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Amended Kalman Filtering with Intermittent Measurements in Target Tracking

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This paper focuses on the state estimation problem of target tracking with intermittent measurements. Leveraged by the posterior measurements, an amended Kalman filter is proposed in this paper to improve the precision of the current estimated state. Both the deduction and proof of the amended Kalman filter are discussed specifically to distinguish amended Kalman filter from the Kalman smoother. Extensive simulations are conducted and the simulation results verify the excellent tracking performance of the amended Kalman filter.

Keywords: state estimation, target tracking, intermittent measurements, amended Kalman filter, tracking performance

1. INTRODUCTION

The state estimation for networked systems has motivated a significant number of researches due to its wide application in many engineering fields, such as target tracking, navigation, and so on [1-3]. However, in many real-world applications, such problems are inescapable as packet dropout, random time delay and uncertain measurements [4-6], which causes some challenges for the optimal state estimation.

To deal with the above problems, a number of methods regarding state estimation have been proposed. The optimal state estimation problem with uncertain measurements is investigated in [7, 8], but its estimation performance is dissatisfactory because only noises are used for updating the state estimation when the measurements are missing. By reorganizing measurements, the delayed system is transformed into a delay-free one in [9] and [10]. By augmenting state space model, an optimal linear estimation method with random measurement delays is proposed in [11]. As for the filtering problem with missing measurements, Sinopoli proposes a modified Kalman filter (MKF) when the measurements' arrival sequence $\{\gamma_k\}$ is known [12]. [13, 14] propose a new model to describe the measurements arrival conditions considering the measurements delay and missing. [15] proposes a recursive estimation method for nonlinear stochastic systems with multi-step transmission delays, multiple packet dropouts and correlated noises. Recently, fusion estimation methods in networked systems with random time delay, packet dropout

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and uncertain measurements have been investigated in [1, 2, 16-19].

In target tracking system, as we know, there are few references considering the random time delay, packet dropout and uncertain measurements. Usually, the missing measurement or uncertain measurement is related with detection probability in target tracking systems, such as radar tracking system [20]. And most methods mentioned above are unsuitable in tracking systems because the spectrum radius of state transmission matrix is not less than 1.

Kalman filter is one of famous state estimation methods, and it has been widely used in target tracking, control, integrated navigation, and so on [21, 22]. Motivated by the above discussion, we investigate the state estimation with intermittent measurements in target tracking system based on improved Kalman filter presented in [12]. Generally speaking, the performance of current estimated state will be deteriorated when the current measurements are missing. Thus, this paper uses the posterior measurements to improve the precision of the current estimated state whether the current measurements are missing or not. This method sounds like Kalman smoother, but unlike Kalman smoother, this method improves tracking performance by minimizing the covariance of innovation rather than the covariance of estimated state.

To the best of our knowledge, almost all the state estimation methods obtain optimal estimated state by minimizing the covariance of estimated state. However, minimizing the covariance of estimated state will deteriorate the precision of estimated state in some special situations, such as measurements missing. Based on Kalman filter, Kalman smoother uses the latter measurements to further improve the precision of estimated state by minimizing the covariance of estimated state. But the method minimizing the covariance of estimated state leads Kalman smoother to have a poor precision in optimal estimation while measurements missing. In this case, we must pay attention to the measurements. Therefore, the method proposed in this paper improves tracking performance by minimizing the covariance of innovation because the innovation reflects the difference between measurements and predicted state. The innovation covariance includes more measurements information than estimated state covariance.

The method in this paper is developed for scalar measurements but is also valid for vector measurements. The case of vector measurements is obviated in this paper since the development of the method is very complex. The rest of this paper is organized as follows. Section 2 presents the problem under investigation. In Section 3, the amended Kalman filter (AKF) is presented based on the minimum innovation covariance, and the differences between AKF and Kalman smoother are also discussed in this section. In Section 4, target tracking examples with CA model are given.

2. PROBLEM FORMULATION

Consider the following discrete-time linear dynamical system:

$$x_{k+1} = \Phi_{k+1|k} x_k + w_k \tag{1}$$

$$z_k = H_k x_k + v_k \tag{2}$$

where $x_k \in R^n$ is the state vector, $z_k \in R^1$ is the measurement scalar. $w_k \in R^n$ and $v_k \in R^1$ are the process and measurement noise, respectively. $\Phi_{k+1|k}$ and H_k are constant matrices with suitable dimensions.

