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Photon Statistics, Tomographic Entropy of a Single Qubit-Radiation Field in the Even Binomial Distribution

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In this article we present a scheme of the interaction between a single qubit (SQ) and radiation field (RF) follows the even "binomial distribution" (BD). The density matrix of the SQ-RF system is given in terms of the number of photon transition and RF distribution parameters. The photon statistics and topographical properties of the RF are quantified by the evolution of Mandel parameter and tomographic entropy respectively. We explore the relation between the tomographic entropy and Wehrl entropy of the RF of the estimators of model parameters in the even BD. The results show that the SQ-RF correlation is connected by the tomographical and statistical properties of the proposed system.

Keywords: even binomial distribution, mandel parameter, Wehrl entropy, tomographic entropy, photon statistics

1. INTRODUCTION

In recent years, some studies have shed light on the properties of the probability distributions as an initial states of the RF play a central role in quantum statistics and technology of quantum information. So different studies have been introduced different discrete distribution as an initial quantum states of the RF. In this respect the excited binomial states (BSs) and the negative BSs of the have been introduced by Wang and Fu [1]. The results showed that both the excited negative BS and BS are normalized in terms of hyper-geometric functions. Also, Agarwal and Tara [2] have studied the non-classical properties such as sub-Poissonian photon statistics and squeezing of the RF in excited coherent states. Additionally, Joshi and Lawande had been introduced the squeezed BS and nonlinear squeezed BS and discussed their nonclassical properties [3]. Moreover, another class of the BS which obey the BD is the even BSs and nonlinear even BSs has been also introduced [4]. It found that the classical and statistical properties of these states as an initial distribution of the RF have different richer structure of the RF. More recently, the link between the statistical properties of the RF in the even deformed BS and nonlocal correlation between a two qubits has been studied [5].

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Recently, there are many applications of two level systems in quantum computers, communications, quantum cryptography, and quantum metrology [6-9]. Some of these applications are depend on the interaction of the superconducting qubits and different types of the RF distributions [10]. Also, the modern applications of the two-level system have been pointed out that from different types of quantum algorithms [11-14]. The mathematical model of field-qubit interaction is known as the Jaynes Cummings Model (JCM) which has different applications in the quantum information theory. The statistical properties of the RF during the interaction processes have taken into account in different investigations [15-20]. As known some measures have been employed in the classical phase space. In this respect, Wehrl entropy (WE) [21] has been used as a quantifier of the dynamical properties of a SQ-RF system in phase space by [22]. The WE was introduced as a classical entropy of a quantum state and can give additional insights into the dynamics of the system [21, 23, 24]. Also, WE was successfully used to quantify the classicality of the OF initially in Poissonian distribution [26]. [27] have used WE to measure the entanglement in phase space. The results showed that the degree of the inter-mode correlation was strongly depends on the photon number difference in two-mode Fock states. Recently, the minimum of WE among the quantum states with a given von Neumann entropy in terms of the RF basis donated by and it has proven that was achieved by thermal Gaussian states. This result determines the relation between the von Neumann and the Wehrl entropies by [28]. Another quantifier of the SQ-RF state in phase space which depends of the state tomography is called tomographic entropy (TE). In this regard the TE in the context of a multi-level atom was introduced by V.I. Man'ko [29, 30]. Also, the tomographic properties of the spin spin-state tomography is studied by generalizing the techniques of measuring two-particle spin states in [31-33]. More recently, the TE of the SQ-RF system initially in the excited BD and non-linear BD has investigated [34]. The results are shown the behavior of the TE the same as von Neumann entropy with different amplitude.

Recently the dynamical properties of tomographic entropy and its relation with the quantum Fisher information of the RF obeys the excited BS and excited negative BS has been investigated [35]. Also, this relation has only demonstrated the RF in the excited BS. Therefore, this work investigates the relation between the trimorphic properties due to the TE and the statistical properties of the RF in the even BD quantified by the Mandel parameter. We further study the effect of the number of photons transition between the SQ and the RF system.

