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An Analytical Approach on User Rate Maximization in Two-Way Relay Communication using NOMA

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As more and more users are using wireless devices, the demand for higher data rate has increased. Therefore, new technologies are being developed to meet the needs. One such technique is Non-Orthogonal Multiple Access (NOMA). NOMA is an essential technology for 5G wireless communication as it increases spectrum efficiency and reduces system latency. In the present work, a model for increasing the user data rate in a two-way relay communication employing NOMA scheme is proposed. The outage probability and outage ground reference expressions are derived for the system involving end-to-end communication via rayleigh fading channels. Based upon the above outage probabilities an efficient power allocation technique is employed which shows higher sum rate of 0.36bps/Hz compared with random power allocation.

Keywords: NOMA, ergodic capacity, outage probability, power allocation, relay

1. INTRODUCTION

Recent developments in wireless industry have led to new benchmarks such as high data rate, massive connectivity and low latency. In 4G, OFDMA has been adopted but spectrum efficiency is underutilized due to orthogonal subcarriers. NOMA operates over the power domain. Superposition coding and successive interference cancellation at transmitter and receiver respectively enhances the performance of NOMA [1]. It is a versatile scheme which can help users to meet the 5G standards. It also improves spectral efficiency by multiplexing several users on same resource block with different power level.

In general, transmitted signal which propagates in various paths causes multi path fading leading to loss of information. Relay based cooperative communication overcomes the fading problem. Its advantages include coverage extension, capacity improvement and diversity gain of wireless system [2]. Cooperative communication [3] in wireless communication system helps the users/agents to enhance their achievable quality of service (QoS) via cooperation among themselves. This cooperation exists at the cost of coding efficiency and transmit power of each user via relaying. This relaying was typically one way or half duplex mode of information exchange between each user/agent.

One-way relaying mechanisms are confronted by spectrum underutilization issues [4]. To achieve the QoS enhancements among the users, efficient spectrum utilization tech-

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niques are needed and one such technique is two-way relaying or full duplex mode of information exchange.

2. LITERATURE SURVEY

2.1 Related Works

P. Liu *et al.* [5] the authors have conducted performance analysis on bidirectional relay communication system employing decode and forward protocol and the results show that the two-way relaying outperforms one-way relaying system in spectrum utilization.

C. Luo *et al.* [6] proposed a Hybrid Demodulate-Forward (HDMF) protocol for Two-Way Relay Channel (TWRC). Proposed HDMF has lower error rate compared to generic protocols. The results also revealed the spectrum efficiency of relay channels with unbalanced bilateral traffic can be improved significantly but with increased cost and computation complexity.

X. Liang *et al.* [7] investigated outage analysis of the system employing decode and forward protocol in a two-way relayed communication. In this work, it was assumed that interference is present only at end users/agents and relay node was assumed to be noisy without any external interference.

Z. Yang *et al.* [8] proposed a two-user uplink and downlink non-orthogonal multiple access (NOMA) scenario with dynamic power allocation. The expressions for average rate and probability of outage to achieve dynamic power allocation is derived for the D-NOMA systems.

N. Vucic *et al.* [9] exploited a difference of convex (DC) structure in the problem of sum rate maximization in an interference limited system. The power allocation and interference filtering are optimized jointly using an iterative algorithm which solves a sequence of convex problems. The proposed technique ensures to achieve a large portion of the globally optimal sum rate.

C. Li *et al.* [10] analyzed a decode and forward two-way relay full duplex relay system on the following system metrics: outage probability and positioning of relay nodes. A closed form expression for outage probability is derived and validated. This system takes into account the effect of self-interference from the local transmit antenna to the local receive antenna. As a result of the self-interference, it was shown that the transmit power cannot be increased forever to enhance the system outage probability. The authors have also proposed the optimal location for relay node placement based on the outage probability and fractional power used by the relay node.

M. Shipon Ali *et al.* [11] formulated a non-linear programming problem which is solved in two steps: (1) Grouping users into clusters; (2) Optimizing their respective power allocations. The results show that for both downlink and uplink NOMA, user clusters with more distinctive channel gains provide impressive throughput gains. The limitations prevail when the cluster size increases beyond a certain threshold. Issues such as SIC error processing and inter-cell interference increases and thus reducing the performance.

Y. Liu *et al.* [12] proposed a tractable expression for the outage probability in single-cell uplink NOMA systems. The outage-constrained min-max power allocation problems for both NOMA and OMA systems were solved using bisection method and iterative algorithm respectively. The results show that NOMA could achieve the same level of fairness as OMA in terms of uplink transmission power and also can save significant power.