When the measurement z_k is transmitted to a processing unit through an unreliable network, there exist possible data losses. In [12], The arrival of measurement at time k is defined as a binary random variable γ_k, with probability $p_{\gamma_k}(1) = \lambda$ and with γ_k independent of γ_s if $k \neq s$. The measurement noise v_k is defined in the following way:

$$p(v_k \mid \gamma_k) = \begin{cases} N(0, R), & \gamma_k = 1 \\ N(0, \sigma^2 I), & \gamma_k = 0 \end{cases}. \tag{3}$$

When the measurements are missing, $\sigma \to \infty$.

Assumption 1: w_k and v_k are uncorrelated white noises with zero mean and covariance matrices $Q \geq 0$ and $R > 0$ (Note: R is a scalar in this paper).

Assumption 2: The initial state x_0 is independent of w_k and v_k, and

$$\mathrm{E}(x_0) = \mu_0,\ \mathrm{E}[(x_0 - \mu_0)(x_0 - \mu_0)'] = P_0. \tag{4}$$

Under these assumptions, the equations of Kalman filter are modified as follows:

$$\hat{x}_{k+1|k} = \Phi_{k+1|k}\hat{x}_{k|k} \tag{5}$$

$$P_{k+1|k} = \Phi_{k+1|k}P_{k|k}\Phi'_{k+1|k} + Q \tag{6}$$

$$\hat{x}_{k+1|k+1} = \hat{x}_{k+1|k} + \gamma_{k+1}K_{k+1}(z_{k+1} - H_{k+1}\hat{x}_{k+1|k}) \tag{7}$$

$$P_{k+1|k+1} = P_{k+1|k} - \gamma_{k+1}K_{k+1}H_{k+1}P_{k+1|k} \tag{8}$$

where $K_{k+1} = P_{k+1|k}H'_{k+1}(H_{k+1}P_{k+1|k}H'_{k+1} + R)^{-1}$ is the Kalman gain matrix.

Considering the measurements $\{z_k\}$ and their arrival sequence $\{\gamma_k\}$, the minimum state-error variance filter will be obtained by using Eqs. (7) and (8). However, the estimated state $\hat{x}_{k|k}$ has a big error when measurement is missing at time k, and its covariance $P_{k|k}$ is also enlarged.

Augmenting state space model is a widely used state estimation method when measurements are missing, such as [2, 13-15], but these methods require the stability of $\Phi_{k+1|k}$. However, the state transmission matrix $\Phi_{k+1|k}$ is unstable due to $\rho(\Phi_{k+1|k}) = 1$ in target tracking, which leads to a monotonically increasing estimation error.

As commented above, we propose a new algorithm to improve the precision of estimated states at time k when measurement is missing by using posterior measurements in target tracking. By minimizing covariance of innovation, the new algorithm obtains the weighting matrix to improve the estimation precision using posterior measurements, which can improve the estimation precision when measurement is missing, and the estimation precision is almost equal to OSFL smoother when measurement is not missing. For the new algorithm adopts different minimum criterion and posterior measurements, the precision of estimated state can be improved. The principle and deducing of the new algorithm can be found in next section.

3. AMENDED KALMAN FILTER

3.1 The Principle of Amended Kalman Filter

Given the modified Kalman filter (MKF) Eqs. (5)-(8), we now study a method using k+1th measurement to improve the precision of estimated states at time kth.

When the k+1th measurement and the estimated state of KF at time kth have been obtained, the kth estimated state can be amended as follows.

$$\widehat{x}_{k|k} = \hat{x}_{k|k} + \gamma_{k+1} C_{k+1} \tilde{z}_{k+1} \tag{9}$$

where C_{k+1} is the $n \times 1$ weighting matrix, $\tilde{z}_{k+1} = z_{k+1} - H_{k+1}\hat{x}_{k+1|k}$ is the innovation and its variance is $S_{k+1} = H_{k+1}P_{k+1|k}H'_{k+1} + R > 0$.

