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Difference Measure Method of Risk Probability Distribution Based on Moment Generating Function and Fuzzy Data Stream Clustering

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The research of the difference measure method for risk probability distribution plays a key role in the early warning decision-making management of retail supply chain unconventional emergency. However, the common difference measure indices are established by the specific density function or distribution law of the risk probability distribution. In Knight uncertain environment, only the moments of the risk probability distribution can be obtained. This study proposes the difference moment measure method of risk probability distribution based on moment generating function and fuzzy data stream clustering for the retail supply chain unconventional emergency. The big data statistical analysis is performed on the risk assessment indices to obtain the moments of the risk probability distribution for unconventional emergency. The difference of moment generating functions for unconventional emergency risk is measured by the distance function in the real vector space of infinite dimensional moments and then the difference between the real distribution and the reference distribution of the risk probability for unconventional emergency is further measured by the moments. The main contribution of this study is that we propose a new difference measure method of risk probability distribution for unconventional emergency based on cloud model method, moment generating function theory, functional function and big data fuzzy statistics technology in Knight uncertain and big date environments, which can overcome the drawbacks of the existing difference measure methods for probability distributions.

Keywords: difference moment measure method, risk probability distribution, moment generating function, fuzzy data stream clustering algorithm, unconventional emergency

1. INTRODUCTION

Since the beginning of the 21st century, unconventional emergencies have frequently occurred [1], such as “SARS”, “A H1N1” flu, New Orleans hurricane, Indian Ocean tsunami, Wenchuan earthquake and “9.11” terrorist incident. Characterized by emergency, major destructiveness, coupling, derivation, and major social influences, such incidents cause tremendous damage to human life, property, and physical and mental health [2]. The emergency management of unconventional emergency has become highly prioritized topic to governments and academia. The risk early warning decision of unconventional emergency in retail supply chain is associated with Knight uncertain and big data environment [3]. The robust multi-criteria early warning decision method for maximum and minimum expected utility needs to select a suitable difference measure index of risk probability distributions to measure the deviation between the real distribution and the

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reference distribution of the risk probability for unconventional emergency, which describes the Knight uncertainty faced by decision makers of retail supply chain.

At present, commonly used indices for measuring the differences in probability distributions are Csiszar *f*-metrics (which include common L^1 distances, relative entropy, and Heringer distance measures) and Bregman metrics (including common L^2 distance and relative entropy measures). Latif N. used Csiszar *f*-divergence to reduce the results for majorization inequality in the form of the Kullback-Leibler divergence and Shannon entropy [4]. Chen G. S. used Csiszar's *f*-divergence of two probability measures to study generalizations of the Holder's inequality [5]. Kumar P. proposed a symmetric measure belonging to the class of Csiszar's *f*-divergences [6]. Stummer W. introduced scaled Bregman distances of probability distributions covering not only the distances of discrete and continuous stochastic observations, but also the distances of random processes and signals [7]. The common indices are established by the specific density function or distribution law of the probability distribution, which measure the differences between probability distributions [8, 9]. However, in the Knight uncertain environment, the specific probability distribution form of risk is often difficult to obtain, and only the moments of the probability distribution can be obtained [10].

Some scholars use the moments to generate moment generating function. Wen L. M. deduced the estimates of the risk moments and then estimated moment-generating functions (MGFs) of risks by Buhlmann's credibility procedure [11]. Stefano D. M. proposed the moment generating function expression on its analyticity domain [12]. Murakami H. studied the moment generating function (MGF) of various Jonckheere Terpstratype statistics by using the MGF with higher order moments [13].

Some scholars use clustering algorithms for statistical analysis. Peters G. provided a framework, DCC Dynamic Clustering Cube, to categorize existing dynamic granular clustering algorithms. Furthermore, the DCC Framework can be used as a research map and starting point for new developments in this area [14]. Lingras P. W. described a concept of meta-clustering that clusters a set of granules using clustering information from another or the same set of networked granules. Cluster membership of one granule can affect another granules cluster membership, resulting in a recursive meta-clustering process [15]. Chen J. R. proposed a novel dynamic evolutionary clustering algorithm based on time weight and latent attributes. The network model is established by introducing the forgetting function to score matrix [16]. Abdullahi S. studied an unsupervised two-stage clustering approach for forest structure classification [17]. Bi W. J. proposed a big data clustering algorithm that ensures profit maximization and mitigates the risk of customer churn [18]. Memon K. H. has studied generalized kernel weighted fuzzy *c*-means clustering algorithm with local information [19]. Vignati F. presented an enhanced technique for hierarchical agglomerative clustering. The data weighting depends dynamically on the degree of advancement of the clustering procedure [20]. The duration, type and structure of connections between individuals in real-world populations play a crucial role in how diseases invade and spread. Barnard R. C. incorporated the heterogeneities into a model by considering a dual-layer static-dynamic multiplex network with tunable clustering [21]. Yang M. S. used feature-weighted entropy to research a feature-reduction fuzzy clustering algorithm and simultaneously reduce these irrelevant feature components [22].