Q. Sun *et al.* [13] studied the maximization of ergodic capacity in a rayleigh fading MIMO-NOMA system with transmitter channel state information (CSIT). A suboptimal power allocation scheme with constraints on the total transmit power and minimum rate of the weak user is proposed to maximize the ergodic capacity of the system. The results show that the proposed algorithm achieves significant gains than OMA schemes.

F. Fang *et al.* [14] proposed two algorithms to optimize channel assignment and maximize the energy efficiency for the downlink NOMA network. A low-complexity suboptimal algorithm to determine energy-efficient sub channel assignment and power proportional factors for same sub channel multiplexed users is proposed. Eventually, an algorithm for allocation of power across sub channels to further maximize energy efficiency is also proposed. The result provides significant improvement in terms of sum rate and energy efficiency over conventional OFDM technique.

D.-T. Do *et al.* [15] studied the problem of hardware impairment aware design to evaluate the performance of CR-NOMA. Closed form expressions were provided to validate performance of NOMA over conventional OMA. A specific insight was given into the performance of the system at high transmit SNR for a CR-NOMA network. Optimal powers were allocated to two NOMA users to satisfy quality of service requirements with the constraints on the SNR thresholds.

D.-T. Do *et al.* [16] investigated on performance at each vehicle in vehicle to everything (V2X) communication using NOMA technology. A full duplex relay assisted broadcasting transmission scheme is added to the results of [15] and compared with half duplex relay assisted NOMA scheme results. The results showed an improvement in spectral efficiency with lower latency and higher reliability for the communications from base station to vehicles in the considered system model.

X. Yue *et al.* [17] investigated a two-way relay NOMA system where two NOMA users communicate via a half-duplex decode and forward relay node. Closed form expressions were derived for system outage with perfect and imperfect successive interference cancellation at the destination. Numerical results were provided to show 1. Superiority of two-way relay NOMA over two-way relay OMA system. 2. Impact of interference at relay on the achievable ergodic rates of the users.

C.-B. Le *et al.* [18] analyzed system performance of the vehicle to everything (V2X) network using cognitive radio based non-orthogonal multiple access (CR-NOMA) scheme. The author analyzed how CR-NOMA delivers high spectral efficiency to V2X networks in terms of two metrics: 1. Outage probability and 2. Bit error rate.

J.-H. Lee *et al.* [19], the authors have derived the outage probability expression for full duplex two-way relay system with and without buffer aiding at the relay node. This system assumes that self-interference within the local transmit and receive antennas is mitigated. In contrary to [10], the results show that outage probability achieves an outage floor due to the absence of self-interference. The authors focused on outage analysis alone and not on the optimal power allocation for QoS enhancement of users.

The following observations are made from above works:

- A system consisting of decode and forward relay node with full duplex communication [5, 6] using NOMA technology is able to achieve 5G signaling rates.
- Spectral efficiency can be increased with perfect SIC [7] at destinations.
- Optimal allocation of power is needed at the relay nodes [8] to mask the interference

available on the channel of the weaker user.

- Power allocation and interference filtering can be achieved simultaneously with iterative algorithms through convex optimization [11-14].
- Systems are analyzed based on outage probabilities which in turn guarantees users' in meeting their QoS requirements [15-18].
- Outage probability cannot be improved just by increasing the SNR. There always exists an upper constraint on the SNR levels [10, 19].
- System described in [10] analyzed the outage performance and proposed an optimal location of relay nodes but did not offer optimal power allocation in the uplink and downlink at the relay node to meet the enhanced QoS requirements of the end users.

2.2 Contribution

In this paper, outage performance of cooperative communication system consisting of stationary source, destination and relay node is analyzed. The contributions made are as follows:

- Self-interference is mitigated by perfect SIC at the relay nodes.
- Self-interference does not have an effect on the outage floor. Hence a two-way full duplex relay system without the impact of self-interference is considered and analyzed here.
- In our work, we have analyzed the outage performance and ergodic capacity of the system with the help of the expressions derived and validated the same. Optimal power allocation with successive iteration is proposed to maximize the user achievable sum rate with the help of convex optimization.

2.3 Outline

The remainder of this paper is organized as follows: Section 3 describes the system model which is assumed based on the above observations. Section 4 analyses the system under consideration in terms expressions for outage probabilities and ergodic capacity of users. Section 5 discusses the simulation results for the analytical expressions obtained followed by conclusion from the work presented in Section 6.

3. SYSTEM MODEL

A two-way relay communication system based on NOMA is considered as shown in Fig. 1 with users u_1, u_2. A decode and forward relay r is employed to establish communication between users u_1, u_2. Channel between the users is assumed to be rayleigh faded. The communication protocol consists of uplink phase [8, 22] and downlink phases as described below.

