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Analytic Form of Expression for a Capacity in an Adaptive Spatial Modulation Systems

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Estimation of channel capacity plays a crucial role in analyzing the performance of spatial modulation (SM) systems. So far, no analytic form of expression has been derived for SM systems with more than two transmit antennas. In this paper, the analytic form of expression is derived for computing the channel capacity using the Huffman mapping scheme for an adaptive SM system with four transmit antennas. Gauss hypergeometric function is exploited to derive the analytic form of expression for a 4×1 system. Analytical results show the validation of the derived expression.

Keywords: spatial modulation, adaptive spatial modulation, capacity, Gauss hypergeometric function, analytic form, Huffman mapping

1. INTRODUCTION

Recently, spatial modulation (SM) is an emerging wireless communication concept in multiple-input multiple-output (MIMO) systems. In SM scheme [1], the information symbols can be transmitted in two groups. The first group is mapped to the signal symbol, and the other group is mapped to the spatial symbol (antenna index). In SM only one antenna is enabled at any instant, and hence only one set of radio frequency (RF) front end is necessary at the transmitter, which reduces the hardware complexity and improves system energy efficiency [2, 3].

The SM capacity behaviors are broadly studied when the channel state information (CSI) is not known at the transmitter. Compared to conventional MIMO systems [4], the SM capacity of a Rayleigh fading channel is improved for a complex Gaussian model when the number of transmit antennas are more than two. In [5], the capacity of SM upper bound is derived in closed form, and it is shown that the upper bound is achievable with channel distribution information at the transmitter. Later in [6], various forms of SM systems are included to analyze the capacity of a tight lower and upper bound to achieve the maximum data rate. Then in [7], a precoding scheme is developed to improve the performance of SM capacity for finite alphabet input. Numerical results in [7] illustrates that the SM with finite alphabet input shows significant improvement over the SM with Gaussian inputs. Among all amplitude-phase modulation (APM) inputs, complex

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Gaussian input system gives better result at high SNR whereas constant modulus system achieves better SM capacity at low SNR region [8]. For the Gaussian mixture model, the SM capacity bounds (lower and upper) are proposed in [9] using Huffman coding.

The SM capacity for a complex Gaussian model is expressed in analytic form for 2×1 SM system by Shi *et al*. in [10]. With increasing the number of antennas at the transmitter, it is very hard to derive the bounds of the closed form expression for capacity [9]. Motivated by this, in this paper, we derive the analytic expression of a capacity for 4×1 SM system using the Huffman coding. Special function such as Gauss hypergeometric function is used to derive the analytic form of expression for a Gaussian distributed signal.

Gauss Hypergeometric Function: Gauss hypergeometric function is a special function which is represented using hypergeometric series. The integral form of hypergeometric function in [11, 12] is represented using gamma function as

$$ {}_2F_1(a,b;c;z) = \frac{\Gamma(c)}{\Gamma(b)\Gamma(c-b)} \int_0^1 t^{b-1}(1-t)^{c-b-1}(1-zt)^{-a}dt; \quad \mathbb{R}(c) > \mathbb{R}(a) > 0. \tag{1} $$

2. SYSTEM MODEL

Consider a generic multiple-input single-output (MISO) system having N transmit antennas and one receive antenna, denoted by $N \times 1$ system. During each transmission at the transmitter $\log MN$ data bits will be transmitted. The first group activates one antenna out of N antennas and it contains $\log N$ bits. The second group having $\log M$ bits and modulates a signal constellation symbol from M-QAM (Quadrature Amplitude Modulation) constellation diagram. Let i $(i = 1, 2, \cdots, N)$ be the index of the activated antenna which is determined by the spatial symbol $\theta_{\text{ch}} \in h_i$, θ be the signal symbol corresponding to the activated antenna with power σ_Θ^2, and ϕ be the received symbol and they are related by

$$ \phi = h\theta + v \tag{2} $$

where, $h = [h_1, h_2, \cdots, h_N]^T$ denotes channel coefficient between the transmitter and receiver denoted by $N \times 1$ vector, and $v \sim \mathcal{CN}(0, \sigma_V^2)$ indicates the received complex additive white Gaussian noise (AWGN) with zero mean and one-sided spectral density of $\sigma_V^2 = 1$.

The SM input system consists of two independent spaces: signal domain space denoted by Θ and spatial domain space denoted by Θ_{ch}. The output of SM space is denoted as Φ. Thus, the SM mutual information between the signal spaces can be expressed as [13, 14]

$$ I(\Theta;\Phi) = I(\Theta;\Phi|\Theta_{\text{ch}}) + I(\Theta_{\text{ch}};\Phi), \tag{3} $$

where $I(\Theta;\Phi|\Theta_{\text{ch}})$ represents the mutual information between input symbol space and output symbol space, and $I(\Theta_{\text{ch}};\Phi)$ represents the mutual information between spatial symbol space and output symbol space.

Due to the randomness of the spatial symbol, each channel will have different probabilities for transmitting the signal symbols. In case of adaptive SM, the probability of selecting the ith transmit antenna is denoted by $p_i = p(\theta_{\text{ch}} = i)$, where $i = 1, 2, \cdots, N$ and satisfies $\sum_{i=1}^{N} p(\theta_{\text{ch}} = i) = 1$, whereas in conventional spatial modulation the probability distribution is uniform. The instantaneous capacity of SM [10] is given by

$$C_s = \max_{p(\theta)} I(\Theta;\Phi|\Theta_{\text{ch}}) = \sum_{i=1}^{N} p(\theta_{\text{ch}} = i) \log \left[\frac{\sigma_i^2}{\sigma_V^2} \right] \tag{4}$$

where $\log[\cdot]$ denotes the base-2 logarithm. Same notation is used throughout the paper.

