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New Analytical Solutions of Wick-Type Stochastic Schamel KdV Equation Via Modified Khater Method

ABDEL-HALEEM ABDEL-ATY[1,2], MOSTAFA M.A. KHATER[3], A. M. ZIDAN[4,5] AND RAGHDA A. M. ATTIA[3,6]

[1]*Department of Physics*
University of Bisha
Bisha, 61922 Saudi Arabia
[2]*Physics Department*
[5]*Mathematics Department*
Al-Azhar University
Assiut, 71524 Egypt
[3]*Department of Mathematics*
Jiangsu University
Zhenjiang, 212013 P.R. China
[4]*Department of Mathematics*
King Khalid University
Abha, 61413 Saudi Arabia
[6]*Department of Basic Science*
Higher Technological Institute
Ramadan City, 44634 Egypt
E-mail: amabdelaty@ub.edu.sa; zidan.math90@azhar.edu.eg;
{*mostafa.khater2024; raghda.attia2024*}*@yahoo.com*

This research employs a new analytical scheme to construct novel traveling wave solutions of the Wick-type stochastic Schamel KdV equation. This equation explains the electrostatic potential for a particular electron distribution in velocity space. It is also used to explain the nonlinear interaction of ion-acoustic waves when electron trapping. By using the Hermite transform, inverse Hermite transforms, and white noise analysis allows us for applying the modified Khater method to this model. Many novel solutions are obtained and sketched to discuss more physical properties of the model.

Keywords: Wick-type stochastic Schamel KdV equation, modified Khater method, analytical traveling wave solutions, solitary waves, partial differential equations

1. INTRODUCTION

Recently, many vital phenomena are formulated in nonlinear partial differential equations form [1-8]. These equations explain and discuss more detail of each model by solving them with analytical or numerical schemes. According to the importance and effectiveness of this study, many researchers in different fields have been attracted to investigate more and more of the character of each phenomenon. So that, many analytical and numerical schemes have been formulated such as the simplest equation method, modified tanh-function method, B-spline method, iterative method, and other methods [9-25].

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This paper studies the analytical solutions of the Wick-type stochastic Schamel KdV equation by using the modified Khater method. This equation describes the nonlinear interaction of ion-acoustic waves when electron trapping is present, and also it governs the electrostatic potential for a particular electron distribution in velocity space. Our model is a generalized model of the Schamel-Korteweg-de Vries equation that is given by [16-18]

$$\varphi_t + \left(a\,\varphi^{0.5} + b\,\varphi\right)\varphi_x + c\,\varphi_{xxx} = 0, \tag{1}$$

where $\varphi = \varphi(x,t)$ is unknown function of the space variable (x) and time (t) while a, b, c are respectively, represent the activation trapping, the convection, and the dispersion coefficients. Eq. (1) has various applications in different fields such as plasma physics and optical fibre. Additionally, the Schamel KdV model is considered as a generalized form of the generalized KdV model under the following condition [a = 0] [29, 30]. Moreover, it also incorporates the Schamel equation when [b = 0] [31, 32]. On the other hand, the Wick-type stochastic Schamel-KdV equation with variable coefficients is one of the most important stochastic PDEs. This equation is given by [33-35]

$$\varphi_t + \left[\beta_1(t) \otimes \varphi^{\frac{1}{2}} + \beta_2(t) \otimes \varphi\right] \otimes \varphi_t + \beta_3(3) \otimes \varphi_{xxx} = 0, \tag{2}$$

where $\otimes$ is the Wick product on the Kondratiev distribution space $(\delta)_{-1}$ and $\beta_1(t), \beta_2(t), \beta_3(t)$ are valued functions [36].

The paper is organized as follows: Section (2), applies the modified Khater method on the suggested model to get novel solitary wave solutions of it [37, 38]. Section (3), explains the summary of all the steps of our paper is detailed.