3.1 Uplink NOMA

Communication between users u_1 and u_2 take place via relay r and hence it is inherently assumed there is no line of sight path existing between the users. Symbols s_1 and s_2

are transmitted respectively by user u_1 and u_2 simultaneously. As a result, there is superposition of symbols [8, 22] at the relay and it is termed as z_r and given as follows

$$z_r = k_1 h_{u_1 r} s_1 + k_2 h_{u_2 r} s_2 + n_r \tag{1}$$

where $k_1 = \sqrt{b_1 P_s}$ and $k_2 = \sqrt{b_2 P_s}$ are the powers allocated to u_1 and u_2 with the constraint $0 < b_1, b_2 < 1$ and $b_1 + b_2 = 1$. Power constraints [8] are placed to ensure interference mitigation in the considered NOMA system. The rayleigh faded channel [25] coefficient is denoted as $h_{u_i r} \sim \mathcal{CN}(0, \sigma^2_{u_i r})$ for the path between *user* u_i − relay r, where $i \in \{1, 2\}$. Channel is assumed to include complex additive gaussian white noise and it is denoted as $n_r \sim \mathcal{CN}(0, \sigma^2)$.

In generally as [13], it is assumed that the system consists of 2 users with user u_1 being the strongest user due to its proximity to the relay and user u_2 being the weakest user as it is situated at a farther distance from the relay and hence $\sigma^2_{u_1 r} \geq \sigma^2_{u_2 r}$. The decoding of the symbols [23, 24] takes place at the relay with the strongest user u_1's symbol decoded first followed by the weakest user u_2's symbol considering the u_1's data as interference. Hence the SNRs for decoding respective user's symbol is given as

$$\gamma^r_{s_2} = b_2 \rho_s \beta_{u_2 r} \tag{2}$$

$$\gamma^r_{s_1} = \frac{b_1 \rho_s \beta_{u_1 r}}{b_2 \rho_s \beta_{u_2 r} + 1}$$

where $\rho_s = P_s/\sigma^2$, $\beta_{u_1 r}$ and $\beta_{u_2 r}$ are channel gain coefficients and its distribution are given as follows

$$f_{\beta_{u_i r}}(s) = \frac{1}{\sigma^2_{u_i r}} e^{-(s/\sigma^2_{u_i r})}, i \in \{1, 2\}. \tag{3}$$

The relay is assumed to successfully decode the symbols only if the rate of transmission of users u_1, u_2 are below the threshold given as

$$\log_2(1 + a_2 \rho_s \beta_{u_2 r}) \quad \text{and} \quad \log_2(1 + \frac{a_1 \rho_s \beta_{u_1 r}}{a_2 \rho_s \beta_{u_2 r}}). \tag{4}$$

3.2 Broadcast Relay

The decoded symbols at the relay are combined linearly and broadcasted as superimposed signal [8] to both users in downlink with total relay power P_r as shown in Fig. 1. The received symbols due to the broadcast for the i^{th} user is given as

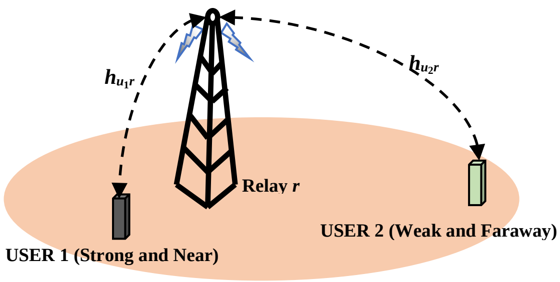

Fig. 1. Uplink phase and downlink phase of two-way relay based NOMA system.

$$z_{u_i} = h_{u_i r}(k_3 s_1 + k_4 s_2) + n_{u_i} \qquad i = 1, 2 \tag{5}$$

where $k_3 = \sqrt{b_3 P_r}$ and $k_4 = \sqrt{b_4 P_r}$. b_3, $b_4 > 0$ with $b_3 + b_4 = 1$ are the allocated fractional power [8] for symbols s_1 and s_2 for the downlink transmission from relay to users u_2 and u_1 respectively. Users u_1 and u_2 cancel the interferences in the received signal z_{u_1} and z_{u_2} by initially removing their own transmitted symbols. Successful decoding of symbols s_1 at u_2 and s_2 at u_1 is possible only if the SNRs satisfy the following

$$\begin{aligned} \gamma_{s_1}^{u_2} &= b_3 \rho_r \beta_{u_2 r} \\ \gamma_{s_2}^{u_1} &= b_4 \rho_r \beta_{u_1 r} \end{aligned} \tag{6}$$

where $\rho_r = P_r/\sigma^2$. Therefore, the successful transmission rates for u_1 and u_2 are obtained as

$$C_{u_1} = \tfrac{1}{2}\log_2(1 + \min(b_2 \rho_s \beta_{u_2 r}, b_4 \rho_r \beta_{u_1 r})) \tag{7}$$

$$C_{u_2} = \tfrac{1}{2}\log_2(1 + \min(\tfrac{b_1 \rho_s \beta_{u_1 r}}{b_2 \rho_s \beta_{u_2 r} + 1}, b_3 \rho_r \beta_{u_2 r})) \tag{8}$$