This article is formulated as: The SQ-RF model when the RF follows the even BD will be introduced in Section 2. The formula for the Mandel parameter, the TE and the WE will be presented in Section 3. Section 4 investigates the dynamics of the TE and von Neumann entropy in terms of the RF basis donated by S_{RF} as quantifiers of the entanglement between the RF and SQ and its relation with the Mandel parameter and the WE. Finally, we will give a summary of the obtained results in Section 5.

2. SQ AND RF IN THE EVEN BD

We consider an important model of the interaction between an RF defined by the annihilation (creation) operator $\hat{a}$ ($\hat{a}^{\dagger}$) and SQ defined by the atomic operators $\hat{R}_{01} =$

$|q_0\rangle\langle q_1|$, where $|q_0\rangle$ ($|q_1\rangle$) denotes the upper (lower) qubit state and $\hat{R}_{10} = |q_1\rangle\langle q_0|$ as [36]

$$\hat{H}_{in} = \beta \left\{ \hat{a}^k \sqrt{\hat{a}^\dagger \hat{a}} \hat{R}_{01} + \sqrt{\hat{a}^\dagger \hat{a}} \hat{a}^{\dagger k} \hat{R}_{10} \right\} \tag{1}$$

where the factor $\sqrt{\hat{a}^\dagger \hat{a}}$ is the intensity-dependent SQ-RF function with the coupling constant β.

The even BD is the initial distribution of the RF corresponds to the even BSs. So, the density matrix of the RF in the even BD can be formulated as

$$\rho_{RF}(0) = \sum_{n=0}^{M/2} \sum_{m=0}^{M/2} \sqrt{b_n(M,\eta) b_m(M,\eta)} \, |2n\rangle \langle 2m| \tag{2}$$

where $b_n(M,\eta) = \left\{ \frac{2\eta^{4n}(1-\eta^2)^{M-2n}}{1+(1-2\eta^2)^M} \right\} \left(\frac{M!}{(2n)!(M-2n)!} \right)$ and η, M are the parameter of the probability mass function of the even BD.

We assume that SQ-RF initial is formulated as $|w(0)\rangle = |w_{RF}(0)\rangle \otimes |w_{SQ}(0)\rangle$, where the SQ is in its upper state $|w_{SQ}(0)\rangle = |q_0\rangle$. This implies that the even BD is given by

$$|w(0)\rangle = \sum_{n=0}^{M/2} \sqrt{b_n(M,\eta)} |2n\rangle \otimes |q_0\rangle, \tag{3}$$

where b_n is the amplitude of the even BS.

The final SQ-RF state can be obtained by solving the Schrödinger equation,

$$i\hbar \frac{d|w(t)\rangle}{dt} = \hat{H}_{int} |w(t)\rangle, \qquad \hbar = 1$$

which can be expressed as

$$|w(t)\rangle = \sum_{n=k}^{M/2} \sum_{\ell=0}^{1} \sqrt{b_n(M,\eta)} \left\{ i^\ell \cos\left[t(n+1) + \frac{\ell\pi}{2} \right] \right\} |n+\ell, q_\ell\rangle. \tag{4}$$

The density matrix is given by $\rho(t) = |w(t)\rangle\langle w(t)|$, and hence, the SQ (RF) density matrix can be obtained by taking the trace over the RF (SQ) basis, $\rho_{SQ[RF]}(t) = tr_{RF[SQ]} \{\rho(t)\}$, according to

$$\begin{aligned} \rho_{RF}(t) &= \sum_{n_1=k}^{M/2} \sum_{n_2=k}^{M/2} \sum_{\ell=0}^{1} \sqrt{b_{n_1}(M,\eta) b_{n_2}(M,\eta)} \left\{ \cos\{t(n_1+1) + \frac{\ell\pi}{2}\} \right\} \\ &\quad \times \left\{ \cos\left[t(n_2+1) + \frac{\ell\pi}{2} \right] \right\} |n_1+\ell\rangle \langle n_2+\ell| . \end{aligned} \tag{5}$$

Eq. (5) shows the change of the RF distribution within the interaction time. The SQ density matrix is similarly obtained, yielding

$$\begin{aligned} \rho_{SQ}(t) &= \sum_{n=k}^{M/2} \sum_{\ell_1=0}^{1} \sum_{\ell_2=0}^{1} b_n(M,\eta) \left\{ i^{\ell_1} \cos\left[t(n+1) + \frac{\ell_1\pi}{2} \right] \right\} \\ &\quad \times \left\{ (-i)^{\ell_2} \cos\left[t(n+1) + \frac{\ell_2\pi}{2} \right] \right\} |q_{\ell_1}\rangle\langle q_{\ell_2}|. \end{aligned} \tag{6}$$

In the next section, we define the Mandel parameter, the WE and the TE in terms of the density matrices of the SQ and RF.

