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Anti Anticipate Synchronization of Chaotic Complex Non-linear Structures With Secure Communication Applications

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Through this article we portray the anti anticipate synchronization (AAS) for pair similar non-linear chaotic complex structures. A plan is intended to realize AAS of chaotic conduct from these structures based on the Lyapunov function. To confirm the viability of the constructed scheme, the AAS of pair similar complex Lü structures is drawn as an example. Numerical calculations are determined to show the usefulness of the controller's theoretical explanations. A basic implementation of secure communication is accomplished depending on the results of AAS.

Keywords: chaotic, anti anticipate synchronization, error function, Lyapunov function, complex

1. INTRODUCTION

Following Lorenz figured out the first chaotic structure portrayed with real factors [1], so numerous chaotic structures including real factors were described [2-5]. Fowler *et al*. implemented the Lorenz complex form in 1982 as a generalisation of the Lorenz structure with real factors [6]. Mahmoud *et al*. developed and researched several unstable, complex structures during the last few years [7-13]. It is fully understood that there is a much broader application of unstable, complex non-linear structures. Concerning illustration during they include electromagnetic field amplitudes [14]. Also there is extra instance, during the chaotic complex structure is being utilized for correspondence, wherever the quantity of factors can be multiplied to maximize the complexity and reliability of the data sent [15].

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Recently a number of different synchronization regimes have been suggested for chaotic structures with complex factors [16-20]. If we describe the state vectors of pair chaotic complex structures as $\mathbf{x}(t)$ and $\mathbf{y}(t)$, the pair structures are attaining complete synchronization (CS) with error function $\lim_{t\longrightarrow\infty}\|\mathbf{y}(t)-\mathbf{x}(t)\|=0$ [16]. Anti synchronization (AS) is characterized when an error occurs $\lim_{t\longrightarrow\infty}\|\mathbf{y}(t)+\mathbf{x}(t)\|=0$. This designates that the state factors of pair chaotic complex interactive structures have the same amplitude but are different in the indication [17]. Lag synchronization (LS) infers that only the state factors of the pair chaotic complex structures are synchronised with positive lag time τ_0, *i.e.*, $\lim_{t\longrightarrow\infty}\|\mathbf{y}(t)-\mathbf{x}(t\ -\tau_0)\|=0$ [18, 19]. The anticipate synchronization is achieve if we define the error function as $\lim_{t\longrightarrow\infty}\|\mathbf{y}(t)-\mathbf{x}(t+\tau)\|=0$, where τ is the anticipate time [20].

Anticipated synchronization is a fascinating phenomenon that has recently been released that enables one to anticipate or predict the dynamics non-linear structures using a specific structure acronical master-slave synchronization [20, 21]. In theory, there may be an arbitrary broad forecast horizon acquired through a conveyor belt of anticipated synchronized exact slave reproductions of the actual structure [22]. The anticipating synchronization has a lot of applications in semiconductor lasers, chaotic laser diodes and cryptographic purposes. In this work we ask what will happen if the structure of slave anticipated actions of an opposite-shaped master structure. We will deal with a new kind of synchronizations that has never been studied in the literature. We may call this kind of synchronizations “AAS”.

Throughout this article, we wish to debate the probability of achieving AAS among pair similar chaotic complex structures that were not actually introduced in the literature. In AAS the aggregate of master structure with anticipate time $\mathbf{x}(t+\tau)$ and the slave structure $\mathbf{y}(t)$ is vanishing when $t\longrightarrow\infty$. The principle of AAS is not yet observed in real (or complex) chaotic (or hyperchaotic) structures, either. So it is necessary to adopt the concept of AAS when the master and slave structures are similar. The use of AAS contributes to the assumption that the receiving device requires the message to be sent before it is delivered, and in the analysis of secure communications that did not appear before.

The paper’s structure is as follows. Section 2 includes a definition of the non-linear structure of n-dimensional chaotic complexes structures. Section 3 sets out the nature of the conceptual scheme for achieving AAS in chaotic dynamic non-linear structures. In Section 4, as a contrast to Section 3, we are addressing AAS of pair similar chaotic Lü structures. Based the results of section 4 the a simple application of secure communication is displayed. Ultimately, Section 6 outlines the principal findings of our inquiries.