4. ANALYSIS OF SYSTEM PERFORMANCE

4.1 Probability of System Outage

Let $\tilde{R}_1$ and $\tilde{R}_2$ denote the desired transmission rates for users u_1 and u_2 respectively. A system is classified to be in outage when at least one of its participating users is experiencing outage. This can be expressed as

$$\begin{aligned} P(\xi) &= 1 - P(C_{u_1} \geq \tilde{R}_1, C_{u_2} \geq \tilde{R}_2), \\ &= 1 - P(\beta_{u_1 r} \geq \tfrac{R_1}{b_4 \rho_r}, q_1 \leq \beta_{u_2 r} \leq \psi), \end{aligned} \tag{9}$$

where the quantities $\tilde{R}_1$, $\tilde{R}_2$, and q_1 are defined as

$$\begin{aligned} R_1 &= 2^{2\tilde{R}_1} - 1, \\ R_2 &= 2^{2\tilde{R}_2} - 1, \\ q_1 &= \max(\tfrac{R_2}{b_3 \rho_r}, \tfrac{R_1}{b_2 \rho_s}), \\ \psi &= \tfrac{b_1 \rho_s \beta_{u_1 r} - R_2}{b_2 R_2 \rho_s}. \end{aligned} \tag{10}$$

ψ is a function of $\beta_{u_1 r}$. The above outage probability shows that outage of one user affects the decoding efficacy of another user in the downlink. Hence it becomes obvious to consider the outage analysis as both uplink and downlink transmission. The expression in Eq. (9) can be simplified as

$$\begin{aligned} P(\xi) &= P(\xi \mid \psi < q_1)P(\psi < q_1) + P(\xi \mid \psi \geq q_1)P(\psi \geq q_1) \\ &= 1 - P(q_1 \leq \beta_{u_2 r} \leq \psi, \beta_{u_1 r} \geq q_2) \end{aligned} \tag{11}$$

where the quantity q_2 is a constant defined

$$q_2 = \max\left(\frac{b_2 R_2^2}{b_1 b_3 \rho_r} + \frac{R_2}{b_1 \rho_S}, \frac{R_1 R_2 + R_2}{b_1 \rho_S}, \frac{R_1}{b_4 \rho_r}\right). \tag{12}$$

The expression for average probability of outage for the system is obtained as follows

$$\begin{aligned} P(\xi) &= 1 - \frac{1}{\sigma_{u_1 r}^2}\int_{q_2}^{\infty}\left(\frac{1}{\sigma_{u_2 r}^2}\int_{q_1}^{\psi} e^{\frac{-\beta_{u_2 r}}{\sigma_{u_2 r}^2}} d\beta_{u_2 r}\right) e^{\frac{-\beta_{u_1 r}}{\sigma_{u_1 r}^2}} d\beta_{u_1 r} \\ &= 1 - e^{\left(\frac{q_2}{\sigma_{u_1 r}^2} + \frac{q_1}{\sigma_{u_2 r}^2}\right)} + \frac{e^{-\left(\frac{q_2}{\sigma_{u_1 r}^2} + \frac{q_2 b_1}{b_2 R_2 \sigma_{u_2 r}^2} - \frac{1}{b_2 \rho_S \sigma_{u_2 r}^2}\right)}}{1 + \frac{b_1 \sigma_{u_1 r}^2}{b_2 R_2 \sigma_{u_2 r}^2}} \end{aligned} \tag{13}$$

where the terms q_1 and q_2 are defined in Eqs. (10) and (12), respectively. At high SNR, the probability of outage for system $P(\xi)$ is independent of the SNR terms ρ_s and ρ_r, the user u_1's QoS rate requirement $\tilde{R}_1$ and fractional power factors at relay b_3 or b_4.

