2}\end{bmatrix}.$$

$$(P\cup Q)\cup R=\begin{bmatrix}0.7e^{i0.1} & 0.5e^{i0.1}\\ 0.6e^{i0.3} & 0.9e^{i0.2}\end{bmatrix}\cup\begin{bmatrix}0.4e^{i0.3} & 0.3e^{i0.5}\\ 0.6e^{i0.1} & 0.7e^{i0.2}\end{bmatrix}=\begin{bmatrix}0.7e^{i0.1} & 0.5e^{i0.1}\\ 0.6e^{i0.1} & 0.9e^{i0.2}\end{bmatrix}.$$

$P\cup(Q\cup R)=(P\cup Q)\cup R.$

Distributive law:

$$P\cup(Q\cap R)=\begin{bmatrix}0.5\ e^{i0.7} & 0.3e^{i0.1}\\ 0.6e^{i0.3} & 0.2e^{i0.5}\end{bmatrix}\cup\begin{bmatrix}0.4e^{i0.3} & 0.3e^{i0.5}\\ 0.2e^{i0.4} & 0.7e^{i0.2}\end{bmatrix}=\begin{bmatrix}0.5e^{i0.3} & 0.3e^{i0.1}\\ 0.6e^{i0.3} & 0.7e^{i0.2}\end{bmatrix}.$$

$$(P\cup Q)\cap(P\cup R)=\begin{bmatrix}0.7e^{i0.1} & 0.5e^{i0.1}\\ 0.6e^{i0.3} & 0.9e^{i0.2}\end{bmatrix}\cap\begin{bmatrix}0.5e^{i0.3} & 0.3e^{i0.1}\\ 0.6e^{i0.1} & 0.7e^{i0.2}\end{bmatrix}=\begin{bmatrix}0.5e^{i0.3} & 0.3e^{i0.1}\\ 0.6e^{i0.3} & 0.7e^{i0.2}\end{bmatrix}.$$

4. SIMILARITY MEASURE FOR COMPLEX FUZZY MATRIX

In this section, we have proposed a new similarity measure for the complex fuzzy matrix and studied its computational feature with the help of a suitable numerical example. In the literature, it may be noted that the following necessary conditions for the proposed similarity measure must be satisfied:

Definition 5 *A real valued mapping: $\hat{S}: P \times Q \to [0,1]$ is known as a similarity measure between two complex fuzzy matrices $P = \mu_P(x_{ij}) = r_P(x_{ij})\, e^{i\omega_P(x_{ij})}$ and $Q = \mu_Q(x_{ij}) = r_Q(x_{ij})\, e^{i\omega_Q(x_{ij})}$, if $\hat{S}$ satisfies the following axioms:*

(i) $\hat{S}(P,Q) = \hat{S}(Q,P)$;

(ii) $\hat{S}(P,Q) = 1 \Longleftrightarrow (P,Q) = (Q,P)$;

(iii) $\hat{S}(P,Q) = 0 \Longleftrightarrow x_{ij} \in U$, *where* $r_P(x_{ij}) = 1, \quad r_Q(x_{ij}) = 0$ *or* $r_P(x_{ij}) = 0, \quad r_Q(x_{ij}) = 1$ *and* $\omega_P(x_{ij}) = 2\pi, \quad \omega_Q(x_{ij}) = 0$ *or* $\omega_P(x_{ij}) = 0, \quad \omega_Q(x_{ij}) = 2\pi$

(iv) For three modified complex fuzzy matrices P, Q and R subject to $P \subseteq Q \subseteq R$, we get, $\hat{S}(P,Q) \le \hat{S}(P,R)$ or $\hat{S}(P,Q) \le \hat{S}(R,Q)$.