In order to achieve the capacity, the signal symbol θ assumed to be an independent and identically distributed zero mean complex Gaussian random variable with the probability distribution function (PDF) of

$$p(\theta) = \frac{1}{\pi\sigma_\Theta^2} \exp\left[\frac{-|\theta|^2}{\sigma_\Theta^2} \right]. \tag{5}$$

When the ith channel is selected to transmit the signal symbol, the received signal will satisfy complex Gaussian distribution with PDF given by

$$p(\phi|\theta_{\text{ch}} = i) = \frac{1}{\pi\sigma_i^2} \exp\left[\frac{-|\phi|^2}{\sigma_i^2} \right] \tag{6}$$

where $\sigma_i^2 = |h_i|^2\sigma_\Theta^2 + \sigma_V^2$ $(i = 1, 2, ..., N)$ indicates the variance of the signal received from the ith channel and $|\cdot|$ denotes the modulus operator. Since, every channel is selected with different probability, the PDF of the received signal can be written as

$$p(\phi) = \sum_{i=1}^{N} p(\theta_{\text{ch}} = i)\ p(\phi|\theta_{\text{ch}} = i). \tag{7}$$

Further, the capacity due to spatial symbol space [4] can be calculated from Eq. (3) as

$$C_{\text{ch}} = I(\Theta_{\text{ch}};\Phi) = \sum_{i=1}^{N} \int_0^\infty p(\theta_{\text{ch}} = i)\ p(\phi|\theta_{\text{ch}} = i) \log\left[\frac{p(\phi|\theta_{ch} = i)}{p(\phi)} \right] d\phi. \tag{8}$$

In conclusion, the instantaneous capacity of a SM can be obtained as

$$C_{\text{SM}} = C_{\text{s}} + C_{\text{ch}} \tag{9}$$

where C_{s} and C_{ch} are given in Eqs. (4) and (8), respectively.

3. ADAPTIVE SPATIAL MODULATION

In this section, Huffman coding based adaptive spatial modulation is developed using a probability vector, denoted by **p**. Huffman coding assigns the spatial symbols to the binary codes based on the frequency of occurrence of each symbol. The spatial information bits are aligned to its equivalent transmit antenna according to the designed Huffman code. Smaller codeword indicates that its corresponding antenna has more chance to be activated whereas longer codeword has less chance to be activated by its corresponding

Table 1. Huffman mapping for $\mathbf{p} = \left[\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{8}\right]$.

Bit pattern	Enabled Antenna	Probability ($\mathbf{p}$)
0	Tx-1 (i=1)	$\frac{1}{2}$
10	Tx-2 (i=2)	$\frac{1}{4}$
110	Tx-3 (i=3)	$\frac{1}{8}$
111	Tx-4 (i=4)	$\frac{1}{8}$

antenna. The codewords in the generated codebook are no way a prefix of any other codeword.

The main objective of adaptive spatial modulation is to find the probability vector $\mathbf{p}$ that improves the system performance. The optimization problem can be formulated as follows

$$\mathbf{P} := \begin{cases} \max\limits_{\mathbf{p}} & f(\mathbf{p}) \\ \text{s.t} & \mathbf{p} \in \mathbb{P} \end{cases} \tag{10}$$

where $\mathbf{p}$ is a probability vector and $f(\mathbf{p})$ is merely a performance metric of capacity. The probability of selecting the ith transmit antenna is denoted by p_i, where $i = 1, 2, \cdots, N$ and satisfies $\sum_{i=1}^{N} p_i = 1$.

The Huffman mapping scheme for 4 transmit antenna is as follows: If '0' is the first detected bit, tx-1 is chosen. If, on the other hand, the first detected bit is a '1,' then proceed to the second bit. If the second bit is a '0,' select tx-2; otherwise, if the second bit is a '1,' proceed to the third bit. If the third bit is '0,' choose tx-3; otherwise, choose tx-4 and these are tabulated in Table 1. The number of bits required to represent these transmit antennas is 1,2,3, and 3, and their activation probabilities are 50%, 25%, 12.5%, and 12.5%, respectively. These probability vectors are digital in nature, and every possible combination of probability vectors is revealed as codebook. Depending on the activation probability of each transmit antenna, one of the corresponding favourable probability vector $\mathbf{p}$ is selected from codebook $\mathbb{P}$. From [9], the proposed Huffman coding scheme for the feasible domain can be designed as

$$\mathbb{P} = \left\{ \mathbf{p} \mid \sum_{i=1}^{N} p_i = 1, \quad p_i \in \left\{0, 1, 2^{-1}, \cdots, 2^{-\zeta}\right\} \right\} \tag{11}$$

where $\zeta (0 \le \zeta \le N-1)$ is an integer which is associated with transmission codebook. Larger value of ζ will give better performance but it affects the feedback load to the receiver.

4. CAPACITY IN ANALYTIC FORM

In the literature, there is no analytic form of expression for estimating the capacity of a 4×1 adaptive SM system. In this paper, we derive the capacity expression based on Huffman mapping scheme.