Theorem 1: For system (1)-(3) which satisfies Assumptions 1 and 2,
(a) the covariance of amended innovation $\widehat{S}_{k+1}$ has a minimum value when

$$C_{k+1} = (H_{k+1}\Phi_{k+1|k})^{+} S_{k+1}^{-1} H_{k+1} P_{k+1|k} H'_{k+1} \tag{10}$$

where $(\bullet)^{+}$ represents Moore-Penrose pseudo-inverse.
(b) the covariance of amended estimated error is

$$\widehat{P}_{k|k} = P_{k|k} + \gamma_{k+1}^{2} C_{k+1} S_{k+1} C'_{k+1} - \gamma_{k+1} P_{k|k} \Phi'_{k+1|k} H'_{k+1} C'_{k+1} - \gamma_{k+1} C_{k+1} H_{k+1} \Phi_{k+1|k} P_{k|k}. \tag{11}$$

***Proof*:** (a) For system (1)-(3), when the estimated state is amended according to Eq. (9), the prediction error equation is as follows:

$$\begin{aligned} \widehat{\tilde{x}}_{k+1|k} &= x_{k+1} - \widehat{x}_{k+1|k} = x_{k+1} - \Phi_{k+1|k}(\hat{x}_{k|k} + \gamma_{k+1} C_{k+1} \tilde{z}_{k+1}) \\ &= \tilde{x}_{k+1|k} - \gamma_{k+1} \Phi_{k+1|k} C_{k+1} \tilde{z}_{k+1} = \tilde{x}_{k+1|k} - \gamma_{k+1} A_{k+1} \tilde{z}_{k+1} \end{aligned} \tag{12}$$

where $A_{k+1} = \Phi_{k+1|k} C_{k+1}$.

From Eq. (12), the covariance of prediction error is deduced as follows:

$$\begin{aligned} \widehat{P}_{k+1|k} &= \mathrm{cov}(\widehat{\tilde{x}}_{k+1|k}) = \mathrm{E}[(\tilde{x}_{k+1|k} - \gamma_{k+1} A_{k+1} \tilde{z}_{k+1})(\tilde{x}_{k+1|k} - \gamma_{k+1} A_{k+1} \tilde{z}_{k+1})'] = \mathrm{E}(\tilde{x}_{k+1|k} \tilde{x}'_{k+1|k}) \\ &\quad + \gamma_{k+1}^{2} \mathrm{E}(A_{k+1} \tilde{z}_{k+1} \tilde{z}'_{k+1} A'_{k+1}) - \gamma_{k+1} \mathrm{E}(\tilde{x}_{k+1|k} \tilde{z}'_{k+1} A'_{k+1}) - \gamma_{k+1} \mathrm{E}(A_{k+1} \tilde{z}_{k+1} \tilde{x}'_{k+1|k}) \end{aligned} \tag{13}$$

Because

$$\mathrm{E}[\tilde{x}_{k+1|k} \tilde{x}'_{k+1|k}] = P_{k+1|k} \tag{14}$$

$$\mathrm{E}(A \tilde{z}_{k+1} \tilde{z}'_{k+1} A'_{k+1}) = A_{k+1} S_{k+1} A'_{k+1} \tag{15}$$

$$\mathrm{E}(\tilde{x}_{k+1|k} \tilde{z}'_{k+1} A'_{k+1}) = \mathrm{E}[\tilde{x}_{k+1|k} (H_{k+1} \tilde{x}_{k+1|k} + v_{k+1})' A'_{k+1}] = P_{k+1|k} H'_{k+1} A'_{k+1} \tag{16}$$

$$\mathrm{E}(A_{k+1} \tilde{z}_{k+1} \tilde{x}'_{k+1|k}) = \mathrm{E}[A_{k+1} (H_{k+1} \tilde{x}_{k+1|k} + v_{k+1}) \tilde{x}'_{k+1|k}] = A_{k+1} H_{k+1} P_{k+1|k} \tag{17}$$

Then, Eq. (13) can be rewritten as

$$\widehat{P}_{k+1|k} = P_{k+1|k} + \gamma_{k+1}^2 A_{k+1} S_{k+1} A'_{k+1} - \gamma_{k+1} P_{k+1|k} H'_{k+1} A'_{k+1} - \gamma_{k+1} A_{k+1} H_{k+1} P_{k+1|k}. \tag{18}$$

The amended innovation is $\widehat{\tilde{z}}_{k+1} = z_{k+1} - H_{k+1}\widehat{x}_{k+1|k}$ and its covariance is

$$\begin{aligned}\widehat{S}_{k+1} &= H_{k+1}\widehat{P}_{k+1|k} H'_{k+1} + R \\ &= S_{k+1} + \gamma_{k+1}^2 H_{k+1} A_{k+1} S_{k+1} A'_{k+1} H'_{k+1} - \gamma_{k+1} B A'_{k+1} H'_{k+1} - \gamma_{k+1} H_{k+1} A_{k+1} B \\ &= S_{k+1} + \gamma_{k+1}^2 L_{k+1} S_{k+1} L'_{k+1} - \gamma_{k+1} B_{k+1} L'_{k+1} - \gamma_{k+1} L_{k+1} B_{k+1}\end{aligned} \tag{19}$$

where $B_{k+1} = H_{k+1}P_{k+1|k}H'_{k+1}$ and $L_{k+1} = H_{k+1}A_{k+1}$.