Further, we propose a new similarity measure for the complex fuzzy matrices which is supposed to be very helpful in obtaining the solutions to various decision-making problems as follows:

Definition 6 *Suppose there are two complex fuzzy matrices P and Q on the universe of discourse U. The complex form of P and Q can be written as follows: $P = \mu_P(x_{ij}) = r_P(x_{ij})\, e^{i\omega_P(x_{ij})}, Q = \mu_Q(x_{ij}) = r_Q(x_{ij})\, e^{i\omega_Q(x_{ij})}$.*
The similarity measure of two CFMs P and Q, denoted by $\hat{S}(P,Q)$, is defined as follows:

$$\hat{S}(P,Q) = \frac{1}{2mn} \sum_{j=1}^{n} \sum_{i=1}^{m} \left(\frac{|P \cap Q|}{|P \cup Q|} \left[\hat{S}^r(P,Q) + \frac{\hat{S}^{\omega}(P,Q)}{2\pi} \right] \right); \tag{4.1}$$

where $$\hat{S}^r(P,Q) = 1 - \sum_{j=1}^{n} \sum_{i=1}^{m} \max\left(|r_P(x_{ij}) - r_Q(x_{ij})|\right);$$

$$\hat{S}^{\omega}(P,Q) = 2\pi - \sum_{j=1}^{n} \sum_{i=1}^{m} \max\left(|\omega_P(x_{ij}) - \omega_Q(x_{ij})|\right).$$

Theorem 3 *The proposed similarity measure $\hat{S}(P,Q)$ given by equation (4.1) is a valid similarity measure.*

Proof: In view of the axioms listed in Definition 6 and also to validate the proposed similarity measure, we prove the axioms one by one below:

(i) $|P \cap Q| = |Q \cap P|$ & $|P \cup Q| = |Q \cup P|$.

$$\hat{S}^r(P,Q) = 1 - \sum_{j=1}^{n} \sum_{i=1}^{m} \max\left(|r_P(x_{ij}) - r_Q(x_{ij})|\right) = 1 - \sum_{j=1}^{n} \sum_{i=1}^{m} \max\left(|r_Q(x_{ij}) - r_P(x_{ij})|\right) = \hat{S}^r(Q,P).$$

Similarly, $\hat{S}^{\omega}(P,Q)=\hat{S}^{\omega}(Q,P) \implies \hat{S}(P,Q)=\hat{S}(Q,P)$.

(ii) Let $P=Q$. Then, $\frac{|P\cap Q|}{|P\cup Q|}=1$.

$$\hat{S}^{r}(P,Q)=1-\sum_{j=1}^{n}\sum_{i=1}^{m}\max\left(\left|r_P\left(x_{ij}\right)-r_P\left(x_{ij}\right)\right|\right)=1,$$

$$\hat{S}^{\omega}(P,Q)=1-\sum_{j=1}^{n}\sum_{i=1}^{m}\max\left(\left|\omega_P\left(x_{ij}\right)-\omega_P\left(x_{ij}\right)\right|\right)=2\pi.$$

Substituting all the values in the proposed similarity measure. Then, $\hat{S}(P,Q)=1$.

(iii) Substitute $r_P\left(x_{ij}\right)=1, \quad r_Q\left(x_{ij}\right)=0$ and $\omega_P\left(x_{ij}\right)=2\pi, \quad \omega_Q\left(x_{ij}\right)=0$. Then,

$$\hat{S}^{r}(P,Q)=\hat{S}^{\omega}(P,Q)=0 \implies \hat{S}(P,Q)=0.$$

(iv) When $P\subseteq Q\subseteq R$.

Then, $r_P\left(x_{ij}\right)\le r_Q\left(x_{ij}\right)\le r_R\left(x_{ij}\right)$ and $\omega_P\left(x_{ij}\right)\le\omega_Q\left(x_{ij}\right)\le\omega_R\left(x_{ij}\right)$.

$$\implies \max\left(\left|r_P\left(x_{ij}\right)-r_R\left(x_{ij}\right)\right|\right)\le\max\left(\left|r_P\left(x_{ij}\right)-r_Q\left(x_{ij}\right)\right|\right).$$

$$\implies \max\left(\left|\omega_P\left(x_{ij}\right)-\omega_R\left(x_{ij}\right)\right|\right)\le\max\left(\left|\omega_P\left(x_{ij}\right)-\omega_Q\left(x_{ij}\right)\right|\right) \implies \hat{S}(P,Q)\le\hat{S}(P,R).$$

Hence, all the axioms for similarity measures are satisfied.