3. PHOTON STATISTICS OF THE RF IN THE EVEN BD

3.1 Mandel Parameter

As known that the statistical properties of the RF during the time evolution are exchanged between Possionian, sup and supper-Poissonian according to the value of Mandel parameter. The Mandel parameter is defined as [37, 38]

$$P_m(t) = \frac{\langle \hat{n}^2 \rangle - \langle \hat{n} \rangle (1 + \langle \hat{n} \rangle)}{\langle \hat{n} \rangle} \tag{7}$$

where $\langle \hat{n} \rangle = tr\{\hat{a}^\dagger \hat{a} |w(t)\rangle\langle w(t)|\}$ is the average photon number. The RF distribution is being super-Poissonian for positive values $P_m(t) > 0$ while sub-Poissonian distribution for $-1 \leq P_m(t) < 0$ and Poissonian for $P_m(t) = 0$.

3.2 Tomographic Entropy

The tomographic entropy is used as a quantifier for the tomographic properties of the SQ-RF state in phase space. The TE in terms of the phase space parameters ϑ, φ of the SQ is defined as [29]

$$S_T(t) = -\frac{1}{\sqrt{2\pi}} \int_0^{2\pi} \int_0^{\pi} \sum_{\ell=-\frac{1}{2}}^{\ell=\frac{1}{2}} \xi_\ell(\vartheta, \varphi, t) \ln \xi_\ell(\vartheta, \varphi, t) \sin \vartheta d\vartheta d\varphi \tag{8}$$

where $\xi_{\frac{1}{2}}(\vartheta, \varphi, t)$ and $\xi_{-\frac{1}{2}}(\vartheta, \varphi, t)$ are given by

$$\xi_\pm(\vartheta, \varphi, t) = \frac{1}{2}\left\{1 + (\rho_{q_0q_0}(t) - \rho_{q_1q_1}(t)) \cos(\vartheta)\right\} \pm \sin(\vartheta) Re\left(\rho_{q_0q_1}(t) \exp(i\varphi)\right) \tag{9}$$

where $\rho_{q_jq_s}(t) = \langle q_j | \rho_{SQ}(t) | q_s \rangle, \quad s, j = 0, 1.$

To quantify the RF-SQ nonlocal correlation, we use the von Neumann entropy of the field basis S_{RF} given by [39-41]

$$S_{RF} = Tr\{\hat{\rho}_{RF} \ln(1/\hat{\rho}_{RF})\} = -\sum_{j=1}^{L} \gamma_j \ln \gamma_j, \tag{10}$$

where γ_j are the eigenvalues of the RF density matrix $\hat{\rho}_{RF}(t)$. Field entropy varies from zero for separable states to $\ln(2)$ for maximally correlated states.

3.3 Wehrl Entropy

All information of the quantum system is involved in its density matrix ρ, which related with the initial distribution and can be represented by the Husimi quasi-distribution function $H_\rho(\psi) = \frac{1}{\pi}\langle\psi|\hat{\rho}_{RF}(t)|\psi\rangle$, where $|\psi\rangle$ is the coherent state. $H_\rho(\psi)$ acts the projection of the distribution of the RF on the Poissonian distribution formulated coherent state. Therefore, the corresponding WE in terms of the the Husimi Q-function is defined as [21]

$$S_W = -\int_\Omega H_\rho(\psi)\ln H_\rho(\psi)\,d\psi \tag{11}$$

where $\psi = |\psi|\exp(i\Phi)$. Hence, the WE based on Eq. (11) is given by [26]:

$$S_W = -\int_0^{2\pi}\left\{\int_{\mathbf{0}}^{\infty}\left(H_\rho(\psi)\ln H_\rho(\psi)\right)|\psi|\,\mathbf{d}\,|\psi|\right\}d\Phi. \tag{12}$$

In the next section we will discuss the photon statistics of the RF in the even BD and its correlation with the tomographical and entanglement properties of the proposed system.