2. APPLICATION

Using the Hermite transform with the following wave transformation [$\varphi = \varphi(x,t) = \varphi(\wp)$, $\wp = \lambda\,\left[x + \int_0^t \alpha(x,t)dt\right] + c$] on Eq. (2), leads to

$$-\alpha\,\varphi\,\varphi' + \left[\beta_1\,\varphi^2 + \beta_2\,\varphi^3\right]\,\varphi' + \beta_3\,\lambda^3\,\left[\varphi\,\varphi''' + 3\,\varphi'\,\varphi''\right] = 0, \tag{3}$$

where k, c are arbitrary constants. Calculating the balance value between the highest order derivative term and nonlinear term of Eq. (3), and using the general suggested form of solutions of the modified Khater method, obtain the general solution of Eq. (3) that is given by

$$\varphi(\wp) = \sum_{i=1}^{n} a_i k^{if(\wp)} + \sum_{i=1}^{n} b_i k^{-if(\wp)} + a_0 = a_1 k^{f(\wp)} + a_0 + b_1 k^{-f(\wp)}, \tag{4}$$

where a_0, a_1, b_1 are arbitrary constants. Additionally, $f(\wp)$ is the solution function of the following ODE

$$f'(\wp) = \frac{\delta\,k^{f(\wp)} + \rho\,k^{-f(\wp)} + \chi}{\ln(k)}, \tag{5}$$

where δ, ρ, χ are arbitrary constants. Substituting Eq. (4) along (5) and its derivatives into Eq. (3), give a polynomial of $k^{f(\wp)}$. Gathering the coefficients of the same power of $[k^{i\,f(\wp)}, (i=-3,-2,-1,0,1,2,3)]$, gives a system of algebraic equations. Solving this system by using Mathematica 11.3, yields

Case 1: $\left[a_0 \to -\frac{4\beta_1}{5\beta_2}, a_1 \to -\frac{4\beta_1\delta}{5\beta_2\chi}, b_1 \to -\frac{4\beta_1\rho}{5\beta_2\chi}, \alpha \to -\frac{16\beta_1^2(\chi^2-4\delta\rho)}{75\beta_2\chi^2}, \lambda \to -\frac{\left(\frac{2}{5}\right)^{2/3}\beta_1^{2/3}}{\sqrt[3]{3}\sqrt[3]{\beta_2}\sqrt[3]{\beta_3}\chi^{2/3}}\right]$.

Thus, the solutions of Eq. (2) are given in the following forms:

For $[\chi^2-4\delta\rho<0\,\&\,\delta\neq 0]$

$$\varphi_1(x,t) = -\frac{2\beta_1\left(\chi^2-4\delta\rho\right)\sec^2\left(\frac{1}{2}\wp\sqrt{4\delta\rho-\chi^2}\right)}{5\beta_2\chi\left(\chi-\sqrt{4\delta\rho-\chi^2}\tan\left(\frac{1}{2}\wp\sqrt{4\delta\rho-\chi^2}\right)\right)}, \tag{6}$$

$$\varphi_2(x,t) = -\frac{2\beta_1\left(\chi^2-4\delta\rho\right)\csc^2\left(\frac{1}{2}\wp\sqrt{4\delta\rho-\chi^2}\right)}{5\beta_2\chi\left(\chi-\sqrt{4\delta\rho-\chi^2}\cot\left(\frac{1}{2}\wp\sqrt{4\delta\rho-\chi^2}\right)\right)}. \tag{7}$$

For $[\chi^2-4\delta\rho>0\,\&\,\delta\neq 0]$

$$\varphi_3(x,t) = -\frac{2\beta_1\left(\chi^2-4\delta\rho\right)\operatorname{sech}^2\left(\frac{1}{2}\wp\sqrt{\chi^2-4\delta\rho}\right)}{5\beta_2\chi\left(\sqrt{\chi^2-4\delta\rho}\tanh\left(\frac{1}{2}\wp\sqrt{\chi^2-4\delta\rho}\right)+\chi\right)}, \tag{8}$$

$$\varphi_4(x,t) = \frac{2\beta_1\left(\chi^2-4\delta\rho\right)\operatorname{csch}^2\left(\frac{1}{2}\wp\sqrt{\chi^2-4\delta\rho}\right)}{5\beta_2\chi\left(\sqrt{\chi^2-4\delta\rho}\coth\left(\frac{1}{2}\wp\sqrt{\chi^2-4\delta\rho}\right)+\chi\right)}. \tag{9}$$