By substituting Eqs. (6) and (7) in Eq. (8), we get the below expression

$$C_{\mathrm{ch}} = I(\Theta_{\mathrm{ch}};\Phi) = \sum_{i=1}^{N}\int_0^\infty p_i \frac{1}{\pi\sigma_i^2}\exp\left[\frac{-|\phi|^2}{\sigma_i^2}\right]\log\left[\frac{\frac{1}{\pi\sigma_i^2}\exp\left[\frac{-|\phi|^2}{\sigma_i^2}\right]}{\sum_{i=1}^{N} p_i \frac{1}{\pi\sigma_i^2}\exp\left[\frac{-|\phi|^2}{\sigma_i^2}\right]}\right]d\phi.$$

The terms are expanded based on their activation probabilities for $N=4$ as shown below

$$\begin{aligned} C_{\mathrm{ch}} = &\int_0^\infty \frac{p_1}{\pi\sigma_1^2}\exp\left[\frac{-|\phi|^2}{\sigma_1^2}\right]\log\left[\frac{\frac{1}{\pi\sigma_1^2}\exp\left[\frac{-|\phi|^2}{\sigma_1^2}\right]}{\frac{p_1}{\pi\sigma_1^2}\exp\left[\frac{-|\phi|^2}{\sigma_1^2}\right]+\frac{p_2}{\pi\sigma_2^2}\exp\left[\frac{-|\phi|^2}{\sigma_2^2}\right]+\sum_{i=3}^{4}\frac{p_3}{\pi\sigma_i^2}\exp\left[\frac{-|\phi|^2}{\sigma_i^2}\right]}\right]d\phi \\ &+\int_0^\infty \frac{p_2}{\pi\sigma_2^2}\exp\left[\frac{-|\phi|^2}{\sigma_2^2}\right]\log\left[\frac{\frac{1}{\pi\sigma_2^2}\exp\left[\frac{-|\phi|^2}{\sigma_2^2}\right]}{\frac{p_1}{\pi\sigma_1^2}\exp\left[\frac{-|\phi|^2}{\sigma_1^2}\right]+\frac{p_2}{\pi\sigma_2^2}\exp\left[\frac{-|\phi|^2}{\sigma_2^2}\right]+\sum_{i=3}^{4}\frac{p_3}{\pi\sigma_i^2}\exp\left[\frac{-|\phi|^2}{\sigma_i^2}\right]}\right]d\phi \\ &+\sum_{i=3}^{4}\int_0^\infty \frac{p_i}{\pi\sigma_i^2}\exp\left[\frac{-|\phi|^2}{\sigma_i^2}\right]\log\left[\frac{\frac{1}{\pi\sigma_i^2}\exp\left[\frac{-|\phi|^2}{\sigma_i^2}\right]}{\frac{p_1}{\pi\sigma_1^2}\exp\left[\frac{-|\phi|^2}{\sigma_1^2}\right]+\frac{p_2}{\pi\sigma_2^2}\exp\left[\frac{-|\phi|^2}{\sigma_2^2}\right]+\sum_{i=3}^{4}\frac{p_3}{\pi\sigma_i^2}\exp\left[\frac{-|\phi|^2}{\sigma_i^2}\right]}\right]d\phi \end{aligned} \tag{12}$$

Now convert the above equation to the polar co-ordinate system and then again convert to rectangular system then we get the expression as shown in below

$$\begin{aligned} C_{\mathrm{ch}} = &\underbrace{\int_0^\infty \frac{p_1}{\sigma_1^2}\exp\left[\frac{-r}{\sigma_1^2}\right]\log\left[\frac{\frac{1}{\pi\sigma_1^2}\exp\left[\frac{-r}{\sigma_1^2}\right]}{\frac{p_1}{\pi\sigma_1^2}\exp\left[\frac{-r}{\sigma_1^2}\right]+\frac{p_2}{\pi\sigma_2^2}\exp\left[\frac{-r}{\sigma_2^2}\right]+\sum_{i=3}^{4}\frac{p_3}{\pi\sigma_i^2}\exp\left[\frac{-r}{\sigma_i^2}\right]}\right]dr}_{\text{I}} \\ &\underbrace{+\int_0^\infty \frac{p_2}{\sigma_2^2}\exp\left[\frac{-r}{\sigma_2^2}\right]\log\left[\frac{\frac{1}{\pi\sigma_2^2}\exp\left[\frac{-r}{\sigma_2^2}\right]}{\frac{p_1}{\pi\sigma_1^2}\exp\left[\frac{-r}{\sigma_1^2}\right]+\frac{p_2}{\pi\sigma_2^2}\exp\left[\frac{-r}{\sigma_2^2}\right]+\sum_{i=3}^{4}\frac{p_3}{\pi\sigma_i^2}\exp\left[\frac{-r}{\sigma_i^2}\right]}\right]dr}_{\text{II}} \\ &\underbrace{+\sum_{i=3}^{4}\int_0^\infty \frac{p_i}{\sigma_i^2}\exp\left[\frac{-r}{\sigma_i^2}\right]\log\left[\frac{\frac{1}{\pi\sigma_i^2}\exp\left[\frac{-r}{\sigma_i^2}\right]}{\frac{p_1}{\pi\sigma_1^2}\exp\left[\frac{-r}{\sigma_1^2}\right]+\frac{p_2}{\pi\sigma_2^2}\exp\left[\frac{-r}{\sigma_2^2}\right]+\sum_{i=3}^{4}\frac{p_3}{\pi\sigma_i^2}\exp\left[\frac{-r}{\sigma_i^2}\right]}\right]dr}_{\text{III}} \end{aligned} \tag{13}$$