4. DISCUSSION OF RESULTS

Here, we discuss the dynamical behavior of the TE, WE and NE based on the evolution of Eqs. (8), (12) and (10) respectively. The behavior of these quantities will be compared with the behavior of the Mandel parameter as a quantifier of the photon statistics of the RF follows the even BD. We examine the influence of the number of photons transition between the SQ and RF via one photon (*i.e.* $k = 1$) in Fig. 1 and two photon ($k = 2$) in Fig. 2. According to the obtained results of figure 1. The field exchange between Poissonian and sub-Poissonian for one-photon case and completely sub-Poissonian which the sharing between the qubit and field by two photons. The comparison between figure (a) and figure (b) depicts a strong correlation between the photon statistics of the field by the Mandel parameter and its classicality quantified by the WE. Interestingly, $P_m = 0$ at the periodic time corresponding to the field in its classical state where $S_W = 1 + \ln\pi$. From figure (d) we have $S_{RF} = 0$ at the periodic time for $k = 1$ which mean that SQ-RF in separable state when the field is Poissonian. By increasing the number of photons as $k = 2$ the situation is different where the field is go far from the classical state and being more quantum mechanically (see Fig. 2). Also, a high amount of SQ-RF entanglement is obtained by increases the number of photons transition. Additionally the comparison between figure (c), figure (d) show that the von Neumann entropy and TF have the same behavior with different amplitude. This similarity emphases the ability of TE to quantify the nonlocal correlation between the qubit and field distribution during the time evolution.

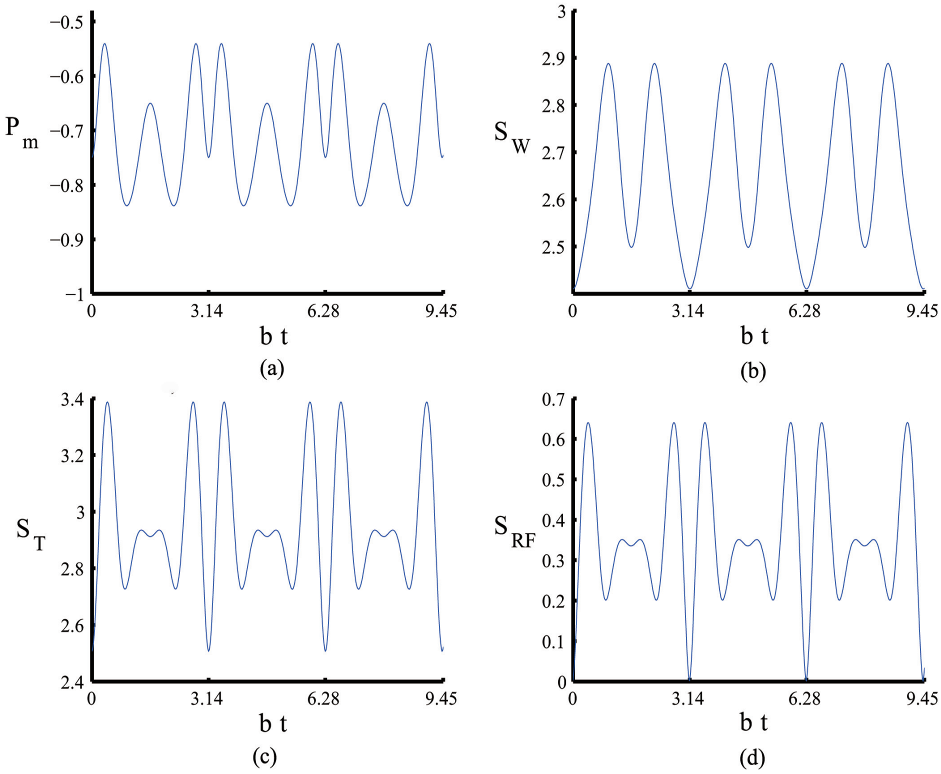

Fig. 1. The evolution of: (a) Mandel parameter P_m; (b) WE S_W; (c) TE S_T and (d) RE entropy S_{RF} of a field mode interacting with an SQ. The RF is initially follows the even BD with parameter $M = 4$ and $\eta = 0.5$ for one-photon transition $k = 1$.