2. A CHAOTIC COMPLEX NON-LINEAR STRUCTURES

If it is deterministic, a complex dynamic structure is called chaotic, has an intermittent behavior and displays sensible dependence on the primary requirements. To ensure that the device is dissipative, the total of Lyapunov exponents requirement signifies negative. It has wide possible in non-linear orbits, safe information, lasers, neural nepairrks, biological structures and so on. This is because of the chaotic systems is unstable complex structures with high capacity characteristics, high protection and high efficiency. Work on chaotic dynamic, non-linear structures is therefore now extremely important [23, 24].

Find the chaotic complex non-linear structure as obeys:

$$\dot{\mathbf{x}} = \Phi\mathbf{x} + \mathbf{h}(\mathbf{x}), \tag{1}$$

where the state of complex vector is $\mathbf{x} = (x_1,\ x_2, ...,\ x_n)^T,\ \mathbf{x} = \mathbf{x}^r + j\mathbf{x}^i = (u_1 + ju_2,\ u_3 + ju_4, ...,\ u_{2n-1} + ju_{2n})^T,\ j^2 = -1,\ T$ transposing denotes. The real (or complex) matrix of structure parameters is $\Phi \in R^{n\times n}$ and $\mathbf{h} = (h_1,\ h_2, ...,\ h_n)^T$ is the form of the non-linear complex functions and the r and i superscripts stand for the real and imaginary parts of the state complex vector $\mathbf{x}$.

Throughout this paper, by designing a control scheme, we investigate the AAS phenomenon of pair similar form Eq. (1) structures with known parameters. We have numerically tested its validity.

Remark 1: Many complex chaotic structures can be represented by Eq. (1), like complex Lorenz, Chen and Lü structures [14]. To show the results of our scheme of pair similar type structures (1), we pick, as an example, the chaotic complex Lü structure that was implemented and studied in our research [14]. The equations of chaotic Lü structure with complex factors are fromed as:

$$\begin{aligned} \dot{x} &= \rho(y - x), \\ \dot{y} &= \nu y - xz, \\ \dot{z} &= 1/2\,(\bar{x}y + x\bar{y}) - \mu z, \end{aligned} \tag{2}$$

where $\mathbf{x} = (x_1, x_2, x_3)^T = (x, y, z)^T, \rho, \mu$ and ν are positive parameters, $x = u_1 + ju_2$, $y = u_3 + ju_4$ are complex functions, and $u_l\ (l = 1, ..., 4)$, $z = u_5$ are real functions. Dots are time-related derivatives and an overbar signifies complex conjugate factors.

The chaotic Lü complex structure is a specific autonomous 5-dimensional, flowing structure. In the case of $\rho = 40$, $\mu = 5$, $\nu = 22$ structure (2) has chaotic attractor, observe Fig. 1. For more complex attributes on this structure please see [14].

3. A SCHEME FOR CREATING A COMPLEX AAS CONTROLLER

Suppose pair similar chaotic non-linear structures with complex factor of the arrangement (1). We express the master structure by the index m as:

$$\dot{\mathbf{x}}_m = \dot{\mathbf{x}}_m^r + j\dot{\mathbf{x}}_m^i = \Phi\mathbf{x}_m + \mathbf{h}(\mathbf{x}_m). \tag{3}$$

The controlled slave structure with the index s is formed as:

$$\dot{\mathbf{x}}_s = \dot{\mathbf{x}}_s^r + j\dot{\mathbf{x}}_s^i = \Phi\mathbf{x}_s + \mathbf{h}(\mathbf{x}_s) + \mathbf{V}, \tag{4}$$

where $\mathbf{V} = (V_1, V_2, ..., V_n)^T = \mathbf{V}^r + j\mathbf{V}^i$, $\mathbf{V}^r = (v_1, v_3, ..., v_{2n-1})^T$, $\mathbf{V}^i = (v_2, v_4, ..., v_{2n})^T$.

Definition: Pair similar complex dynamical structures in a master-slave configuration can display AAS If a complex error vector δ exists, set such as:

$$\delta = \delta^r + j\delta^i = \lim_{t\longrightarrow\infty} \|\mathbf{x}_s(t) + \mathbf{x}_m(t\ + \tau)\| = \mathbf{0}, \tag{5}$$