Consider the first term *i.e.* I ,

$$\int_0^\infty \frac{p_1}{\sigma_1^2}\exp\left[\frac{-r}{\sigma_1^2}\right]\log\left[\frac{\frac{1}{\pi\sigma_1^2}\exp\left[\frac{-r}{\sigma_1^2}\right]}{\frac{p_1}{\pi\sigma_1^2}\exp\left[\frac{-r}{\sigma_1^2}\right]+\frac{p_2}{\pi\sigma_2^2}\exp\left[\frac{-r}{\sigma_2^2}\right]+\sum_{i=3}^{4}\frac{p_3}{\pi\sigma_i^2}\exp\left[\frac{-r}{\sigma_i^2}\right]}\right]dr$$

$$
\begin{aligned}
&= p_1 \log \frac{1}{\pi\sigma_1^2} - p_1 \log \frac{p_1}{\pi\sigma_1^2} - p_1 \log \frac{p_2}{\pi\sigma_2^2} + \frac{p_1\sigma_1^2}{\sigma_2^2}\log e - p_1 \log \frac{1}{\pi}\left(\frac{p_3}{\sigma_3^2}+\frac{p_4}{\sigma_4^2}\right) \\
&+ \log e \int_0^\infty \frac{\left(\frac{p_3}{\sigma_3^4}\exp\left[\frac{-r}{\sigma_3^2}\right] + \frac{p_4}{\sigma_4^4}\exp\left[\frac{-r}{\sigma_4^2}\right]\right) p_1 \exp\left[\frac{-r}{\sigma_1^2}\right]}{\frac{p_3}{\sigma_3^2}\exp\left[\frac{-r}{\sigma_3^2}\right] + \frac{p_4}{\sigma_4^2}\exp\left[\frac{-r}{\sigma_4^2}\right]} dr
\end{aligned} \tag{14}
$$

Further simplification of the above Eq. (16) (see the Appendix for proof), we get

$$
= -p_1 \log \frac{p_1 p_2}{\pi\sigma_2^2} + \frac{p_1\sigma_1^2}{\sigma_2^2}\log e - p_1 \log \frac{1}{\pi}\left(\frac{p_3}{\sigma_3^2}+\frac{p_4}{\sigma_4^2}\right) + \frac{\alpha p_1 \sigma_1^2}{\sigma_{\max}^2}\log e \tag{15}
$$

Now consider the second term, *i.e.* II

$$
\begin{aligned}
&\int_0^\infty \frac{p_2}{\sigma_2^2}\exp\left[\frac{-r}{\sigma_2^2}\right] \log\left[\frac{\frac{1}{\pi\sigma_2^2}\exp\left[\frac{-r}{\sigma_2^2}\right]}{\frac{p_1}{\pi\sigma_1^2}\exp\left[\frac{-r}{\sigma_1^2}\right] + \frac{p_2}{\pi\sigma_2^2}\exp\left[\frac{-r}{\sigma_2^2}\right] + \sum_{i=3}^4 \frac{p_3}{\pi\sigma_i^2}\exp\left[\frac{-r}{\sigma_i^2}\right]}\right] dr \\
&= p_2 \log \frac{1}{\pi\sigma_2^2} - p_2 \log \frac{p_1}{\pi\sigma_1^2} - p_2 \log \frac{p_2}{\pi\sigma_2^2} + \frac{p_2\sigma_2^2}{\sigma_1^2}\log e - p_2 \log \frac{1}{\pi}\left(\frac{p_3}{\sigma_3^2}+\frac{p_4}{\sigma_4^2}\right) \\
&+ \log e \int_0^\infty \frac{\left(\frac{p_3}{\sigma_3^4}\exp\left[\frac{-r}{\sigma_3^2}\right] + \frac{p_4}{\sigma_4^4}\exp\left[\frac{-r}{\sigma_4^2}\right]\right) p_2 \exp\left[\frac{-r}{\sigma_2^2}\right]}{\frac{p_3}{\sigma_3^2}\exp\left[\frac{-r}{\sigma_3^2}\right] + \frac{p_4}{\sigma_4^2}\exp\left[\frac{-r}{\sigma_4^2}\right]} dr
\end{aligned} \tag{16}
$$

After simplifying the above Eq. (16), we get

$$
= -p_2 \log \frac{p_1 p_2}{\pi\sigma_1^2} + \frac{p_2\sigma_2^2}{\sigma_1^2}\log e - p_2 \log \frac{1}{\pi}\left(\frac{p_3}{\sigma_3^2}+\frac{p_4}{\sigma_4^2}\right) + \frac{\beta p_2 \sigma_2^2}{\sigma_{\max}^2}\log e \tag{17}
$$

Now consider the third term, *i.e.* III

$$
\sum_{i=3}^4 \int_0^\infty \frac{p_i}{\sigma_i^2}\exp\left[\frac{-r}{\sigma_i^2}\right] \log\left[\frac{\frac{1}{\pi\sigma_i^2}\exp\left[\frac{-r}{\sigma_i^2}\right]}{\frac{p_1}{\pi\sigma_1^2}\exp\left[\frac{-r}{\sigma_1^2}\right] + \frac{p_2}{\pi\sigma_2^2}\exp\left[\frac{-r}{\sigma_2^2}\right] + \sum_{i=3}^4 \frac{p_3}{\pi\sigma_i^2}\exp\left[\frac{-r}{\sigma_i^2}\right]}\right] dr
$$

$$
\begin{aligned}
&= p_3 \log \frac{1}{\pi\sigma_3^2} + p_4 \log \frac{1}{\pi\sigma_4^2} - (p_3+p_4)\log \frac{p_1}{\pi\sigma_1^2} + \frac{(p_3\sigma_3^2 + p_4\sigma_4^2)}{\sigma_1^2}\log e \\
&-(p_3+p_4)\log \frac{p_2}{\pi\sigma_2^2} - (p_3+p_4)\log \frac{1}{\pi}\left(\frac{p_3}{\sigma_3^2}+\frac{p_4}{\sigma_4^2}\right) + \frac{(p_3\sigma_3^2 + p_4\sigma_4^2)}{\sigma_2^2}\log e \\
&+ \log e \int_0^\infty \frac{\left(\frac{p_3}{\sigma_3^4}\exp\left[\frac{-r}{\sigma_3^2}\right] + \frac{p_4}{\sigma_4^4}\exp\left[\frac{-r}{\sigma_4^2}\right]\right)\left(p_3 \exp\left[\frac{-r}{\sigma_3^2}\right] + p_4 \exp\left[\frac{-r}{\sigma_4^2}\right]\right)}{\frac{p_3}{\sigma_3^2}\exp\left[\frac{-r}{\sigma_3^2}\right] + \frac{p_4}{\sigma_4^2}\exp\left[\frac{-r}{\sigma_4^2}\right]} dr
\end{aligned} \tag{18}
$$