When $\gamma_{k+1} = 0$, $\widehat{S}_{k+1} = S_{k+1}$, which indicates the z_{k+1} is missing, thus $C_{k+1} = 0$.

When $\gamma_{k+1} = 1$, to satisfy the minimum of $\widehat{S}_{k+1}$, it requires that

$$\frac{\partial \widehat{S}_{k+1}}{\partial L_{k+1}} = \gamma_{k+1}^2 (S_{k+1} + S'_{k+1}) L_{k+1} - \gamma_{k+1} B_{k+1} - \gamma_{k+1} B'_{k+1} = 0. \tag{20}$$

Note $\gamma_{k+1}^2 = \gamma_{k+1} = 1$ and the symmetry of S_{k+1} and B_{k+1}, we have the following equation

$$L_{k+1} = S_{k+1}^{-1} B_{k+1}. \tag{21}$$

Thus, the expression of C_{k+1} can be written as follows:

$$C_{k+1} = (H_{k+1}\Phi_{k+1|k})^{+} S_{k+1}^{-1} B_{k+1}. \tag{22}$$

$\widehat{S}_{k+1}$ has a minimum value when $C_{k+1} = (H_{k+1}\Phi_{k+1|k})^{+} S_{k+1}^{-1} H_{k+1} P_{k+1|k} H'_{k+1}$.

(b) The estimated error equation is

$$\widehat{\tilde{x}}_{k|k} = x_k - \widehat{x}_{k|k} = x_k - (\hat{x}_{k|k} + \gamma_{k+1} C_{k+1}\tilde{z}_{k+1}) = \tilde{x}_{k|k} - \gamma_{k+1} C_{k+1}\tilde{z}_{k+1}. \tag{23}$$

From Eq. (23), the covariance of estimated error is deduced as follows:

$$\begin{aligned}\widehat{P}_{k|k} &= \mathrm{cov}(\widehat{\tilde{x}}_{k|k}) = \mathrm{E}[(\tilde{x}_{k|k} - \gamma_{k+1} C_{k+1}\tilde{z}_{k+1})(\tilde{x}_{k+1|k} - \gamma_{k+1} C_{k+1}\tilde{z}_{k+1})'] \\ &= \mathrm{E}(\tilde{x}_{k|k}\tilde{x}'_{k|k}) + \gamma_{k+1}^2 \mathrm{E}(C_{k+1}\tilde{z}_{k+1}\tilde{z}'_{k+1}C'_{k+1}) - \gamma_{k+1}\mathrm{E}(\tilde{x}_{k|k}\tilde{z}'_{k+1}C'_{k+1}) - \gamma_{k+1}\mathrm{E}(C_{k+1}\tilde{z}_{k+1}\tilde{x}'_{k|k}).\end{aligned} \tag{24}$$

Because

$$\mathrm{E}(\tilde{x}_{k|k}\tilde{x}'_{k|k}) = P_{k|k} \tag{25}$$

$$\mathrm{E}(C_{k+1}\tilde{z}_{k+1}\tilde{z}'_{k+1}C'_{k+1}) = C_{k+1} S_{k+1} C'_{k+1} \tag{26}$$

$$\begin{aligned}\mathrm{E}(\tilde{x}_{k|k}\tilde{z}'_{k+1}C'_{k+1}) &= \mathrm{E}[\tilde{x}_{k|k}(H_{k+1}x_{k+1} + v_{k+1} - H_{k+1}\hat{x}_{k+1|k})'C'_{k+1}] \\ &= \mathrm{E}[\tilde{x}_{k|k}(H_{k+1}\Phi_{k+1|k}x_k + H_{k+1}w_k + v_{k+1} - H_{k+1}\Phi_{k+1|k}\hat{x}_{k|k})'C'_{k+1}] \\ &= \mathrm{E}[\tilde{x}_{k|k}(H_{k+1}\Phi_{k+1|k}\tilde{x}_{k|k} + H_{k+1}w_k + v_{k+1})'C'_{k+1}] = P_{k|k}\Phi'_{k+1|k}H'_{k+1}C'_{k+1}\end{aligned} \tag{27}$$

$$\mathrm{E}(C_{k+1}\tilde{z}_{k+1}\tilde{x}'_{k|k}) = \mathrm{E}[C_{k+1}(H_{k+1}\Phi_{k+1|k}\tilde{x}_{k|k} + H_{k+1}w_k + v_{k+1})\tilde{x}'_{k|k}] = C_{k+1}H_{k+1}\Phi_{k+1|k}P_{k|k} \tag{28}$$