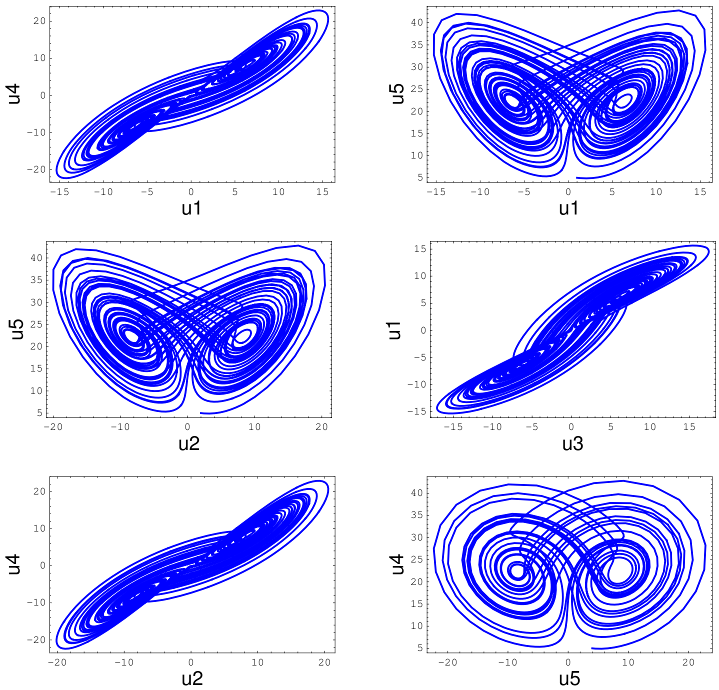

Fig. 1. Some attractors of the complx Lü system in some planes.

where $\delta = (\delta_1, \delta_2, ..., \delta_n)^T$, $\mathbf{x}_m(t)$ and $\mathbf{y}_s(t)$ are the region complex vectors of the master and slave structures, even if they are similar, $\delta^r = \lim_{t\longrightarrow\infty} \|\mathbf{x}_s^r(t)+\mathbf{x}_m^r(t+\tau)\| = 0$ *and* $\delta^i = \lim_{t\longrightarrow\infty} \left\|\mathbf{x}_s^i(t)+\mathbf{x}_m^i(t+\tau)\right\| = 0$, $\delta^r = (\delta_{u_1}, \delta_{u_3}, ..., \delta_{u_{2n-1}})^T$, $\delta^i = (\delta_{u_2}, \delta_{u_4}, ..., \delta_{u_{2n}})^T$, and τ is the positive anitcipate time.

Remark 2: If we set $\tau = 0$ in Eq. (5) we describe AS among structures (3) and (4) [17].

Remark 3: When $\delta = \lim_{t\longrightarrow\infty} \|\mathbf{y}_s(t) - \mathbf{x}_m(t+\tau)\|$ and $\tau = 0$, we get CS of structures (3) and (4) [16], while if $\tau < 0$ we concern LS of the same structures [18].

Theorem 1: If non-linear controller is outlined as:

$$\begin{aligned}\mathbf{V} =& \mathbf{V}^r + j\mathbf{V}^i = -\Phi\mathbf{x}_s(t) - \mathbf{h}(\mathbf{x}_s(t)) - \Phi\mathbf{x}_m(t+\tau) - \mathbf{h}(\mathbf{x}_m(t+\tau)) - k\delta \\ =& -\Phi\mathbf{x}_s^r(t) - \mathbf{h}^r(\mathbf{x}_s(t)) - \Phi\mathbf{x}_m^r(t+\tau) - \mathbf{h}^r(\mathbf{x}_m(t+\tau)) - k\delta^r \\ &+ j[-\Phi\mathbf{x}_s^i(t) - \mathbf{h}^i(\mathbf{x}_s(t)) - \Phi\mathbf{x}_m^i(t+\tau) - \mathbf{h}^i(\mathbf{x}_m(t+\tau)) - k\delta^i],\end{aligned} \tag{6}$$

then the slave structure (4) anti-anticipate synchronize the master structure (3), where $k > 0$.