Further simplify the above Eq. (18), we get

$$\begin{aligned}
&= p_3 \log \frac{1}{\pi\sigma_3^2} - (p_3+p_4)\log \frac{p_1 p_2}{\pi^2\sigma_1^2\sigma_2^2} + (p_3\sigma_3^2 + p_4\sigma_4^2)\left(\frac{1}{\sigma_1^2}+\frac{1}{\sigma_2^2}\right)\log e \\
&+ p_4 \log \frac{1}{\pi\sigma_4^2} - (p_3+p_4)\log \frac{1}{\pi}\left(\frac{p_3}{\sigma_3^2}+\frac{p_4}{\sigma_4^2}\right) + \frac{\gamma(\sigma_4^2-\sigma_3^2)^2}{\sigma_3^2\sigma_4^2}\log e
\end{aligned} \tag{19}$$

By combining Eqs. (15), (17) and (19), we have

$$\begin{aligned}
&- p_1 \log \frac{p_1 p_2}{\pi\sigma_2^2} + \frac{p_1\sigma_1^2}{\sigma_2^2}\log e - p_1 \log \frac{1}{\pi}\left(\frac{p_3}{\sigma_3^2}+\frac{p_4}{\sigma_4^2}\right) + \frac{\alpha p_1\sigma_1^2}{\sigma_{\max}^2}\log e - p_2 \log \frac{p_1p_2}{\pi\sigma_1^2} \\
&+ \frac{p_2\sigma_2^2}{\sigma_1^2}\log e - p_2 \log \frac{1}{\pi}\left(\frac{p_3}{\sigma_3^2}+\frac{p_4}{\sigma_4^2}\right) + \frac{\beta p_2\sigma_2^2}{\sigma_{\max}^2}\log e + p_3 \log \frac{1}{\pi\sigma_3^2} + p_4 \log \frac{1}{\pi\sigma_4^2} \\
&- (p_3+p_4)\log \frac{p_1p_2}{\pi^2\sigma_1^2\sigma_2^2} + (p_3\sigma_3^2+p_4\sigma_4^2)\left(\frac{1}{\sigma_1^2}+\frac{1}{\sigma_2^2}\right)\log e + \frac{\gamma(\sigma_4^2-\sigma_3^2)^2}{\sigma_3^2\sigma_4^2}\log e \\
&- (p_3+p_4)\log \frac{1}{\pi}\left(\frac{p_3}{\sigma_3^2}+\frac{p_4}{\sigma_4^2}\right).
\end{aligned}$$

Now further reduce the above equation, we have

$$\begin{aligned}
C_{\mathrm{ch}} &= \frac{\alpha p_1\sigma_1^2}{\sigma_{\max}^2}\log e + \frac{\beta p_2\sigma_2^2}{\sigma_{\max}^2}\log e + \frac{\gamma(\sigma_4^2-\sigma_3^2)^2}{\sigma_3^2\sigma_4^2} - p_1 \log \frac{p_1p_2}{\pi\sigma_2^2} - p_2 \log \frac{p_1p_2}{\pi\sigma_1^2} \\
&+ p_4 \log \frac{1}{\pi\sigma_4^2} + \frac{p_1\sigma_1^4 + p_2\sigma_2^4 + p_3\sigma_1^2\sigma_3^2 + p_4\sigma_1^2\sigma_4^2 + p_3\sigma_2^2\sigma_3^2 + p_4\sigma_2^2\sigma_4^2}{\sigma_1^2\sigma_2^2}\log e \\
&+ p_3 \log \frac{1}{\pi\sigma_3^2} - (p_3+p_4)\log \frac{p_1p_2}{\pi^2\sigma_1^2\sigma_2^2} - \log \frac{1}{\pi}\left(\frac{p_3}{\sigma_3^2}+\frac{p_4}{\sigma_4^2}\right)
\end{aligned} \tag{20}$$

where α, β and γ are given by

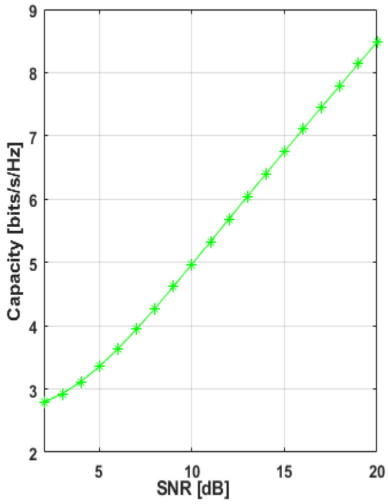
$$\alpha = \begin{cases} {}_2F_1\left(1, \frac{\sigma_{\max}^2\sigma_{\min}^2}{\sigma_1^2(\sigma_{\max}^2-\sigma_{\min}^2)}; 1 + \frac{\sigma_{\max}^2\sigma_{\min}^2}{\sigma_1^2(\sigma_{\max}^2-\sigma_{\min}^2)}; \frac{-p_4}{p_3}\frac{\sigma_{\max}^2}{\sigma_{\min}^2}\right) & ; \sigma_3^2 > \sigma_4^2 \\ {}_2F_1\left(1, \frac{\sigma_{\max}^2\sigma_{\min}^2}{\sigma_1^2(\sigma_{\max}^2-\sigma_{\min}^2)}; 1 + \frac{\sigma_{\max}^2\sigma_{\min}^2}{\sigma_1^2(\sigma_{\max}^2-\sigma_{\min}^2)}; \frac{-p_3}{p_4}\frac{\sigma_{\max}^2}{\sigma_{\min}^2}\right) & ; \sigma_3^2 < \sigma_4^2 \end{cases}$$