5. CONCLUSION

Entanglement estimation and tomographical properties of partite system are important in quantum statistics and the technology of quantum information. So, it is important to explore different properties of state tomography and entanglement in the point of view of photon statistics of the field during the interaction processes. In this article, we have introduced a model of the interaction between a qubit and field mode following the even BD. Accordingly, the TE, WE and Mandel parameter as indicators of photon statistics of the field have been investigated. We found that the field becomes non-classical when the number of photons field mode increases. Also, the photon statistics of the field is strongly connected with the topographical properties of the SQ-RF state and entanglement. Additionally, the classicality of the field mode was related with the photon statistics of the field where the field is more quantum mechanically as the Mandel parameter is sub-Poissonian distribution. A high a mount of some kind of quantum correlation between the SQ and RF can by obtained for two-photon processes compared with one photon case where the TF presented a richer structure for the proposed system.

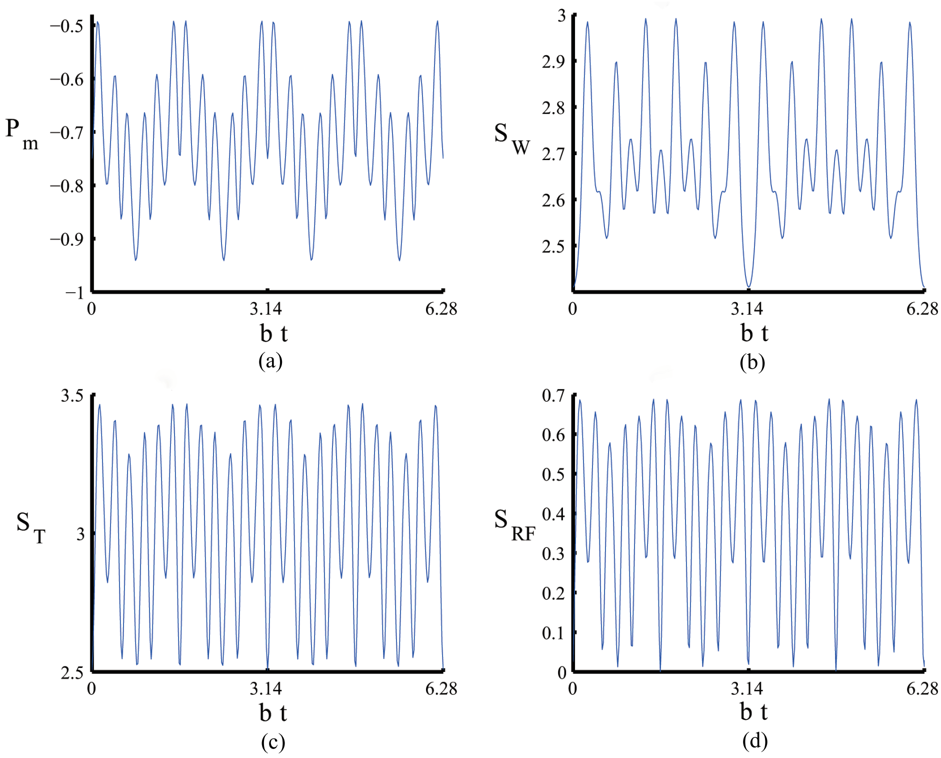

Fig. 2. The same as Fig. 1 but for two-photon transition $k = 2$.

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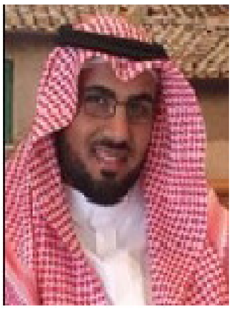

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