Proof: By using the meaning of AAS:

$$\delta = \delta^r + j\delta^i = \mathbf{x}_s(t) + \mathbf{x}_m(t+\tau). \tag{7}$$

So,

$$\begin{aligned}\dot{\delta}=&\dot{\delta}^r+j\dot{\delta}^i=\dot{\mathbf{x}}_s(t)+\dot{\mathbf{x}}_m(t+\tau)\\&=\dot{\mathbf{x}}_s^r(t)+\dot{\mathbf{x}}_m^r(t+\tau)+j[\dot{\mathbf{x}}_s^i(t)+\dot{\mathbf{x}}_m^i(t+\tau)],\end{aligned}\tag{8}$$

and by utilizing complex structures (3) and (4), we gain the error dynamical structure as:

$$\begin{aligned}\dot{\delta}=&\dot{\delta}^r+j\dot{\delta}^i=\Phi\mathbf{x}_s^r(t)+\mathbf{h}^r(\mathbf{x}_s(t))+\Phi\mathbf{x}_m^r(t+\tau)+\mathbf{h}^r(\mathbf{x}_m(t+\tau))+\mathbf{L}^r\\&+j[\Phi\mathbf{x}_s^i(t)+\mathbf{h}^i(\mathbf{x}_s(t))+\Phi\mathbf{x}_m^i(t+\tau)+\mathbf{h}^i(\mathbf{x}_m(t+\tau))]+\mathbf{L}^i.\end{aligned}\tag{9}$$

By dividing the real and the imaginary components in Eq. (9), the structure of the error is composed as:

$$\begin{cases}\dot{\delta}^r=\Phi\mathbf{x}_s^r(t)+\mathbf{h}^r(\mathbf{x}_s(t))+\Phi\mathbf{x}_m^r(t+\tau)+\mathbf{h}^r(\mathbf{x}_m(t+\tau))+\mathbf{L}^r,\\ \dot{\delta}^i=\Phi\mathbf{x}_s^i(t)+\mathbf{h}^i(\mathbf{x}_s(t))+\Phi\mathbf{x}_m^i(t+\tau)+\mathbf{h}^i(\mathbf{x}_m(t+\tau))]+\mathbf{L}^i.\end{cases}\tag{10}$$

For this structure, we can now describe the function of Lyapunov by the subsequent positive clear amount:

$$\begin{aligned}L(t)=&\tfrac{1}{2}[(\delta^r)^T\delta^r+(\delta^i)^T\delta^i],\\=&\tfrac{1}{2}\left(\sum_{l=1}^{n}\delta_{u_{2l-1}}^2+\sum_{l=1}^{n}\delta_{u_{2l}}^2\right).\end{aligned}\tag{11}$$

Keep in mind that the complete time speculative of $L(t)$ across the path of the error structure (10) is as obeys:

$$\begin{aligned}\dot{L}(t)=&(\dot{\delta}^r)^T\delta^r+(\dot{\delta}^i)^T\delta^i,\\=&(\Phi\mathbf{x}_s^r(t)+\mathbf{h}^r(\mathbf{x}_s(t))+\Phi\mathbf{x}_m^r(t+\tau)+\mathbf{h}^r(\mathbf{x}_m(t+\tau))+\mathbf{V}^r)^T\delta^r\\&+(\Phi\mathbf{x}_s^i(t)+\mathbf{h}^i(\mathbf{x}_s(t))+\Phi\mathbf{x}_m^i(t+\tau)+\mathbf{h}^i(\mathbf{x}_m(t+\tau))+\mathbf{V}^i)^T\delta^i.\end{aligned}\tag{12}$$

Through replacing of (6) about $\mathbf{V}^r$, $\mathbf{V}^i$ in Eq. (12) we get:

$$\begin{aligned}\dot{L}(t)=&-k[(\delta^r)^T\delta^r+(\delta^i)^T\delta^i]\\=&-k\left(\sum_{l=1}^{n}\delta_{u_{2l-1}}^2+\sum_{l=1}^{n}\delta_{u_{2l}}^2\right).\end{aligned}\tag{13}$$

Considering that $L(t)$ is a sure defined function and its derivative is negatively defined, the error structure (9) is stable according to the famous Lyapunov hypothesis, which indicates that $\delta_{u_{2l}}$ and $\delta_{u_{2l-1}}$ perform to zero as $t\to\infty$, $l=1,2,...,n.$ As a result, the Slave Structure and Master Structure states will be globally anti-anticipated synchronized with anticipatory time. This is completing the proof.

Certainly, our scheme is highlighted by employing it in Section 4 for a pair of similar chaotic Lü structures with complex factors.