$$\beta = \begin{cases} {}_2F_1\left(1, \frac{\sigma_{\max}^2\sigma_{\min}^2}{\sigma_2^2(\sigma_{\max}^2-\sigma_{\min}^2)}; 1 + \frac{\sigma_{\max}^2\sigma_{\min}^2}{\sigma_2^2(\sigma_{\max}^2-\sigma_{\min}^2)}; \frac{-p_4}{p_3}\frac{\sigma_{\max}^2}{\sigma_{\min}^2}\right) & ; \sigma_3^2 > \sigma_4^2 \\ {}_2F_1\left(1, \frac{\sigma_{\max}^2\sigma_{\min}^2}{\sigma_2^2(\sigma_{\max}^2-\sigma_{\min}^2)}; 1 + \frac{\sigma_{\max}^2\sigma_{\min}^2}{\sigma_2^2(\sigma_{\max}^2-\sigma_{\min}^2)}; \frac{-p_3}{p_4}\frac{\sigma_{\max}^2}{\sigma_{\min}^2}\right) & ; \sigma_3^2 < \sigma_4^2 \end{cases}$$

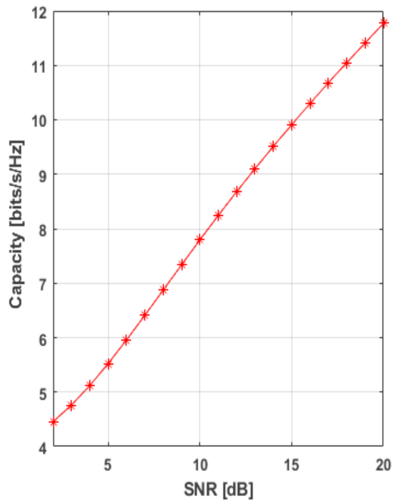
$$\gamma = \begin{cases} p_4\ {}_2F_1\left(1, \frac{\sigma_{\max}^2}{\sigma_{\max}^2-\sigma_{\min}^2}; 1 + \frac{\sigma_{\max}^2}{\sigma_{\max}^2-\sigma_{\min}^2}; \frac{-p_4}{p_3}\frac{\sigma_{\max}^2}{\sigma_{\min}^2}\right) & ; \sigma_3^2 > \sigma_4^2 \\ p_3\ {}_2F_1\left(1, \frac{\sigma_{\max}^2}{\sigma_{\max}^2-\sigma_{\min}^2}; 1 + \frac{\sigma_{\max}^2}{\sigma_{\max}^2-\sigma_{\min}^2}; \frac{-p_3}{p_4}\frac{\sigma_{\max}^2}{\sigma_{\min}^2}\right) & ; \sigma_3^2 < \sigma_4^2 \end{cases}$$

where, $\sigma_{\max}^2 = \max\{\sigma_3^2, \sigma_4^2\}$ and $\sigma_{\min}^2 = \min\{\sigma_3^2, \sigma_4^2\}$.
From Eq. (4), the capacity due to signal symbol space for $N = 4$ is given by

$$C_{\mathrm{s}} = \sum_{i=1}^{4} p_i \log[\sigma_i^2] - \log[\sigma_V^2] \tag{21}$$



(a) $p_1 = \left[\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}\right]$



(b) $p_2 = \left[\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{8}\right]$

Fig. 1. Capacity comparison when $N = 4$.

By substituting Eqs. (20) and (21) in Eq. (9), we get the analytic form of expression for the capacity

$$C_{SM} = \frac{\alpha p_1 \sigma_1^2}{\sigma_{\max}^2} \log e + \frac{\beta p_2 \sigma_2^2}{\sigma_{\max}^2} \log e + \frac{\gamma(\sigma_4^2 - \sigma_3^2)^2}{\sigma_3^2 \sigma_4^2} - \log \frac{p_1 p_2}{\pi \sigma_1^2 \sigma_2^2} - \log \frac{1}{\pi}\left(\frac{p_3}{\sigma_3^2} + \frac{p_4}{\sigma_4^2}\right)$$
$$+ \frac{p_1 \sigma_1^4 + p_2 \sigma_2^4 + p_3 \sigma_1^2 \sigma_3^2 + p_4 \sigma_1^2 \sigma_4^2 + p_3 \sigma_2^2 \sigma_3^2 + p_4 \sigma_2^2 \sigma_4^2}{\sigma_1^2 \sigma_2^2} \log e - \log[\sigma_V^2]$$

5. RESULTS

In Fig. 1, we plot the capacity curves for conventional spatial modulation and Huffman coding based adaptive spatial modulation for 4×1 system model. In Huffman coding based adaptive spatial modulation one of the possible probability vector $p_2 = \left[\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{8}\right]$ is considered and compared with conventional spatial modulation $p_1 = \left[\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}\right]$. This optimum probability vector is not showing the good performance at all SNR. Actually each antenna activation probability vector gives random amount of information in two domains *i.e.* signal and spatial domain respectively. Adaptive spatial modulation is nothing but considering all possible combinations of probability vectors for 4×1 system to improve the capacity at each SNR. In Fig. 2, we compare the capacity curves of 2×1 conventional spatial modulation, 4×1 conventional spatial modulation and 4×1 adaptive spatial modulation with Huffman mapping. Huffman coding based adaptive spatial modulation gives superior performance compared to conventional spatial modulation.