For $[\chi=\frac{\rho}{2}=\kappa\,\&\,\delta=0]$

$$\varphi_5(x,t) = -\frac{4\beta_1 e^{\kappa\wp}}{5\beta_2\left(e^{\kappa\wp}-2\right)}. \tag{10}$$

For $[\chi=\delta=\kappa\&\rho=0]$

$$\varphi_6(x,t) = \frac{4\beta_1}{5\beta_2\left(e^{\kappa\wp}-1\right)}. \tag{11}$$

For $[\rho=0\,\&\,\chi\neq 0\,\&\,\delta\neq 0]$

$$\varphi_7(x,t) = \frac{8\beta_1}{5\beta_2\left(\delta e^{\wp\chi}-2\right)}. \tag{12}$$

For $[\chi=\delta=0\,\&\,\rho\neq 0]$

$$\varphi_8(x,t) = \frac{1}{2}\left(-\sqrt{k^2-4\beta}+k-\frac{2\sqrt{2}}{\wp}\right). \tag{13}$$

For $[\delta = 0 \,\&\, \chi \neq 0 \,\&\, \rho \neq 0]$

$$\varphi_9(x,t) = \frac{4\beta_1 \chi e^{\wp\chi}}{5\beta_2\rho - 5\beta_2\chi e^{\wp\chi}}. \tag{14}$$

For $[\chi^2 - 4\delta\rho = 0]$

$$\varphi_{10}(x,t) = \frac{2\beta_1\left(4\delta\rho(\wp\chi+2)^2 - \wp\chi^3(\wp\chi+4)\right)}{5\beta_2\wp\chi^3(\wp\chi+2)}. \tag{15}$$

where $\left[\wp = -\frac{\left(\frac{2}{5}\right)^{2/3}\beta_1^{2/3}}{\sqrt[3]{3}\sqrt[3]{\beta_2}\sqrt[3]{\beta_3}\chi^{2/3}}\left[x + \int_0^t\left(-\frac{16\beta_1^2(\chi^2-4\delta\rho)}{75\beta_2\chi^2}\right)dt\right] + c, \beta_1 = \beta_1(x,t), \beta_2 = \beta_2(x,t), \beta_3 = \beta_3(x,t)\right]$.

Case 2: $\left[a_0 \to -\frac{4\beta_1}{5\beta_2}, a_1 \to -\frac{4\beta_1\delta}{5\beta_2\chi}, b_1 \to -\frac{4\beta_1\rho}{5\beta_2\chi}, \alpha \to -\frac{16\beta_1^2(\chi^2-4\delta\rho)}{75\beta_2\chi^2}, \lambda \to -\frac{(-2)^{2/3}\beta_1^{2/3}}{\sqrt[3]{3}5^{2/3}\sqrt[3]{\beta_2}\sqrt[3]{\beta_3}\chi^{2/3}}\right]$.

Thus, the solutions of Eq. (2) are given in the following forms:
For $[\chi^2 - 4\delta\rho < 0 \,\&\, \delta \neq 0]$

$$\varphi_{11}(x,t) = -\frac{2\beta_1\left(\chi^2-4\delta\rho\right)\sec^2\left(\frac{1}{2}\wp\sqrt{4\delta\rho-\chi^2}\right)}{5\beta_2\chi\left(\chi - \sqrt{4\delta\rho-\chi^2}\tan\left(\frac{1}{2}\wp\sqrt{4\delta\rho-\chi^2}\right)\right)}, \tag{16}$$

$$\varphi_{12}(x,t) = -\frac{2\beta_1\left(\chi^2-4\delta\rho\right)\csc^2\left(\frac{1}{2}\wp\sqrt{4\delta\rho-\chi^2}\right)}{5\beta_2\chi\left(\chi - \sqrt{4\delta\rho-\chi^2}\cot\left(\frac{1}{2}\wp\sqrt{4\delta\rho-\chi^2}\right)\right)}. \tag{17}$$