The terms q_1 and q_2 are functions of both ρ_s and ρ_r. Hence, at high SNRs, using the approximation $1 - e^{-x} \simeq x$, $x \ll 1$, the expression is re-written as

$$P(\xi) \simeq \frac{q_2}{\sigma_{u_1 r}^2} + \frac{q_1}{\sigma_{u_2 r}^2} + \frac{1}{1 + \frac{b_1 \sigma_{u_1 r}^2}{b_2 R_2 \sigma_{u_2 r}^2}} + \frac{\left(\frac{1}{b_2 \rho_s \sigma_{u_2 r}^2} - \frac{q_2}{\sigma_{u_1 r}^2} - \frac{q_2 b_1}{b_2 R_2 \sigma_{u_2 r}^2}\right)}{1 + \frac{b_1 \sigma_{u_1 r}^2}{b_2 R_2 \sigma_{u_2 r}^2}} \tag{14}$$

where the term $T_0 = \dfrac{1}{1 + \frac{b_1 \sigma_{u_1 r}^2}{b_2 R_2 \sigma_{u_2 r}^2}}$ is a constant and all other terms in denominator contain some function of the SNR terms. Hence, at higher SNR, except T_0 all other terms can be neglected to obtain the expression for asymptotic outage probability of the system as

$$P^{high}(\xi) = T_0 = \frac{b_2 R_2 \sigma_{u_2 r}^2}{b_1 \sigma_{u_1 r}^2 + b_2 R_2 \sigma_{u_2 r}^2} \tag{15}$$

which is independent of the ρ_s and ρ_r, b_3 or b_4 and $\tilde{R}_1$ constraints.

In the above expression, it is clear that all the terms except $\gamma_{S_1}^r$ are monotonically increasing functions of the SNR. Therefore, at higher SNRs, only the term $\gamma_{S_1}^r$ saturating at $b_1\beta_{u_1 r}/b_2\beta_{u_2 r}$ contributes to the probability of outage for the entire system along with the user u_2's rate requirement R_2.

4.2 Ergodic Capacity of Users

The expression for the ergodic capacity of user u_1 is obtained by averaging over the distribution of S as

$$\begin{aligned} S &= \min(b_2\rho_s\beta_{u_2 r}, b_4\rho_r\beta_{u_1 r}), \\ C_{u_1} &= \tfrac{1}{2}\log_2(1+S), \\ \bar{C}_{u_1} &= \int_0^{\infty} \tfrac{1}{2}\log_2(1+S) f_S(s) dx \\ &= \frac{1}{2\ln 2}\int_0^{\infty} \frac{\bar{F}_S(s)}{1+s} dx. \end{aligned} \tag{16}$$

where $\bar{F}_S(s)$ denotes the complementary cumulative distribution function (CCDF) of S and can be obtained as

$$\begin{aligned}\bar{F}_S(s) &= P(\min(b_2\rho_s\beta_{u_2r}, b_4\rho_r\beta_{u_1r}) \geq s) \\ &= P(\beta_{u_2r} \geq \frac{s}{b_2\rho_S}, \beta_{u_1r} \geq \frac{s}{b_4\rho_r})\end{aligned} \tag{17}$$

β_{u_1r} and β_{u_2r} are independent of each other and hence

$$\bar{F}_S(s) = \exp(-s(\frac{1}{b_2\rho_s\sigma_{u_2r}^2} + \frac{1}{b_4\rho_r\sigma_{u_1r}^2})), s > 0. \tag{18}$$

Upon simplification [20], $\bar{C}_{u_1}$ is determined as

$$\bar{C}_{u_1} = \frac{1}{2\ln 2} \times \exp(\frac{1}{b_2\rho_s\sigma_{u_2r}^2} + \frac{1}{b_4\rho_r\sigma_{u_1r}^2}) Ei(-s(\frac{1}{b_2\rho_s\sigma_{u_2r}^2} + \frac{1}{b_4\rho_r\sigma_{u_1r}^2})), s > 0. \tag{19}$$

where Ei denotes the exponential integral [20] and is defined as $Ei(s) = -\int_{-s}^{\infty}(e^{-t}/t)dt$, $s < 0$. Similarly, the CCDF $\bar{F}_Z(z)$, where

$$Z = \min(\frac{b_1\rho_s\beta_{u_1r}}{b_2\rho_s\beta_{u_2r}+1}, b_3\rho_s\beta_{u_2r}) \tag{20}$$

can be obtained as

$$\bar{F}_z(z) = P(\min(\frac{b_1\rho_s\beta_{u_1r}}{b_2\rho_s\beta_{u_2r}+1}, b_3\rho_s\beta_{u_2r}) \geq z). \tag{21}$$