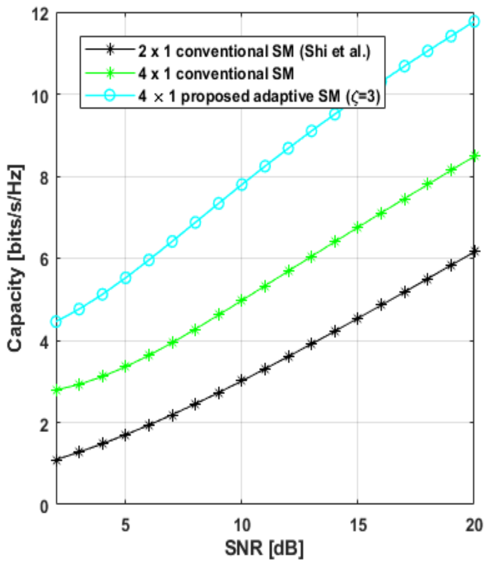


Fig. 2. Capacity comparison of various systems.

6. CONCLUSION

In this paper, conventional spatial modulation and adaptive spatial modulation systems for a Gaussian model are analyzed. Then, the analytic form of expression for adaptive spatial modulation is derived using Gauss Hypergeometric series for 4×1 system and the results are compared with 2×1 SM system. The analytic comparison results show that the adaptive spatial modulation based on the analytic form of expression derived in this paper gives better performance than the conventional spatial modulation.

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APPENDIX

Eq. (13) is

$$p_1 \log \frac{1}{\pi\sigma_1^2} - p_1 \log \frac{p_1}{\pi\sigma_1^2} - p_1 \log \frac{p_2}{\pi\sigma_2^2} + \frac{p_1\sigma_1^2}{\sigma_2^2} \log e - p_1 \log \frac{1}{\pi}\left(\frac{p_3}{\sigma_3^2} + \frac{p_4}{\sigma_4^2}\right)$$
$$+ \log e \int_0^\infty \frac{\left(\frac{p_3}{\sigma_3^4} \exp\left[\frac{-r}{\sigma_3^2}\right] + \frac{p_4}{\sigma_4^4} \exp\left[\frac{-r}{\sigma_4^2}\right]\right) p_1 \exp\left[\frac{-r}{\sigma_1^2}\right]}{\frac{p_3}{\sigma_3^2} \exp\left[\frac{-r}{\sigma_3^2}\right] + \frac{p_4}{\sigma_4^2} \exp\left[\frac{-r}{\sigma_4^2}\right]} dr$$

After simplification, the equation can be reduced to

$$= \underbrace{-p_1 \log\left(\frac{p_1 p_2}{\pi\sigma_2^2}\right) + \frac{p_1\sigma_1^2}{\sigma_2^2} \log e - p_1 \log \frac{1}{\pi}\left(\frac{p_3}{\sigma_3^2} + \frac{p_4}{\sigma_4^2}\right)}_{\mathrm{I}} +$$

$$\underbrace{\frac{p_1 p_3}{\sigma_3^4} \log e \int_0^\infty \frac{\exp\left[\frac{-r}{\sigma_3^2}\right] \exp\left[\frac{-r}{\sigma_1^2}\right]}{\frac{p_3}{\sigma_3^2} \exp\left[\frac{-r}{\sigma_3^2}\right] + \frac{p_4}{\sigma_4^2} \exp\left[\frac{-r}{\sigma_4^2}\right]} dr}_{\mathrm{II}} + \underbrace{\frac{p_1 p_4}{\sigma_4^4} \log e \int_0^\infty \frac{\exp\left[\frac{-r}{\sigma_4^2}\right] \exp\left[\frac{-r}{\sigma_1^2}\right]}{\frac{p_3}{\sigma_3^2} \exp\left[\frac{-r}{\sigma_3^2}\right] + \frac{p_4}{\sigma_4^2} \exp\left[\frac{-r}{\sigma_4^2}\right]} dr}_{\mathrm{III}}$$

consider the part II alone *i.e.*,

$$\frac{p_1 p_3}{\sigma_3^4} \log e \int_0^\infty \frac{\exp\left[\frac{-r}{\sigma_3^2}\right] \exp\left[\frac{-r}{\sigma_1^2}\right]}{\frac{p_3}{\sigma_3^2} \exp\left[\frac{-r}{\sigma_3^2}\right] + \frac{p_4}{\sigma_4^2} \exp\left[\frac{-r}{\sigma_4^2}\right]} dr$$

Let $\frac{1}{\sigma_1^2}=\frac{1}{\sigma_4^2}+\frac{1}{\sigma_s^2}$ and substitute this into above equation

$$\frac{p_1p_3}{\sigma_3^4}\log e\int_0^\infty \frac{\exp\left[\frac{-r}{\sigma_4^2}\right]\exp\left[\frac{-r}{\sigma_3^2}\right]\exp\left[\frac{-r}{\sigma_s^2}\right]}{\frac{p_3}{\sigma_3^2}\exp\left[\frac{-r}{\sigma_3^2}\right]+\frac{p_4}{\sigma_4^2}\exp\left[\frac{-r}{\sigma_4^2}\right]}dr$$
$$=\frac{p_1p_3}{\sigma_3^4}\log e\int_0^\infty \frac{\exp\left[\frac{-r}{\sigma_4^2}\right]\exp\left[\frac{-r}{\sigma_s^2}\right]}{\frac{p_3}{\sigma_3^2}+\frac{p_4}{\sigma_4^2}\exp\left[\frac{-r}{\sigma_4^2}\right]\exp\left[\frac{r}{\sigma_3^2}\right]}dr$$