For $[\chi^2 - 4\delta\rho > 0 \,\&\, \delta \neq 0]$

$$\varphi_{13}(x,t) = -\frac{2\beta_1\left(\chi^2-4\delta\rho\right)\operatorname{sech}^2\left(\frac{1}{2}\wp\sqrt{\chi^2-4\delta\rho}\right)}{5\beta_2\chi\left(\sqrt{\chi^2-4\delta\rho}\tanh\left(\frac{1}{2}\wp\sqrt{\chi^2-4\delta\rho}\right)+\chi\right)}, \tag{18}$$

$$\varphi_{14}(x,t) = \frac{2\beta_1\left(\chi^2-4\delta\rho\right)\operatorname{csch}^2\left(\frac{1}{2}\wp\sqrt{\chi^2-4\delta\rho}\right)}{5\beta_2\chi\left(\sqrt{\chi^2-4\delta\rho}\coth\left(\frac{1}{2}\wp\sqrt{\chi^2-4\delta\rho}\right)+\chi\right)}. \tag{19}$$

For $[\chi = \frac{\rho}{2} = \kappa \,\&\, \delta = 0]$

$$\varphi_{15}(x,t) = -\frac{4\beta_1 e^{\kappa\wp}}{5\beta_2\left(e^{\kappa\wp}-2\right)}. \tag{20}$$

For $[\chi = \delta = \kappa \& \rho = 0]$

$$\varphi_{16}(x,t) = \frac{4\beta_1}{5\beta_2\left(e^{\kappa\wp}-1\right)}. \tag{21}$$

For $[\rho = 0 \,\&\, \chi \neq 0 \,\&\, \delta \neq 0]$

$$\varphi_{17}(x,t) = \frac{8\beta_1}{5\beta_2\left(\delta e^{\wp\chi} - 2\right)}. \tag{22}$$

For $[\chi = \delta = 0 \,\&\, \rho \neq 0]$

$$\varphi_{18}(x,t) = \frac{1}{2}\left(-\sqrt{k^2 - 4\beta} + k - \frac{2\sqrt{2}}{\wp}\right). \tag{23}$$

For $[\delta = 0 \,\&\, \chi \neq 0 \,\&\, \rho \neq 0]$

$$\varphi_{19}(x,t) = \frac{4\beta_1 \chi e^{\wp\chi}}{5\beta_2\rho - 5\beta_2\chi e^{\wp\chi}}. \tag{24}$$

For $[\chi^2 - 4\delta\rho = 0]$

$$\varphi_{20}(x,t) = \frac{2\beta_1\left(4\delta\rho(\wp\chi + 2)^2 - \wp\chi^3(\wp\chi + 4)\right)}{5\beta_2\wp\chi^3(\wp\chi + 2)}. \tag{25}$$

where $\left[\wp = -\frac{(-2)^{2/3}\beta_1^{2/3}}{\sqrt[3]{3}5^{2/3}\sqrt[3]{\beta_2}\sqrt[3]{\beta_3}\chi^{2/3}}\left[x + \int_0^t\left(-\frac{16\beta_1^2(\chi^2 - 4\delta\rho)}{75\beta_2\chi^2}\right)dt\right] + c,\ \beta_1 = \beta_1(x,t),\ \beta_2 = \beta_2(x,t),\ \beta_3 = \beta_3(x,t)\right].$