4. AN EXAMPLE OF AN AAS WITH PAIR SIMILAR COMPLEX CHAOTIC STRUCTURES

Now let's view at the AAS pair of similar chaotic complex Lü structures as an instance for Section 3. The master and the slave structures are thus described as reads, individually:

$$\begin{aligned}\dot{x}_m =&\rho\left(y_m - x_m\right),\\ \dot{y}_m =&\nu y_m - x_m z_m,\\ \dot{z}_m =&1/2\left(\bar{x}_m y_m + x_m \bar{y}_m\right) - \mu z_m,\end{aligned} \tag{14}$$

and

$$\begin{aligned}\dot{x_s} =&\rho\left(y_s - x_s\right) + V_1,\\ \dot{y_s} =&\nu y_s - x_s z_s + V_2,\\ \dot{z_s} =&1/2\left(\bar{x}_s y_s + x_s \bar{y}_s\right) - \mu z_s + V_3,\end{aligned} \tag{15}$$

where $x_m = u_{1m} + ju_{2m}$, $y_m = u_{3m} + ju_{4m}$, $z_m = u_{5m}$, $x_s = u_{1s} + ju_{2s}$, $y_s = u_{3s} + ju_{4s}$ and $z_s = u_{5s}$, "uber bar" refers to complex conjugation, $V_1 = v_1 + jv_2$, $V_2 = v_3 + jv_4$ and $V_3 = v_5$ are the controller, which to decide.

The complicated structures (14) and (15) could be composed as:

$$\begin{pmatrix}\dot{x}_m\\ \dot{y}_m\\ \dot{z}_m\end{pmatrix} = \begin{pmatrix}-\rho & \rho & 0\\ 0 & \nu & 0\\ 0 & 0 & -\mu\end{pmatrix}\begin{pmatrix}x_m\\ y_m\\ z_m\end{pmatrix} + \begin{pmatrix}0\\ -x_m z_m\\ 1/2\left(\bar{x}_m y_m + x_m \bar{y}_m\right)\end{pmatrix}, \tag{16}$$

and

$$\begin{pmatrix}\dot{x}_s\\ \dot{y}_s\\ \dot{z}_s\end{pmatrix} = \begin{pmatrix}-\rho & \rho & 0\\ 0 & \nu & 0\\ 0 & 0 & -\mu\end{pmatrix}\begin{pmatrix}x_s\\ y_s\\ z_s\end{pmatrix} + \begin{pmatrix}0\\ -x_s z_s\\ 1/2\left(\bar{x}_s y_s + x_s \bar{y}_s\right)\end{pmatrix} + \begin{pmatrix}L_1\\ L_2\\ L_3\end{pmatrix}. \tag{17}$$

Therefore, when you compare system structures (16) and (17) amidst the sort of structures (3) and (4) sequentially, we observe:

$$\begin{aligned}\Phi=&\begin{pmatrix}-\rho & \rho & 0\\ 0 & \nu & 0\\ 0 & 0 & -\mu\end{pmatrix},\\ \mathbf{h}(\mathbf{x}_m)=&\begin{pmatrix}0\\ -x_m z_m\\ 1/2\left(\bar{x}_m y_m + x_m \bar{y}_m\right)\end{pmatrix},\ \mathbf{h}(\mathbf{y}_s) = \begin{pmatrix}0\\ -x_s z_s\\ 1/2\left(\bar{x}_s y_s + x_s \bar{y}_s\right)\end{pmatrix}.\end{aligned}$$

The controller is built as: (According to Theorem 1)

$$\mathbf{V} =-\Phi\mathbf{x}_s(t) - \mathbf{h}(\mathbf{x}_s(t)) - \Phi\mathbf{x}_m(t\ +\tau) - \mathbf{h}(\mathbf{x}_m(t\ +\tau)) - k\delta,$$

$$\begin{pmatrix}V_1\\ V_2\\ V_3\end{pmatrix} = \begin{pmatrix}-\rho\left(y_s(t) - x_s(t)\right) - w_s(t) - \rho\left(y_m(t\ +\tau) - x_m(t\ +\tau)\right) - k\delta_1\\ -\nu y_s(t) + \phi_1 - w_s(t) - \nu y_m(t\ +\tau) + \phi_2 - k\delta_2\\ -\phi_3 + \mu z_s(t) - \phi_4 + \mu z_m(t\ +\tau) - k\delta_3\end{pmatrix}$$