Example: Suppose P and Q be two complex fuzzy matrices defined as :

$$P_{2\times2}=\begin{bmatrix}0.6e^{i0.3\pi} & 0.1e^{i0.7\pi}\\ 0.2e^{i0.1\pi} & 0.5e^{i0.4\pi}\end{bmatrix} \;\&\; Q_{2\times2}=\begin{bmatrix}0.5e^{i0.1\pi} & 0.4e^{i0.3\pi}\\ 0.8e^{i0.6\pi} & 0.7e^{i0.2\pi}\end{bmatrix}.$$

Then

$$\hat{S}(P,Q)=\frac{1}{2\times2\times2}\sum_{j=1}^{2}\sum_{i=1}^{2}\left(\frac{|P\cap Q|}{|P\cup Q|}\left[\hat{S}^{r}(P,Q)+\frac{\hat{S}^{\omega}(P,Q)}{2\pi}\right]\right).$$

$$=\frac{1}{8}\sum_{j=1}^{2}\sum_{i=1}^{2}\left(\frac{|r_P(x_{ij})\exp^{i\omega_P(x_{ij})}\cap r_Q(x_{ij})\exp^{i\omega_Q(x_{ij})}|}{|r_P(x_{ij})\exp^{i\omega_P(x_{ij})}\cup r_Q(x_{ij})\exp^{i\omega_Q(x_{ij})}|}\left(\hat{S}^{r}(P(x_{ij}),Q(x_{ij}))+\hat{S}^{\omega}(P(x_{ij}),Q(x_{ij})\right)\right)$$

$$=\frac{1}{8}\Big\{\frac{|r_P(x_{11})\exp^{i\omega_P(x_{11})}\cap r_Q(x_{11})\exp^{i\omega_Q(x_{11})}|}{|r_P(x_{11})\exp^{i\omega_P(x_{11})}\cup r_Q(x_{11})\exp^{i\omega_Q(x_{11})}|}\left(\hat{S}^{r}(P(x_{11}),Q(x_{11}))+\hat{S}^{\omega}(P(x_{11}),Q(x_{11})\right)+$$

$$\frac{|r_P(x_{12})\exp^{i\omega_P(x_{12})}\cap r_Q(x_{12})\exp^{i\omega_Q(x_{12})}|}{|r_P(x_{12})\exp^{i\omega_P(x_{12})}\cup r_Q(x_{12})\exp^{i\omega_Q(x_{12})}|}\left(\hat{S}^{r}(P(x_{12}),Q(x_{12}))+\hat{S}^{\omega}(P(x_{12}),Q(x_{12})\right)+$$

$$\frac{|r_P(x_{21})\exp^{i\omega_P(x_{21})}\cap r_Q(x_{21})\exp^{i\omega_Q(x_{21})}|}{|r_P(x_{21})\exp^{i\omega_P(x_{21})}\cup r_Q(x_{21})\exp^{i\omega_Q(x_{21})}|}\left(\hat{S}^r(P(x_{21}),Q(x_{ij}))+\hat{S}^\omega(P(x_{21}),Q(x_{21})\right)+$$

$$\frac{|r_P(x_{22})\exp^{i\omega_P(x_{22})}\cap r_Q(x_{22})\exp^{i\omega_Q(x_{22})}|}{|r_P(x_{22})\exp^{i\omega_P(x_{22})}\cup r_Q(x_{22})\exp^{i\omega_Q(x_{22})}|}\left(\hat{S}^r(P(x_{22}),Q(x_{22}))+\hat{S}^\omega(P(x_{22}),Q(x_{22})\right)\}.$$

Thus, the value of the proposed similarity measure has been computed to be $\hat{S}(P,Q) = 0.3647$.