Case 3: $\left[a_0 \to \frac{-2}{5\beta_2}\left(\frac{\beta_2\beta_1\chi}{\sqrt{\beta_2^2(\chi^2 - 4\delta\rho)}} + \beta_1\right), a_1 \to \frac{-4\beta_1\delta}{5\sqrt{\beta_2^2\chi^2 - 4\beta_2^2\delta\rho}}, b_1 \to 0, \alpha \to \frac{-16\beta_1^2}{75\beta_2}, \lambda \to \frac{-\sqrt[3]{-\frac{1}{3}}\left(\frac{2}{5}\right)^{2/3}\beta_1^{2/3}}{\sqrt[3]{4\beta_2\beta_3\delta\rho - \beta_2\beta_3\chi^2}}\right].$

Thus, the solutions of Eq. (2) are given in the following forms:

For $[\chi^2 - 4\delta\rho < 0 \,\&\, \delta \neq 0]$

$$\varphi_{21}(x,t) = \frac{2\beta_1\left(\frac{\sqrt{\beta_2^2(\chi^2 - 4\delta\rho)}\tan\left(\frac{1}{2}\wp\sqrt{4\delta\rho - \chi^2}\right)}{\sqrt{4\delta\rho - \chi^2}} - \beta_2\right)}{5\beta_2^2}, \tag{26}$$

$$\varphi_{22}(x,t) = \frac{2\beta_1\left(\frac{\sqrt{\beta_2^2(\chi^2 - 4\delta\rho)}\cot\left(\frac{1}{2}\wp\sqrt{4\delta\rho - \chi^2}\right)}{\sqrt{4\delta\rho - \chi^2}} - \beta_2\right)}{5\beta_2^2}. \tag{27}$$

For $[\chi^2 - 4\delta\rho > 0 \,\&\, \delta \neq 0]$

$$\varphi_{23}(x,t) = \frac{2\beta_1\left(\frac{\sqrt{\beta_2^2(\chi^2 - 4\delta\rho)}\tanh\left(\frac{1}{2}\wp\sqrt{\chi^2 - 4\delta\rho}\right)}{\sqrt{\chi^2 - 4\delta\rho}} - \beta_2\right)}{5\beta_2^2}, \tag{28}$$

$$\varphi_{24}(x,t)=\frac{2\beta_1\left(\frac{\sqrt{\beta_2^2\left(\chi^2-4\delta\rho\right)}\coth\left(\frac{1}{2}\wp\sqrt{\chi^2-4\delta\rho}\right)}{\sqrt{\chi^2-4\delta\rho}}-\beta_2\right)}{5\beta_2^2}. \tag{29}$$

For $[\delta\rho>0\,\&\,\rho\neq 0\,\&\,\delta\neq 0\,\&\,\chi=0]$

$$\varphi_{25}(x,t)=\frac{2}{5}\beta_1\left(-\frac{\sqrt{\delta\rho}\tan\left(\wp\sqrt{\delta\rho}\right)}{\sqrt{\beta_2^2(-\delta)\rho}}-\frac{1}{\beta_2}\right), \tag{30}$$

$$\varphi_{26}(x,t)=\frac{2}{5}\beta_1\left(\frac{\sqrt{\delta\rho}\cot\left(\wp\sqrt{\delta\rho}\right)}{\sqrt{\beta_2^2(-\delta)\rho}}-\frac{1}{\beta_2}\right). \tag{31}$$

For $[\delta\rho<0\,\&\,\rho\neq 0\,\&\,\delta\neq 0\,\&\,\chi=0]$

$$\varphi_{27}(x,t)=\frac{2}{5}\beta_1\left(\frac{\sqrt{-\delta\rho}\tanh\left(\wp\sqrt{-\delta\rho}\right)}{\sqrt{\beta_2^2(-\delta)\rho}}-\frac{1}{\beta_2}\right), \tag{32}$$

$$\varphi_{28}(x,t)=\frac{2}{5}\beta_1\left(\frac{\sqrt{-\delta\rho}\coth\left(\wp\sqrt{-\delta\rho}\right)}{\sqrt{\beta_2^2(-\delta)\rho}}-\frac{1}{\beta_2}\right). \tag{33}$$