The ergodic capacity for user u_2 is obtained as

$$\bar{C}_{u_2} = \frac{1}{2\ln 2}\int_0^{\infty} \frac{e^{-\phi_1 z}e^{-\phi_2 z^2}}{(1+z)(1+\phi_3 z)}dz, \tag{22}$$

variables ϕ_1, ϕ_2 and ϕ_3 are defined as

$$\begin{aligned}\phi_1 &= \frac{1}{b_1\rho_s\sigma_{u_1r}^2} + \frac{1}{b_3\rho_r\sigma_{u_2r}^2} \\ \phi_2 &= \frac{b_2}{b_1b_3\rho_r\sigma_{u_1r}^2} \\ \phi_3 &= \frac{b_2\sigma_{u_2r}^2}{b_1\sigma_{u_1r}^2}\end{aligned} \tag{23}$$

The ergodic capacity expression for u_2 cannot be solved using known identities. An approximate expression is obtained as follows. Using partial fraction expansion $\bar{C}_{u_2}$ is written as

$$\bar{C}_{u_2} = \frac{1}{2\ln 2}\left(\underbrace{\int_1^{\infty}\frac{c_2 e^{vt}e^{-\hat{\phi}_2 t^2}}{t}dt}_{I_1} - \underbrace{\int_1^{\infty}\frac{c_3 e^{\alpha t}e^{-\phi_2 t^2}}{t}dt}_{I_2}\right), \tag{24}$$

variables c_2, c_3, α, v and $\hat{\phi}_2$ are defined as

$$c_2 = \frac{\exp\left(\frac{\phi_1}{\phi_3} - \frac{\phi_2}{\phi_3^2}\right)}{\phi_3 - 1},$$
$$c_3 = \frac{e^{(\phi_1 - \phi_2)}}{(\phi_1 - 1)},$$
$$\alpha = 2\phi_1 - \phi_2,$$
$$v = \frac{2\phi_2}{\phi_3^2} - \frac{\phi_1}{\phi_3},$$
$$\hat{\phi}_2 = \frac{\phi_2}{\phi_3^2}. \tag{25}$$

Using Taylor series expansion of e^{vt} [20] followed by the results, I_1 can be simplified as

$$I_1 = -\frac{C_2}{2} Ei(-\hat{\phi}_2) + \frac{C_2}{2} \sum_{k=1}^{\infty} \frac{v^k \Gamma\left(\frac{k}{2}, \hat{\phi}_2\right)}{k!(\hat{\phi}_2)} \tag{26}$$

where $\Gamma(s, x) = \int_x^{\infty} t^{\tau-1} e^{-t} dt$ denotes the upper incomplete gamma function [20]. Similarly, I_2 can be simplified as

$$I_2 = -\frac{C_3}{2} Ei(-\phi_2) + \frac{C_3}{2} \sum_{r=1}^{r^u} \frac{\alpha^r \Gamma\left(\frac{r}{2}, \hat{\phi}_2\right)}{r!(\hat{\phi}_2)^{\frac{r}{2}}}. \tag{27}$$

Substituting I_1 and I_2, a final expression of the ergodic capacity for *user* u_2 as

$$\bar{C}_{u_2} \simeq \frac{1}{2\ln 2}\left[\frac{C_3}{2} Ei(-\phi_2) - \frac{C_2}{2} Ei(-\hat{\phi}_2) + \frac{C_2}{2} \sum_{k=1}^{\infty} \frac{v^k \Gamma\left(\frac{k}{2}, \hat{\phi}_2\right)}{k!(\hat{\phi}_2)^{\frac{k}{2}}} - \frac{C_3}{2} \sum_{r=1}^{\infty} \frac{\alpha^r \Gamma\left(\frac{r}{2}, \hat{\phi}_2\right)}{r!(\hat{\phi}_2)^{\frac{r}{2}}}\right]. \tag{28}$$

The presence of the factorial term in the denominator makes infinite summation in the ergodic capacity expression of user u_2 to be tightly approximated as

$$\bar{C}_{u_2} \simeq \frac{1}{2\ln 2}\left[\frac{C_3}{2} Ei(-\phi_2) - \frac{C_2}{2} Ei(-\hat{\phi}_2) + \frac{C_2}{2} \sum_{k=1}^{k^u} \frac{v^k \Gamma\left(\frac{k}{2}, \hat{\phi}_2\right)}{k!(\hat{\phi}_2)^{\frac{k}{2}}} - \frac{C_3}{2} \sum_{r=1}^{r^u} \frac{\alpha^r \Gamma\left(\frac{r}{2}, \hat{\phi}_2\right)}{r!(\hat{\phi}_2)^{\frac{r}{2}}}\right]. \tag{29}$$

The constants k^u and r^u denote the highest limit for the summation which can be computed manually depending the required accuracy levels. The sum-rate ergodic capacity for the system is obtained by combining $\bar{C}_{u_1}$ and $\bar{C}_{u_2}$ as $\bar{C}_{sum}$.