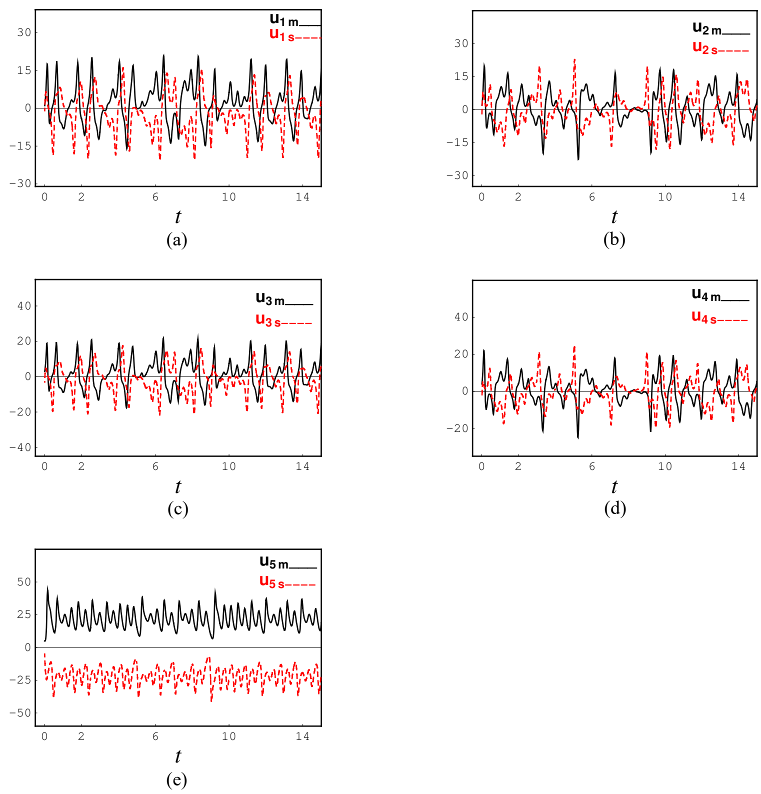

Fig. 2. The states of the master and slave systems when the AAS is achieving.

$$
= \begin{pmatrix} -\rho(u_{3s}(t) - u_{1s}(t) + u_{3m}(t+\tau) - u_{1m}(t+\tau)) - k\delta_{u_1} \\ -\nu(u_{3s}(t) + u_{3m}(t+\tau)) + \phi_5 - k\delta_{u_3} \\ -\phi_7 + \mu(u_{5s}(t) + u_{5m}(t+\tau)) - k\delta_{u_5} \end{pmatrix} + j \begin{pmatrix} -\rho(u_{4s}(t) - u_{2s}(t) + u_{4m}(t+\tau) - u_{2m}(t+\tau)) - k\delta_{u_2} \\ -\nu(u_{4s}(t) + u_{4m}(t)) + \phi_6 - k\delta_{u_4} \\ 0 \end{pmatrix} \tag{18}
$$

where $\phi_1 = x_s(t)z_s(t)$, $\phi_2 = x_m(t+\tau)z_m(t+\tau)$, $\phi_3 = \frac{1}{2}(\bar{x}_s(t)y_s(t) + x_s(t)\bar{y}_s(t))$, $\phi_4 = \frac{1}{2}(\bar{x}_m(t+\tau)y_m(t+\tau) + x_m(t+\tau)\bar{y}_m(t+\tau))$, $\phi_5 = u_{1s}(t)u_{5s}(t) + u_{1m}(t+\tau)u_{5m}(t+\tau)$, $\phi_6 = u_{2s}(t)u_{5s}(t) + u_{2m}(t+\tau)u_{5m}(t+\tau)$, $\phi_7 = u_{1s}(t)u_{3s}(t) + u_{1m}(t+\tau)u_{3m}(t+\tau) + u_{2s}(t)u_{4s}(t) + u_{2m}(t+\tau)u_{4m}(t+\tau)$ and $\delta_{u_l} = u_{lm}(t+\tau) - u_{ls}(t)$, $l = 1,2,3,4,5$.

To check and clarify the usefulness of the suggested scheme, we present the events of the AAS simulation connecting pairs of similar chaotic complex structures in Lü models (14) and (15). We solve structures (14) and (15) with the controller (18) numerically. Parameters are specified as follows $\rho = 42$, $\mu = 6$, $\nu = 25$. The first position of the state vector of the master structure, the first condition of the state vector of the slave structure, the positive anticipate time and k are apprehended as $(x_m(0), y_m(0), z_m(0))^T = (1+2j, 3+4j, 5)^T$, $(x_s(0), y_s(0), z_s(0))^T = (6+8j, 3+4j, 8)^T$, $\tau = 0.2$ and $k = 30$. Issues are presented in Figs. 2-4.