Thus,

$$\widehat{P}_{k|k} = P_{k|k} + \gamma_{k+1}^2 C_{k+1} S_{k+1} C'_{k+1} - \gamma_{k+1} P_{k|k} \Phi'_{k+1|k} H'_{k+1} C'_{k+1} - \gamma_{k+1} C_{k+1} H_{k+1} \Phi_{k+1|k} P_{k|k}. \quad \square$$

The aforementioned proof is similar to our proposed method in [21]. However, in this paper, we pay attention to arrival sequence $\{\gamma_k\}$ and its effect on estimation.

Theorem 2: The innovation covariance of amended Kalman filter is smaller than Kalman filter', *i.e.* $\widehat{S}_{k+1} \le S_{k+1}$.

Proof: According to Eqs. (19) and (22),

$$\widehat{S}_{k+1} = \begin{cases} S_{k+1}, & \gamma_{k+1} = 0 \\ S_{k+1} - B_{k+1}^2 S_{k+1}^{-1} < S_{k+1}, & \gamma_{k+1} = 1 \end{cases} \quad \square$$

Theorem 2 indicates that amended Kalman filter (AKF) has a less innovation covariance than Kalman filter. Actually, when $z_k \in R^m$ $(m > 1)$, the amended Kalman filter also has a less innovation covariance, but the deducing of L_{k+1} is very complex according to matrix analysis if the dimensions of $z_k \in R^m$ $(m > 1)$ are correlated. However, the deducing of L_{k+1} can be operated in each dimension if each dimension of $z_k \in R^m$ $(m > 1)$ is uncorrelated to each other.

3.2 Comparison of Amended Kalman Filter with Kalman Smoother

The Kalman smoother usually includes fixed point smoother, fixed lag smoother and fixed interval smoother [23]. From Eq. (9) of AKF, it is obvious that AKF uses the k+1th measurements to improve the precision of estimated state at time k, which is similar to one step fixed lag (OSFL) smoother [24]. The equations of OSFL smoother are as follows:

$$\hat{X}_{k|k+1} = X_{k|k} + M_{k+1} \tilde{z}_{k+1}, \tag{29}$$

$$P_{k|k+1} = P_{k|k} - M_{k+1} S_{k+1} M'_{k+1}, \tag{30}$$

where

$$M_{k+1} = P_{k|k} \Phi'_{k+1|k} H'_{k+1} S_{k+1}^{-1}. \tag{31}$$

When there exist possible data losses, OSFL smoother cannot be used directly. But we can modify the equations of OSFL smoother based on MKF algorithm as follows:

$$\hat{X}_{k|k+1} = X_{k|k} + \gamma_{k+1} M_{k+1} \tilde{z}_{k+1}, \tag{32}$$

$$P_{k|k+1} = P_{k|k} - \gamma_{k+1} M_{k+1} S_{k+1} M'_{k+1}. \tag{33}$$

Theorem 3: the variance of amended estimated error $\widehat{P}_{k|k}$ has a minimum value when

$$C_{k+1} = P_{k|k} \Phi'_{k+1|k} H'_{k+1} S_{k+1}^{-1}. \tag{34}$$

Proof: From Theorem 1, we can find that the expression of C_{k+1} has no relation with γ_{k+1}, thus γ_{k+1} is omitted in the proof. To satisfy the minimum of $\widehat{P}_{k|k}$, it requires that

$$\frac{\partial \widehat{P}_k}{\partial C_{k+1}} = (S_{k+1} + S'_{k+1})C_{k+1} - 2P_{k|k}\Phi'_{k+1|k}H'_{k+1} = 0. \tag{35}$$

Therefore,

$$C_{k+1} = P_{k|k}\Phi'_{k+1|k}H'_{k+1}S^{-1}_{k+1}.$$ ❑

Obviously, AKF is equal to OSFL smoother when C_{k+1} is calculated by minimizing the covariance of amended estimated error $\widehat{P}_{k|k}$. To distinguish the two kinds of AKF, in this paper AKF calculates C_{k+1} by minimizing the covariance of amended innovation according to Eq. (10). Thus, AKF is different from OSFL smoother because the weighting matrix C_{k+1} is calculated by minimizing the covariance of amended innovation $\widehat{S}_{k+1}$ rather than amended estimated error $\widehat{P}_{k|k}$. Furthermore, OSFL smoother does not use the current smoothed state and covariance to update the estimated state and covariance of next time. And AKF has a better performance than OSFL smoother when the measurements are missing seriously, which can be found in Section 4.