$$\bar{C}_{sum} = \frac{1}{2\ln 2}\left[\frac{C_3}{2} Ei(-\phi_2) - \frac{C_3}{2} Ei(-\hat{\phi}_2) + \sum_{k=1}^{k^u} \frac{C_2}{2} \frac{v^k \Gamma\left(\frac{k}{2}, \hat{\phi}_2\right)}{k!(\hat{\phi}_2)^{\frac{k}{2}}} - \sum_{r=1}^{r^u} \frac{C_2}{2} \frac{v^r \Gamma\left(\frac{r}{2}, \hat{\phi}_2\right)}{r!(\hat{\phi}_2)^{\frac{r}{2}}} \right.$$
$$\left. e^{\left(\frac{1}{b_2\rho_s\sigma_{u_2r}^2} + \frac{1}{b_4\rho_r\sigma_{u_1r}^2}\right)} \times Ei\left(-s\left(\frac{1}{b_2\rho_s\sigma_{u_2r}^2} + \frac{1}{b_4\rho_r\sigma_{u_1r}^2}\right)\right)\right] \tag{30}$$

5. SIMULATION RESULTS

5.1 Probability of System Outage

In this section, various results derived from the previous section are simulated for different variances of user fading channels. Fig. 2 shows the probability of system outage versus SNR for different values of $\sigma^2_{u_1r}$. It is observed that the outage of one user at the relay affects the decoding accuracy of the other user in the downlink. From equations derived earlier, it is observed that the outage probability of user 2 depends on the variance of user 1. Table 1 shows the parameters which are used for obtaining outage probability of the system. The outage floor exists under asymmetrical transmit powers of the relay node for each channel. The outage floor ceases to exist when the total transmit power of the relay node is decreased below 30dB [10]. Fractional relay power factors (b_2 = 0.1 & b_4 = 0.4) are the minimum values required to attain the outage floor and the same has been validated in [10]. Threshold rates ($\tilde{R}_1$ & $\tilde{R}_2$) are standard parameters to compare the achievable rates of users. The channel variance value represents the channel condition of weakest user who is far away from relay node.

Table 1. Parameter for probability of outage for the system.

Parameter	Symbol	Value
Relay Power factor	b_2	0.1
Relay Power factor	b_4	0.4
Fading channel Variances of link from user2 to r	$\sigma^2_{u_2r}$	2
Threshold rate for user 1	$\tilde{R}_1$	1 bps/Hz
Threshold rate for user 2	$\tilde{R}_2$	1 bps/Hz

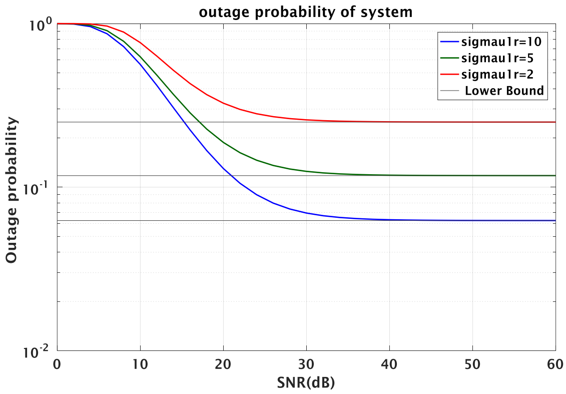

Fig. 2. Probability of outage vs SNR.

The probability of outage reaches the asymptotic floor validating the constant outage probability expression obtained in Eq. (15). With an increase in the value of β_{u_1r}, it is also observed that there is also a monotonic decrease in the asymptotic floor. This is because the relay receives lesser interference from u_2 to decode s_1 with increase in β_{u_1r}. At higher SNRs, $\gamma^r_{s_1}$ saturates. As a result, there is a higher possibility for achieving the minimum rate requirement R_2 at the relay node as desired by weakest use u_2.

In Fig. 2, the probability of outage for the system under consideration for three different values of $\sigma^2_{u_1r}$ is shown. The lower bound asymptotic floor derived represents the limit beyond which the outage probability does not depend on SNR.

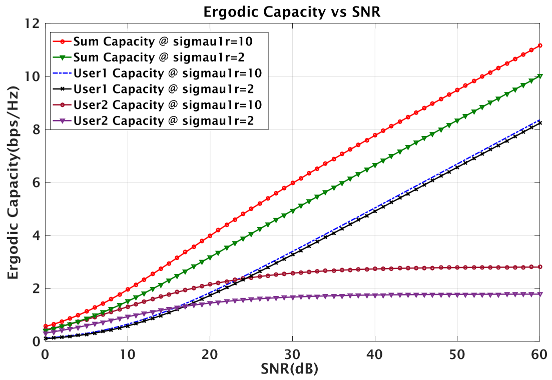

Fig. 3. Ergodic capacity vs SNR.