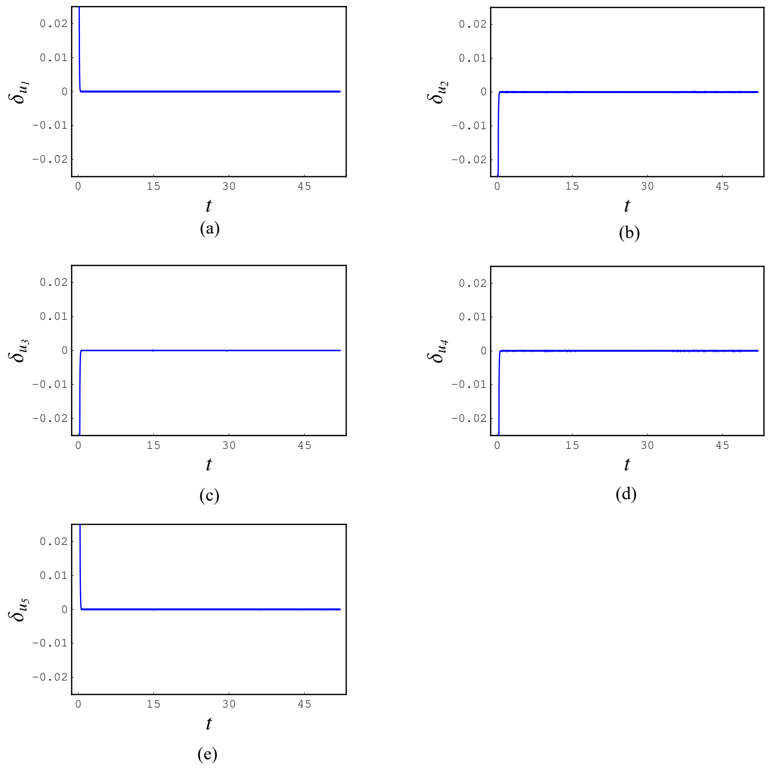

Fig. 3. AAS errors: (a) δ_{u_1} alter t; (b) δ_{u_2} alter t; (c) δ_{u_3} alter t; (d) δ_{u_4} alter t; (e)δ_{u_5} alter t.

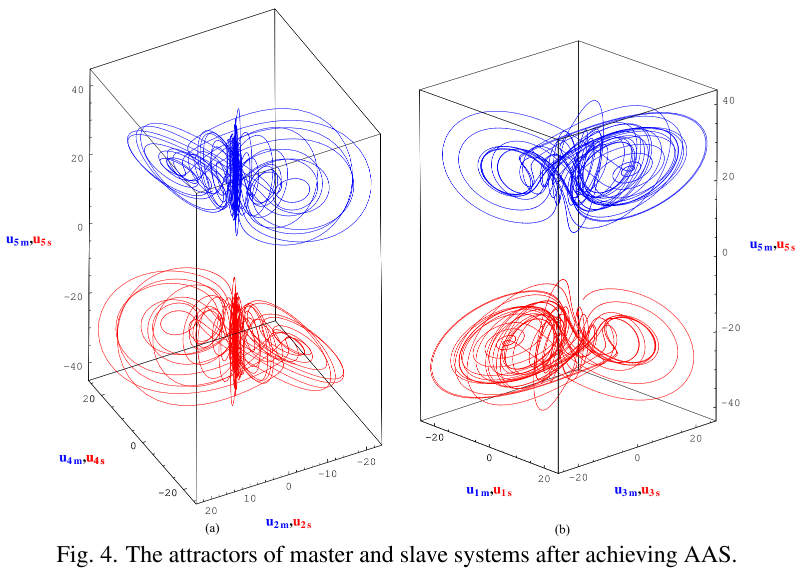

Fig. 4. The attractors of master and slave systems after achieving AAS.

The implementations of Eqs. (14) and (15) are depicted under various original situations in Fig. 2 and demonstrate that AAS is actually done in time t after a very limited interval. Fig. 3 demonstrates that, as predicted from the above theoretical criteria, the AAS errors reduce to zero. From Fig. 4, it is obvious the attractors in (u_2, u_4, u_5) location in Fig. 4 (a) and (u_1, u_3, u_5) location in Fig. 4 (b) of master structure (14) and slave structure (15) have the equivalent scale but configuration reverse with time anticipate $\tau = 0.2$.