For $[\chi=0\,\&\,\rho=-\delta]$

$$\varphi_{29}(x,t)=\frac{2}{5}\beta_1\left(\frac{\rho\coth(\wp\rho)}{\sqrt{\beta_2^2\rho^2}}-\frac{1}{\beta_2}\right). \tag{34}$$

For $[\chi=\delta=\kappa\,\&\,\rho=0]$

$$\varphi_{30}(x,t)=\frac{2}{5}\beta_1\left(\frac{\kappa\coth\left(\frac{\kappa\wp}{2}\right)}{\sqrt{\beta_2^2\kappa^2}}-\frac{1}{\beta_2}\right). \tag{35}$$

For $[\rho=0\,\&\,\chi\neq 0\,\&\,\delta\neq 0]$

$$\varphi_{31}(x,t)=\frac{2}{5}\beta_1\left(\frac{\chi\left(\delta e^{\wp\chi}+2\right)}{\sqrt{\beta_2^2\chi^2}\left(\delta e^{\wp\chi}-2\right)}-\frac{1}{\beta_2}\right). \tag{36}$$

For $[\chi=0\,\&\,\rho=\delta]$

$$\varphi_{32}(x,t)=\frac{2}{5}\beta_1\left(-\frac{1}{\beta_2}-\frac{\rho\tan(C+\wp\rho)}{\sqrt{\beta_2^2\left(-\rho^2\right)}}\right). \tag{37}$$

where $\left[\wp = -\frac{\sqrt[3]{-\frac{1}{3}}\left(\frac{2}{5}\right)^{2/3}\beta_1^{2/3}}{\sqrt[3]{4\beta_2\beta_3\delta\rho-\beta_2\beta_3\chi^2}}\left[x + \int_0^t\left(-\frac{16\beta_1^2}{75\beta_2}\right)dt\right] + c, \beta_1 = \beta_1(x,t), \beta_2 = \beta_2(x,t), \beta_3 = \beta_3(x,t)\right]$.

Case 4: $\left[a_0 \to \frac{-2}{5\beta_2}\left(\frac{\beta_2\beta_1\chi}{\sqrt{\beta_2^2(\chi^2-4\delta\rho)}}+\beta_1\right), a_1 \to 0, b_1 \to \frac{-4\beta_1\rho}{5\sqrt{\beta_2^2\chi^2-4\beta_2^2\delta\rho}}, \alpha \to \frac{-16\beta_1^2}{75\beta_2}, \lambda \to \frac{-\sqrt[3]{-\frac{1}{3}}\left(\frac{2}{5}\right)^{2/3}\beta_1^{2/3}}{\sqrt[3]{4\beta_2\beta_3\delta\rho-\beta_2\beta_3\chi^2}}\right]$.

Thus, the solutions of Eq. (2) are given in the following forms:
For $[\chi^2-4\delta\rho<0\,\&\,\delta\neq 0]$

$$\varphi_{33}(x,t)=\frac{2}{5}\beta_1\left(\frac{4\delta\rho}{\sqrt{\beta_2^2(\chi^2-4\delta\rho)}\left(\chi-\sqrt{4\delta\rho-\chi^2}\tan\left(\frac{1}{2}\wp\sqrt{4\delta\rho-\chi^2}\right)\right)}-\frac{\frac{\beta_2\chi}{\sqrt{\beta_2^2(\chi^2-4\delta\rho)}}+1}{\beta_2}\right), \tag{38}$$

$$\varphi_{34}(x,t)=\frac{8\beta_1\delta\rho}{5\sqrt{\beta_2^2(\chi^2-4\delta\rho)}\left(\chi-\sqrt{4\delta\rho-\chi^2}\cot\left(\frac{1}{2}\wp\sqrt{4\delta\rho-\chi^2}\right)\right)}+\frac{2\beta_1\left(-\frac{\beta_2\chi}{\sqrt{\beta_2^2(\chi^2-4\delta\rho)}}-1\right)}{5\beta_2}. \tag{39}$$