Actually, $\widehat{P}_{k|k}$ is almost equal to $P_{k|k+1}$ when the covariance of process noise is small. This is because

$$\begin{aligned}
\widehat{P}_{k|k} - P_{k|k+1} &= C_{k+1}S_{k+1}C'_{k+1} + M_{k+1}S_{k+1}M'_{k+1} - P_{k|k}\Phi'_{k+1|k}H'_{k+1}C'_{k+1} - C_{k+1}H_{k+1}\Phi_{k+1|k}P_{k|k} \\
&= [(\Phi_{k+1|k})^{+}P_{k+1|k} - P_{k|k}\Phi'_{k+1|k}]H'_{k+1}C'_{k+1} \\
&\quad + [P_{k|k}\Phi'_{k+1|k} - (\Phi_{k+1|k})^{+}P_{k+1|k}]H'_{k+1}S^{-1}_{k+1}H_{k+1}\Phi_{k+1|k}P_{k|k} \\
&= (\Phi_{k+1|k})^{+}QH'_{k+1}C'_{k+1} - (\Phi_{k+1|k})^{+}QH'_{k+1}S^{-1}_{k+1}H_{k+1}\Phi_{k+1|k}P_{k|k} \\
&= (\Phi_{k+1|k})^{+}QH'_{k+1}S^{-1}_{k+1}[(\Phi_{k+1|k})^{+}QH'_{k+1}]'.
\end{aligned} \tag{36}$$

Generally, the diagonal elements of S_{k+1} have a big value, and the diagonal elements of Q have a small value. Thus, the diagonal elements of Eq. (36) have a small value (almost equal to 0). Therefore, $tr(\widehat{P}_{k|k})$ is almost equal to $tr(P_{k|k+1})$(tr is a symbol of trace of matrix), which indicates the precision of AKF is almost equal to OSFL smoother.

4. PERFORMANCE EVALUATION

In this section, we present some cases of target tracking when using amended Kalman Filter with intermittent measurements.

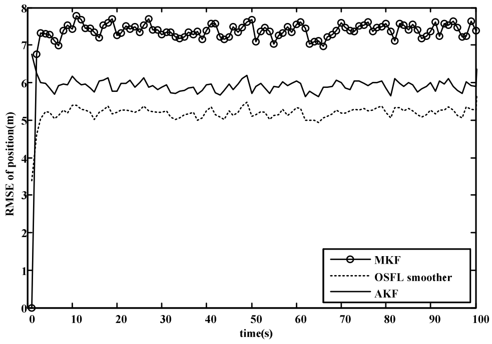
According to CA model in target tracking [21, 25],

$$\Phi_{k+1|k} = \begin{bmatrix} 1 & T & T^2/2 \\ 0 & 1 & T \\ 0 & 0 & 1 \end{bmatrix} \tag{37}$$

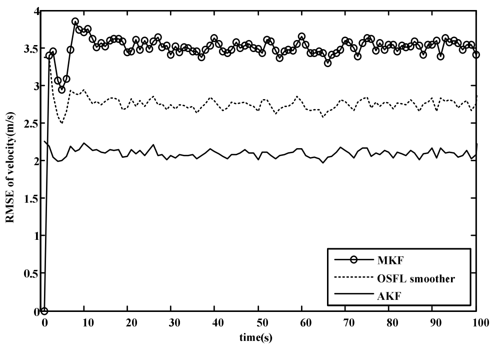
$$H_k = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \tag{38}$$

$$Q = q\begin{bmatrix} T^5/20 & T^4/8 & T^3/6 \\ T^4/8 & T^3/3 & T^2/2 \\ T^3/6 & T^2/2 & T \end{bmatrix} \tag{39}$$

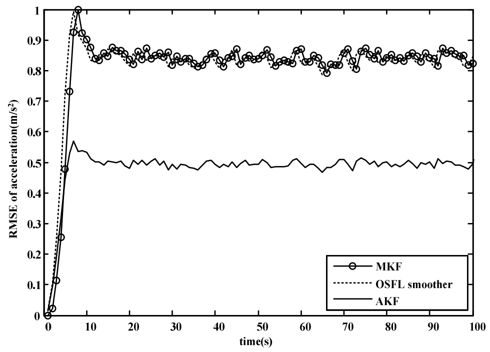
where T is the time interval between two consecutive measurements, and q is the power spectral density of the continuous-time white noise w_k.