5. SECURE COMMUNICATION

The chaotic signs produced by chaotic structures have certain characteristics, like randomness, complexity and sensitive dependence on initial forms, which makes them especially fitting concerning secure communications. Chaotic synchronized and its employees have become a difficult part of non-linear fields, specifical applications for secure communication [25-30]. Within chaotic structures, stable communication transmits a message from the sender to the receiver. As such, the message is immunized or inserted into chaotic structures, distributed, and then detected and recouped by the receiver. Various types of secure communication strategies have been proposed, such as chaotic veiling [26, 27]. The message we have to give it in chaotic veiling is attached to one of chaotic motion to mask it, then the signal is transmitted to the beneficiary.

AAS of complex chaotic structures, where a slave structure state factor anticipating master structure state factor behavior, is an encouraging form of synchronization as it provides outstanding protection in secure communication. We view the structure as the sender structure (14), and the structure as the beneficiary structure (15). For something, we choose the data movement as self-assured as $r(t) = 2\sin 4t \sin 2t$. Take $\hat{r}(t) = r(t) + w_{3m}(t+\tau)$ and suppose that $\hat{r}(t)$ is summed to the factor $w_{2m} \Longrightarrow \bar{r}(t) = \hat{r}(t) + w_{2m}(t+\tau) = r(t) + w_{3m}(t+\tau) + w_{2m}(t+\tau)$. Fig. 5 shows the numerical effects of uses for sec-

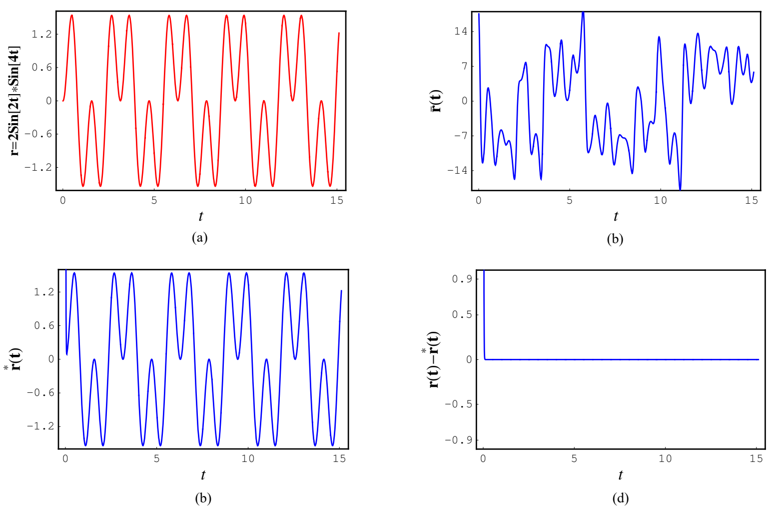

Fig. 5. Secure communication based on thne results of AAS.

ure communication with similar parameters and starting states of Fig. 2. The data signal $r(t)$ and the transmitted signal $\bar{r}(t)$ are pointed respectively in Figs. 5 (a) and (b). The recovered signal for information, whatever is shared by $r^*(t) = \bar{r}(t) + w_{3s}(t) + w_{2s}(t)$, is presented in Figs. 5 (c) and (d) show the mistaken movement between the first and the recouped information signal. From Fig. 5, it is hard to find that after a short transient the data signal $r(t)$ is retrieved precisely.

6. CONCLUSION

A description of the AAS for pair similar chaotic complex structures is given in this paper. A scheme is built to attain AAS of pair similar chaotic non-linear complex structures via Lyapunov functions. Within this scheme, we analytically defined the complex control functions that accomplished AAS. As an example, we employed this scheme for studying AAS of pair similar chaotic Lü structures. The numerical simulations of our examples check all the analytical tests. An excellent deal can be obtained as shown in Figs. 2-4. Drawing on the AAS findings, a secure communications application has been developed. The receiving structure can anticipate the transmitting structure's message via AAS for the first time. The use of this type of synchronization (AAS) leads to the fact that the receiving system expects the message to be sent before it is sent, and this has not appeared before in the study of secure communications.

The suggested control scheme is straightforward to implement and can be implemented in various other non-linear complexes (or real) chaotic or hyperchaotic structures.

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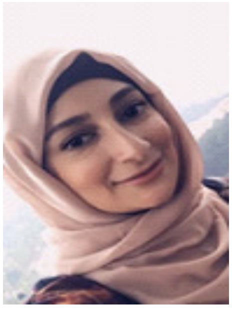

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