For $[\chi^2-4\delta\rho>0\,\&\,\delta\neq 0]$

$$\varphi_{35}(x,t)=\frac{2}{5}\beta_1\left(\frac{4\delta\rho}{\sqrt{\beta_2^2(\chi^2-4\delta\rho)}\left(\sqrt{\chi^2-4\delta\rho}\tanh\left(\frac{1}{2}\wp\sqrt{\chi^2-4\delta\rho}\right)+\chi\right)}-\frac{\frac{\beta_2\chi}{\sqrt{\beta_2^2(\chi^2-4\delta\rho)}}+1}{\beta_2}\right), \tag{40}$$

$$\varphi_{36}(x,t)=\frac{8\beta_1\delta\rho}{5\sqrt{\beta_2^2(\chi^2-4\delta\rho)}\left(\sqrt{\chi^2-4\delta\rho}\coth\left(\frac{1}{2}\wp\sqrt{\chi^2-4\delta\rho}\right)+\chi\right)}+\frac{2\beta_1\left(-\frac{\beta_2\chi}{\sqrt{\beta_2^2(\chi^2-4\delta\rho)}}-1\right)}{5\beta_2}. \tag{41}$$

For $[\delta\rho>0\,\&\,\rho\neq 0\,\&\,\delta\neq 0\,\&\,\chi=0]$

$$\varphi_{37}(x,t)=\frac{2}{5}\beta_1\left(-\frac{\sqrt{\delta\rho}\cot\left(\wp\sqrt{\delta\rho}\right)}{\sqrt{\beta_2^2(-\delta)\rho}}-\frac{1}{\beta_2}\right), \tag{42}$$

$$\varphi_{38}(x,t)=\frac{2}{5}\beta_1\left(\frac{\sqrt{\delta\rho}\tan\left(\wp\sqrt{\delta\rho}\right)}{\sqrt{\beta_2^2(-\delta)\rho}}-\frac{1}{\beta_2}\right). \tag{43}$$

For $[\delta\rho < 0 \,\&\, \rho \neq 0 \,\&\, \delta \neq 0 \,\&\, \chi = 0]$

$$\varphi_{39}(x,t) = \frac{2}{5}\beta_1 \left(-\frac{\sqrt{-\delta\rho}\coth\left(\wp\sqrt{-\delta\rho}\right)}{\sqrt{\beta_2^2(-\delta)\rho}} - \frac{1}{\beta_2} \right), \tag{44}$$

$$\varphi_{40}(x,t) = \frac{2}{5}\beta_1 \left(\frac{\sqrt{\delta}\sqrt{\rho}\tan\left(\sqrt{\delta}\wp\sqrt{\rho}\right)}{\sqrt{\beta_2^2(-\delta)\rho}} - \frac{1}{\beta_2} \right). \tag{45}$$

For $[\chi = 0 \,\&\, \rho = -\delta]$

$$\varphi_{41}(x,t) = \frac{2}{5}\beta_1 \left(-\frac{\rho\tanh(\wp\rho)}{\sqrt{\beta_2^2\rho^2}} - \frac{1}{\beta_2} \right). \tag{46}$$

For $[\chi = \frac{\rho}{2} = \kappa \,\&\, \delta = 0]$

$$\varphi_{42}(x,t) = \frac{2}{5}\beta_1 \left(-\frac{\kappa\left(e^{\kappa\wp}+2\right)}{\sqrt{\beta_2^2\kappa^2}\left(e^{\kappa\wp}-2\right)} - \frac{1}{\beta_2} \right). \tag{47}$$

For $[\chi = 0 \,\&\, \rho = \delta]$

$$\varphi_{43}(x,t) = \frac{2}{5}\beta_1 \left(-\frac{1}{\beta_2} - \frac{\rho\cot(C+\wp\rho)}{\sqrt{\beta_2^2\left(-\rho^2\right)}} \right). \tag{48}$$