(a)

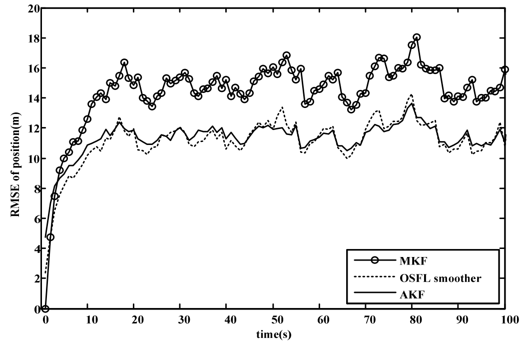


(b)

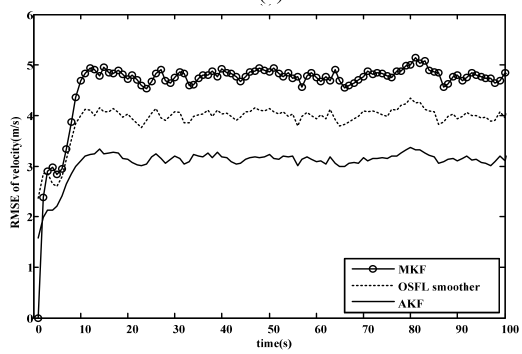


(c)

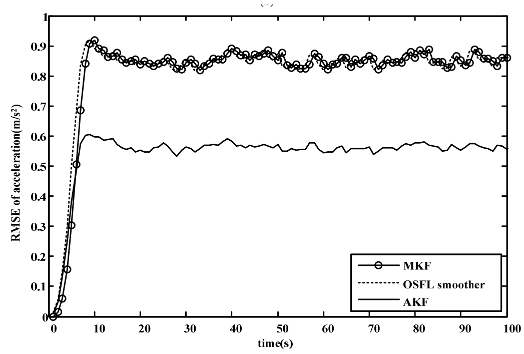
Fig. 1. Tracking results of AKF when $\lambda = 1$.



(a)



(b)



(c)

Fig. 2. Tracking results of AKF when $\lambda = 0.5$.

(a), (b) and (c) are RMSE of position, velocity and acceleration, respectively.

Obviously, the Moore-Penrose pseudo-inverse will lead to a big bias of the estimated velocity and acceleration, because the $(H_{k+1}\Phi_{k+1|k})^{-1}$ does not exist and the measurement only includes position information.

To reduce the bias caused by Moore-Penrose pseudo-inverse, we modify Eq. (10) as follows:

$$\bar{C}_{k+1} = \begin{bmatrix} p_{1,1} + p_{1,2} + p_{1,3} \\ p_{1,2} \\ p_{1,3} \end{bmatrix} \odot L_{k+1} / (p_{1,1} + p_{1,2} + p_{1,3}) \tag{40}$$

where p_{ij} is the ith row jth column element of $P_{k+1|k}$, and $\odot$ means element-by-element multiplication.

We modify Eq. (10) using the covariance of prediction error because it includes the covariance of position ($p_{1,1}$), the cross-covariance of position and velocity ($p_{1,2}$), and the cross-covariance of position and acceleration ($p_{1,3}$).

Assuming $T = 1$s and the movement time of target is 100s. The target begins to move at a constant speed of 20m/s. $R = 100$, $q = 1$, $\mu_0 = [0, 20, 0]'$ and $P_0 = diag(100, 100, 1)$.

Figs. 1 and 2 respectively present the tracking results, when $\lambda = 1$ and $\lambda = 0.5$. The simulations have been operated $M = 1000$ times, and the root mean square errors (RMSE) are used to evaluate the performance of AKF and other algorithms [26].

MKF proposed in [12] is equal to Kalman filter when $\lambda = 1$. It is apparent that the tracking performance of AKF and OSFL smoother are better than MKF whether the measurements are missing or not. The position tracking results of AKF is worse than OSFL smoother, but the velocity and acceleration tracking results of AKF is better than OSFL smoother. The computational time of MKF, OSFL and AKF are 3.3ms, 5.8ms and 6.4ms, respectively. AKF needs more computational time than MKF and OSFL, because AKF amends the estimated state, and the amended estimated state is also used to update the estimated state of the next time.