5. APPLICATION OF COMPLEX FUZZY MATRIX IN IDENTIFICATION OF SIGNAL

In this segment, we have used the concept of the complex fuzzy matrix in detecting the appropriate signal among the various signals transmitted by the transmitter. The methodology used to detect the reference signal is explained below and the applicability of the following methodology is described with the help of an example.

Methodology

Step 1: Suppose $p\ (S_1(x),\ S_2(x),\ S_3(x),\ldots,\ S_p(x))$ number of signals are sent by the transmitter, then each of the p signals are sampled Q times by the receiver. Then, the appropriate signal $S_l(x)\,(l\ varies\ from\ 1\ to\ p)$ is selected with the help of reference signal R, whose value is already known. Let both the signals $S_l(x)$ and R are considered Q times. The absolute value of each j-th signal *i.e.*, $S_j(u)\ (1 \le u \le p)$ in terms of discrete complex fuzzy transform is given by $x_{j,S} = \ \varepsilon_{j,S}e^{i\delta_{j,S}},\ \left(\varepsilon_{j,S},\delta_{j,S} \in \mathbb{R}\ \&\ \varepsilon_{j,S} \ge 1\forall\ S\ (1 \le S \le p)\right)$, where $x_{j,S}$ is the complex Fourier coefficients of signals $S_j(u)$.

Step 2: Now the signals are expressed in form of matrix and is given by $E_{m\times n} = \left[\left|S_j(u)\right|\right]_{Q\times p}$, where signals are denoted by the column of the matrix and Q samples of each signal is considered

$$E = \begin{bmatrix} |S_1(1)| & |S_2(Q)| & \ldots & |S_p(1)| \\ |S_1(2)| & |S_2(Q)| & \ldots & |S_p(2)| \\ . & . & \ldots & . \\ |S_1(Q)| & |S_2(Q)| & \ldots & |S_p(Q)| \end{bmatrix}.$$

Step 3: Similarly, the second matrix is given by $F_{m\times n} = \left[\left|S'_j(u)\right|\right]_{Q\times p}$

$$F = \begin{bmatrix} |S'_1(1)| & |S'_2(Q)| & \ldots & |S'_p(1)| \\ |S'_1(2)| & |S'_2(Q)| & \ldots & |S'_p(2)| \\ . & . & \ldots & . \\ |S'_1(Q)| & |S'_2(Q)| & \ldots & |S'_p(Q)| \end{bmatrix}.$$

Step 4: Next, the product of the above two matrices (E & F) with the complex fuzzy max-min decision matrices are obtained.

Step 5: Finally, the optimal fuzzy set is obtained.
Note: The definition of an optimal set is given in ref [26]. According to which

Definition 7 *"Let $U = u_1, u_2, ..., u_m$ be an initial universe and $Mm[c_{ip}] = [d_{i1}]$. Then, a subset of U can be obtained by using $[d_{i1}]$ as in the following way $opt[d_{i1}](U) = \{u_i : u_i \in U, d_{i1} = 1\}$ which is called an optimum set of U."*

5.1 Application

In this subsection of the current article, we have utilized the notion of complex fuzzy matrix and the proposed methodology given in the above section in finding the reference signals among the five signals obtained by the receiver. Assume that there is a set of five signals $S = \{\psi_1, \psi_2, \psi_3, \psi_4, \psi_5\}$ and every signal is sampled five times each. Let R denote the reference signal from which each signal is compared to obtain the high degree of resemblance between the signal and the reference signal.