For $[\delta = \rho 0 \& \chi \neq 0 \& \neq 0]$

$$\varphi_{44}(x,t) = \frac{2}{5}\beta_1 \left(-\frac{\chi\left(\chi e^{\wp\chi}+\rho\right)}{\sqrt{\beta_2^2\chi^2}\left(\chi e^{\wp\chi}-\rho\right)} - \frac{1}{\beta_2} \right). \tag{49}$$

where $\left[\wp = -\frac{\sqrt[3]{-\frac{1}{3}}\left(\frac{2}{5}\right)^{2/3}\beta_1^{2/3}}{\sqrt[3]{4\beta_2\beta_3\delta\rho-\beta_2\beta_3\chi^2}}\left[x + \int_0^t\left(-\frac{16\beta_1^2}{75\beta_2}\right)dt\right] + c, \beta_1 = \beta_1(x,t), \beta_2 = \beta_2(x,t), \beta_3 = \beta_3(x,t)\right].$

3. CONCLUSION

In this paper, we studied the electrostatic potential for a particular electron distribution in velocity space that is represented by the Wick-type stochastic Schamel KdV equation. The modified Khater method was employed to find exact and solitary wave solutions of this model. The Hermite transform, inverse Hermite transforms, and white noise analysis were also used to convert the nonlinear partial differential equation form of this model to nonlinear ordinary differential equation. Many new solitary wave solutions were constructed in different formula such as trigonometric, hyperbolic, and rational forms.

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A. Abdel-Aty obtained his BSc and MSc (Physics) from Physics Department, Al-Azhar University, Egypt in 2004 and 2009, respectively. After finishing the Master in 2011, Universiti Teknologi Petronas, Malaysia, awarded him a full Ph.D. Scholarship in Theoretical Physics (Quantum Information) and he has awarded the degree in 2015. In 2015, he has awarded Sultana Nahar's prize for the best Ph.D. thesis in Egypt. In 2017, he obtained a scholarship as a visiting researcher at University of Oxford, UK. In 2017, has been elected as a Junior Associate Membership from African Academy of Science, Kenya. He is especially working in theories of quantum measurement, nanomechanical modeling, highly non-classical light, practical information security, and optical implementations of quantum information tasks. His current research interests include quantum resources, optical and atomic implementations of quantum information tasks and protocols, mathematical modeling. He has published more than 60 papers in ISI journals, 4 book chapters and 10 papers in conference proceedings and indexed by Scopus and ISI. He is acting as a managing editor of some journals, such as, Quantum Physics Letters, Int. J. New Horizons in Physics, also he is acting as a Vice-President of Natural Sciences Publishing, USA.

M. M. A. Khater obtained his BSc from Zagazig University, Egypt in 2011 and also obtained his MSc from Mathematical Department, Mansoura University in 2016. After finishing the Master in 2017, Jiangsu University, China, awarded him a full Ph.D. scholarship in control science and engineering. Moreover, Jiangsu University awarded him in 2018 as the best foreigner student in the whole university. He also got a distinguished scholarship from China government. He awarded in 2020 with Obada-prize. He published more than ninety ISI papers and participated in more than ten conferences. He is one of the editor team in four publishing house and a reviewer in more than twenty five ISI journal.

A. M. Zidan obtained his BSc, MSc and Ph.D. from Mathematics Department, Al-Azhar University, Egypt in 2010, 2015 and 2018, respectively. He is especially working in theories of functional analysis, fixed point theory, partial differential equations, fixed point results in different spaces, application of functional analysis in Integral equations, geometrical spaces and non-Gaussian analysis and hyper complex systems. He has published more than 13 papers in ISI journals, and 5 papers in conferences. He is an organizer at the first, second and third international conference on new horizons in basic and applied science in 2013, 2015 and 2017 respectively. He is member of the Egyptian Mathematical Society, 2014 till now.

R. A. M. Attia obtained her BSc, MSc, Ph.D. degrees from Zagazig University, Egypt. She published more than thirty-five ISI papers. She works now as a Postdoctoral in Jiangsu University, China. She participated in many conferences in different countries. She specialized in numerical analysis and partial differential equation.