If $\sigma_3^2>\sigma_4^2$ then $\frac{1}{\sigma_4^2}>\frac{1}{\sigma_3^2}$. If $t=\exp\left[-r(\frac{1}{\sigma_4^2}-\frac{1}{\sigma_3^2})\right]$ then $\exp\left[\frac{-r}{\sigma_1^2}\right]=t^{\frac{\frac{1}{\sigma_4^2}+\frac{1}{\sigma_s^2}}{\frac{1}{\sigma_4^2}-\frac{1}{\sigma_3^2}}}$. Now the equation turn out to be

$$=\frac{p_1p_3}{\sigma_3^4}\log e\frac{\sigma_3^2}{p_3}\frac{1}{\frac{1}{\sigma_4^2}-\frac{1}{\sigma_3^2}}\int_0^1 \frac{t^{\frac{\frac{1}{\sigma_4^2}+\frac{1}{\sigma_s^2}}{\frac{1}{\sigma_4^2}-\frac{1}{\sigma_3^2}}-1}}{1+\frac{p_4\sigma_3^2}{p_3\sigma_4^2}t}dt$$

Replace the integral with the hypergeometric function in Eq. (1), then the equation becomes

$$=\frac{p_1\sigma_1^2}{\sigma_3^2}\log e\ {}_2F_1\left(1,\frac{\sigma_3^2\sigma_4^2}{\sigma_1^2(\sigma_3^2-\sigma_4^2)};1+\frac{\sigma_3^2\sigma_4^2}{\sigma_1^2(\sigma_3^2-\sigma_4^2)};\frac{-p_4\sigma_3^2}{p_3\sigma_4^2}\right)$$

Now, consider the part (III) *i.e.*,

$$\frac{p_1p_4}{\sigma_4^4}\log e\int_0^\infty \frac{\exp\left[\frac{-r}{\sigma_4^2}\right]\exp\left[\frac{-r}{\sigma_1^2}\right]}{\frac{p_3}{\sigma_3^2}\exp\left[\frac{-r}{\sigma_3^2}\right]+\frac{p_4}{\sigma_4^2}\exp\left[\frac{-r}{\sigma_4^2}\right]}dr$$

Let $\frac{1}{\sigma_1^2}=\frac{1}{\sigma_3^2}+\frac{1}{\sigma_s^2}$ and then substitute into the above equation, we get

$$\frac{p_1p_4}{\sigma_4^4}\log e\int_0^\infty \frac{\exp\left[\frac{-r}{\sigma_4^2}\right]\exp\left[\frac{-r}{\sigma_3^2}\right]\exp\left[\frac{-r}{\sigma_s^2}\right]}{\frac{p_3}{\sigma_3^2}\exp\left[\frac{-r}{\sigma_3^2}\right]+\frac{p_4}{\sigma_4^2}\exp\left[\frac{-r}{\sigma_4^2}\right]}dr$$

The equation further reduced to

$$\frac{p_1p_4}{\sigma_4^4}\log e\int_0^\infty \frac{\exp\left[\frac{-r}{\sigma_3^2}\right]\exp\left[\frac{-r}{\sigma_s^2}\right]}{\frac{p_3}{\sigma_3^2}\exp\left[\frac{-r}{\sigma_3^2}\right]\exp\left[\frac{r}{\sigma_4^2}\right]+\frac{p_4}{\sigma_4^2}}dr$$

Further reduce the equation again

$$\frac{p_1p_4}{\sigma_4^4}\log e\int_0^\infty \frac{\exp\left[\frac{-r}{\sigma_3^2}\right]\exp\left[\frac{-r}{\sigma_s^2}\right]}{\frac{p_4}{\sigma_4^2}\left(1+\frac{p_3\sigma_4^2}{p_4\sigma_3^2}\exp\left(-r\left[\frac{1}{\sigma_3^2}-\frac{1}{\sigma_4^2}\right]\right)\right)}dr$$

If $\sigma_3^2 < \sigma_4^2$ then $\frac{1}{\sigma_3^2} > \frac{1}{\sigma_4^2}$. If $t = \exp\left[-r(\frac{1}{\sigma_3^2} - \frac{1}{\sigma_4^2})\right]$ then $\exp\left[\frac{-r}{\sigma_1^2}\right] = t^{\frac{\frac{1}{\sigma_3^2}+\frac{1}{\sigma_s^2}}{\frac{1}{\sigma_3^2}-\frac{1}{\sigma_4^2}}}$. Now the equation turn out to be

$$\frac{p_1}{\sigma_4^2}\log e \frac{1}{\frac{1}{\sigma_3^2}-\frac{1}{\sigma_4^2}} \int_0^1 \frac{t^{\frac{\frac{1}{\sigma_3^2}+\frac{1}{\sigma_s^2}}{\frac{1}{\sigma_3^2}-\frac{1}{\sigma_4^2}}-1}}{1+\frac{p_3\sigma_4^2}{p_4\sigma_3^2}t} dt$$

From Eq. (1) replace the integral with hypergeometric function. Then the equation becomes

$$= \frac{p_1\sigma_1^2}{\sigma_4^2}\log e \;{}_2F_1\left(1, \frac{\sigma_3^2\sigma_4^2}{\sigma_1^2(\sigma_4^2-\sigma_3^2)}; 1+\frac{\sigma_3^2\sigma_4^2}{\sigma_1^2(\sigma_4^2-\sigma_3^2)}; \frac{-p_3\sigma_4^2}{p_4\sigma_3^2}\right)$$

Finally, by combining I ,II and III, we will get Eq. (15), *i.e.*

$$-p_1\log\frac{p_1p_2}{\pi\sigma_2^2} + \frac{p_1\sigma_1^2}{\sigma_2^2}\log e - p_1\log\frac{1}{\pi}\left(\frac{p_3}{\sigma_3^2}+\frac{p_4}{\sigma_4^2}\right) + \frac{\alpha p_1\sigma_1^2}{\sigma_{\max}^2}\log e$$

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