According to Step 2, the matrix E and the second matrix F are obtained. Both the obtained matrices are given below.

$$E = \begin{pmatrix} 0.1 & 0.2 & 0.5 & 0.1 & 0.2 \\ 0.5 & 0.4 & 0.4 & 0.1 & 0.2 \\ 0.4 & 0.2 & 0.2 & 0.2 & 0.3 \\ 0.2 & 0.3 & 0.3 & 0.2 & 0.4 \\ 0.1 & 0.2 & 0.1 & 0.1 & 0.3 \end{pmatrix} \;\&\; F = \begin{pmatrix} 0.2 & 0.1 & 0.3 & 0.1 & 0.1 \\ 0.4 & 0.2 & 0.3 & 0.2 & 0.3 \\ 0.4 & 0.2 & 0.2 & 0.2 & 0.2 \\ 0.1 & 0.5 & 0.3 & 0.5 & 0.4 \\ 0.2 & 0.1 & 0.2 & 0.4 & 0.3 \end{pmatrix}.$$

Next, according to Step 4, the following matrices are obtained:

$$E \times F = \begin{pmatrix} 0.35 & 0.22 & 0.26 & 0.28 & 0.27 \\ 0.47 & 0.28 & 0.42 & 0.34 & 0.35 \\ 0.32 & 0.25 & 0.34 & 0.34 & 0.31 \\ 0.38 & 0.28 & 0.35 & 0.4 & 0.37 \\ 0.21 & 0.15 & 0.2 & 0.24 & 0.22 \end{pmatrix} \;\&$$

$$Mm[E \times F] = [D_{i1}] = (d_{i1}) \forall i \in \{1,2,3,4,5\}.$$

subject to, $d_{11} = \min\{u_{11}, u_{21},\ u_{31},\ u_{41},\ u_{51}\}$; where $u_{11} = 0.35, u_{21} = 0.47,\ u_{31} = 0.32,$ $u_{41} = 0.38,\ u_{51} = 0.21$. Then, $d_{11} = \min\{0.35, 0.47,\ 0.32,\ 0.38,\ 0.21\} = 0.21$.

Similarly, $d_{21} = \min\{0.22, 0.28,\ 0.25,\ 0.28,\ 0.15\} = 0.15,\ d_{31} = \min\{0.26, 0.42,\ 0.34,\ 0.35,\ 0.2\} = 0.2$.

$d_{41} = \min\{0.28, 0.34,\ 0.34,\ 0.4,\ 0.24\} = 0.24$. $d_{51} = \min\{0.27, 0.35,\ 0.31,\ 0.37,\ 0.22\} = 0.22$.

Finally, we obtain the min-max decision matrix which is given below:

$$mM(E \times F) = \begin{pmatrix} d_{11} = 0.21 \\ d_{21} = 0.15 \\ d_{31} = 0.2 \\ d_{41} = 0.24 \\ d_{51} = 0.22 \end{pmatrix}.$$

In the end, the optimum fuzzy set is obtained $[S]$

$$optMm(E \times F)(\alpha) = \left\{ \frac{0.21}{\alpha_1}, \frac{0.15}{\alpha_2}, \frac{0.2}{\alpha_3}, \frac{0.24}{\alpha_4}, \frac{0.22}{\alpha_5} \right\}.$$

Hence, α_4 is the signal. In this manner, the reference signal is identified among the various signals obtained by the receiver. This also validates the proposed methodology.

6. CONCLUSIONS AND FUTURE WORK

The notion of the complex fuzzy matrix has been representationally modified and presented successfully in its exponential form along with various set-theoretic binary operations. Various set-theoretic properties of the fundamental operations related to commutativity, associativity, and distributivity have been well established. Some suitable numerical examples to illustrate these computations have also been included. A new similarity measure for the complex fuzzy matrix has been proposed with proof of its validity. The proposed methodology has been duly implemented in the process of identification of reference signals from a set of signals transmitted from the transmitter. In future, the concept of the entropy using the proposed similarity measure can further be applied as carried out by the Qudah [27] and may be of form $\hat{E}' = \hat{S}(P(x_{ij}), P^c(x_{ij}))$; where $\hat{E}'$ and $\hat{S}$ denote the entropy and similarity measure of the complex fuzzy matrix $P(x_{ij})$ and its complement is denoted by $P^c(x_{ij})$. The proposed measure will be of great use in various problems related to the multi decision-making and signals problems.

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