5.2 Ergodic Capacity of Users

Fig. 3 shows that the ergodic capacity of user 1, user 2 and the sum rate ergodic capacity with respect to SNR for various values of $\sigma^2_{u_1r}$. It is observed that the ergodic capacity of user 2 is less than that of user 1. The results obtained prove the validity of the expression derived henceforth. User 2 is weaker due to smaller channel variance because of which it has upper limitations. It is safer to assume that user 2 is more likely to be in outage than user 1. User 1 is a stronger user with better channel variance. Hence user 1 has greater ergodic capacity than user 2.

A comprehensive plot of upper ceiling in user ergodic capacity for user 2 is observed when channel variance $\sigma^2_{u_1r}$ of stronger user u_1 is varied from weaker to a stronger channel condition. Improvement in channel condition of stronger user actually enhances the ergodic capacity of user u_2. From the result it can be observed that the ergodic capacity of user 2 increases as the SNR increases but reaches a ceiling at SNRs exceeding 30dB due to the fact that outage probability of system is independent of higher SNRs. Ergodic capacity of both the user remains relatively similar at better channel conditions of user u_1.

5.3 Optimized Power Allocation

The optimization problem for sum rate maximization is defined as

Maximize $C_{u_1} + C_{u_2}$
b_1, b_2, b_3, b_4
Subject to $C_{u_1} \geq \tilde{R}_1,\ C_{u_2} \geq \tilde{R}_2,$
$b_1, b_2, b_3, b_4 > 0,\ b_1 + b_2 = 1,\ b_3 + b_4 = 1.$

The solution to above non-convex problem is obtained by converting the problem into a difference of convex equation [21, 26] and obtaining a suboptimal solution for the DC program. Fig. 4 shows the comparison between achievable sum rate when the allocated power is optimized for varying SNRs. An optimal power is allocated for the weaker user when compared to the stronger user. Sum rate achievable is enhanced when optimal power is allocated rather than random power allocation for varying channel conditions between the relay and the strongest user.

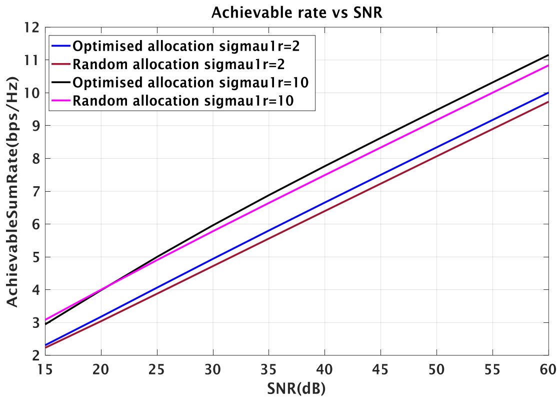

Fig. 4. Achievable rate vs SNR.

Table 2. Fixed parameter for ergodic capacity of the system.

Parameter	Symbol	Value
Relay Power factor	b_2	0.1
Relay Power factor	b_4	0.4
Fading channel Variances of link from u_2 to r	$\sigma^2_{u_2r}$	2
Fading channel Variances of link from u_1 to r	$\sigma^2_{u_1r}$	10
Threshold rate for user 1	$\tilde{R}_1$	1 bps/Hz
Threshold rate for user 2	$\tilde{R}_2$	1 bps/Hz
Upper limit Constants	k^u	60
Upper limit Constants	r^u	60

Table 2 shows the parameters which are used for determining the ergodic capacity of the system for varying channel and SNRs with optimal power allocation. The upper limit constants (k^u & r^u) are chosen from constraints of Eq. (30). When the channel conditions are poor ($\sigma^2_{u_1r}$ = 2), achievable sum rate is higher by 0.15bps/Hz compared to sum rate achievable using randomly allocated powers. When channel conditions are comparatively better ($\sigma^2_{u_1r}$ = 10), SNRs exceeding 25dB yields a higher sum rate of 0.36bps/Hz than random allocation.

6. CONCLUSION

In this paper, analysis for a NOMA two-way relay communication system was presented. Various expressions to analyze the user performance including ergodic capacity, probability of outage and outage floor for higher SNRs were derived and validated. The obtained expressions show that irrespective of changes in stronger user's QoS requirement and relay power factors, the outage probability floor is unaffected. Expressions for achieving minimum QoS rate requirement of both users were obtained and validated. A suboptimal power level is derived using DC programming. With the help of optimized power levels at the relay node, an improvement of 0.36bps/Hz is attained over the achievable sum rate using randomly allocated power levels for each better channel conditions. Furthermore, multiple users can be added to observe the efficacy of rate maximization through convex optimization.

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