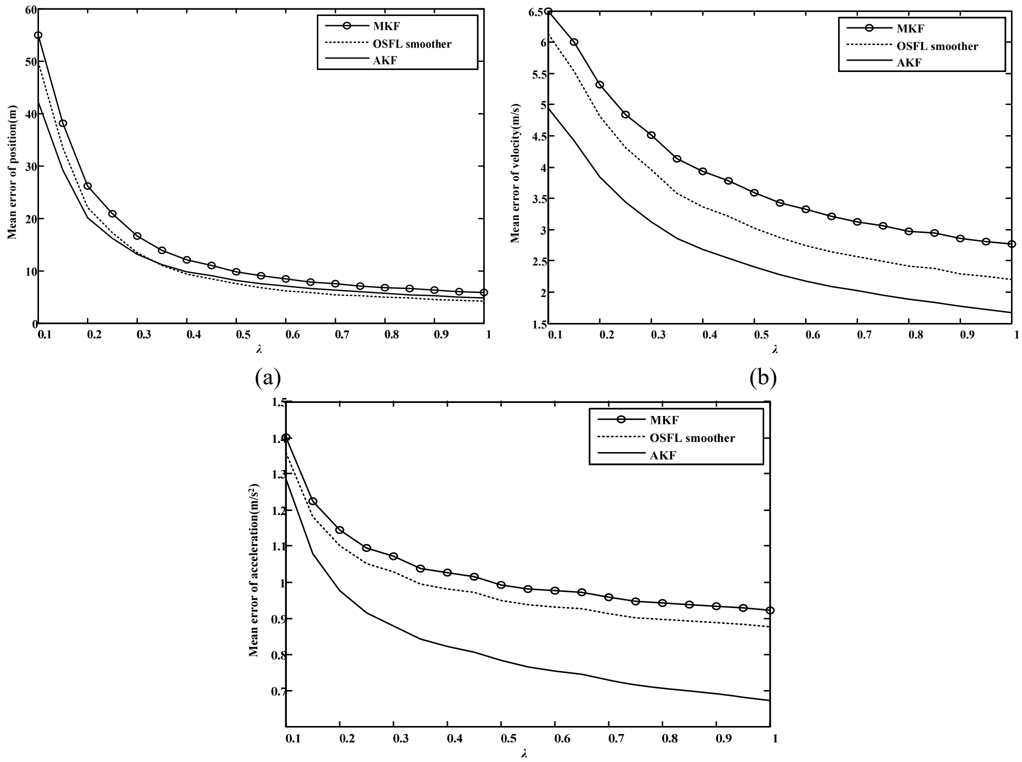
Fig. 3 shows the tracking performance of MKF, OSFL smoother and AKF with different λ. Obviously, the more serious the loss of measurements is, the worse the tracking performance becomes. The tracking results of AKF are better in velocity and acceleration than MKF and OSFL smoother, and when the measurements are missing seriously, the tracking results of AKF are also better in position than MKF and OSFL smoother.

From these examples, it is found that the precision of estimation state can be improved by directly using the next time's measurements when the measurements' arrival sequence $\{\gamma_k\}$ is known. However, it is a challenge issue to directly use more posterior measurements to improve the current estimated precision. Future work will explore this possibility.

5. CONCLUSIONS

Target tracking with intermittent measurements is investigated in this paper. Although there are a large number of references considering the state estimation problem with packet dropouts, most of those methods are unsuitable for target tracking due to the unstable state transmission matrix. Leveraged by posterior measurements, an amended

Kalman filter is proposed to improve the precision of the current estimated state. Both the deduction and proof of this new method are discussed in detail to distinguish it from the Kalman smoother. Simulation results further verify the effectiveness of the proposed method. In our future work, we will continue to optimize the target tracking performance by adopting the proposed method when $z_k \in R^m$ $(m > 1)$ and extend this method by using more posterior measurements.



(a) (b)

(c)

Fig. 3. Tracking results of amended Kalman filter with different λ; (a), (b) and (c) are mean error in position, velocity and acceleration, respectively.

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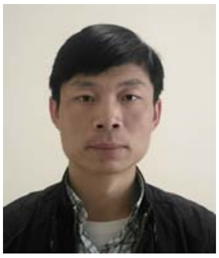

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