Some scholars have studied uncertainties in the system. Kreinovich V. explained that to obtain an adequate solution, we need to take into accounts not only the system of equations and the granules describing uncertainty, and also need to take into accounts the original practical problem [23]. Sanchez M. A. proposed a new method for the formation of fuzzy higher type granular models, which is accomplished by directly discovering uncertainty from a sample of numerical information [24]. DUrso P. reviewed the principal exploratory multivariate analysis methods for imprecise data proposed in the literature, *i.e.*, cluster analysis and other exploratory statistical approaches [25]. Yager R. R. provided an approach to decision making in the face of uncertainty where the uncertain information is expressed on a space of granular objects and the underlying uncertainty is most generally represented by a measure [26]. Alberink I. used the Welch-Satterthwaite equation to quantify uncertainty in estimations of the total weight of drugs in groups of complex matrices, which does not assume constant relative standard deviations [27]. Ghosh S. embarked on functional uncertainty analysis using fuzzy hidden Markov model, which provides with optimal path for hidden failures to mitigate propagation of uncertainty amongst the functional blocks [28]. Jafarzadeh H. proposed a method combing with Quality Function Development (QFD), fuzzy logic, and Data Envelopment Analysis (DEA) to accounts for prioritization, uncertainty and interdependency [29]. Selecting a renewable energy source portfolio is an uncertain multi-criteria decision-making problem. Hocine A. proposed a multi-segment fuzzy goal programming method, which addresses decision-making problems with high levels of uncertainty [30].

The Knight uncertain environment is caused by: (1) scenario uncertainty and multiple scenarios in the future; (2) the uncertainty of decision consequences and strong scenario dependence in retail supply chain unconventional emergency. Research on early warning decision of retail supply chain unconventional emergency risk involves the uncertainty problem of the robust multi-criteria early warning decision model for maximum and minimum expected utility, which shows the uncertainty of random variable probability distribution for unconventional emergency risk. It needs to select a suitable difference measure index of risk probability distribution to measure the deviation between the real distribution and the reference distribution of unconventional emergency risk. The common indices are established by the specific density function or distribution law of the probability distribution, which measure the differences between probability distributions. However, in the Knight uncertain environment, the specific probability distribution form of risk is often difficult to obtain, and only the moments of the risk probability distribution can be obtained. To solve the bottleneck problem of risk probability distribution uncertainty measuring in the robust multi-criteria early warning decision model for the retail supply chain unconventional emergency, this study proposes the difference moment measure method of risk probability distribution based on moment generating function and fuzzy data stream clustering in Knight uncertain and big data environment. It does not depend on the specific probability distribution form of risk random variable for retail supply chain unconventional emergency. It only relies on the non-parametric indices of the moments for the risk probability distribution. In consideration of the big data characteristics of high dimensional and fast changing data stream for the unconventional emergency risk assessment indices, on the basis of the cloud model theory [31] and the fuzzy data stream clustering algorithm, the big data of risk assessment indices are statistically analyzed to obtain the non-parametric indices of the moments for the risk probability distribution of retail supply chain unconventional emergency. Based on the theory of moment generating function and functional theory, the difference between the moment generating functions of retail supply chain unconventional emergency risk is measured by the distance function in the real vector space of infinite dimension moments. The difference between the real distribution and the reference distribution of the risk probability for unconventional emergency is further measured by the moments. The main contribution of this study is that we propose a new difference measure method of risk probability distribution for retail supply chain unconventional emergency based on cloud model method, moment generating function theory, functional function and big data fuzzy statistics technology in Knight uncertain and big date environment, which can overcome the drawbacks of the difference measure methods for probability distributions presented in [5-7]. At present, the research results about the difference moment measure method of risk probability distribution for retail supply chain unconventional emergency have not yet been reported based on moment generating function theory and fuzzy data stream clustering algorithm.

This paper is organized as follows. In Section 2, the cloud model theory and the fuzzy data stream clustering algorithm are used to statistically analyze the big data of risk assessment indices for retail supply chain unconventional emergency, and to obtain the non-parametric indices of the moments for the risk probability distribution of unconventional emergency. In Section 3, the definition and properties of the moment generating function generated by the moments of the risk probability distribution for retail supply chain unconventional emergency are analyzed. Based on the theory of moment generating function and functional theory, the difference between the moment generating functions of retail supply chain unconventional emergency risk is measured by the distance function in the real vector space of infinite dimension moments. The difference degree between the risk probability distributions of unconventional emergency is measured afterward, which constructs a difference moment measure index of risk probability distribution for retail supply chain unconventional emergency. This study provides a convenient nonparametric index for the difference measuring of the risk probability distributions of retail supply chain unconventional emergency in Knight uncertain environment. In Section 4, the feasibility of the difference moment measure method of risk probability distributions is verified based on the example simulation experiments for retail supply chain unconventional emergency.

2. MEASURE MODEL OF RISK PROBABILITY DISTRIBUTION MOMENT BASED ON FUZZY DATA STREAM CLUSTERING

A measure model of risk probability distribution moment for retail supply chain unconventional emergency is built based on the fuzzy data stream clustering algorithm in this study. The summary data cloud droplets of the risk assessment indices for retail supply chain unconventional emergency are extracted by the cloud model summary data generation algorithm based on multi data fusion method. The data are then used as a set constituted by n risk assessment samples at the time t. n is the width of the window. Each sample includes m index attribute values. According to the fuzzy optimization theory, the index values of each sample are normalized to obtain the normalized matrix of the index attribute values at the time t.

$$R^t = (r_{ij}^t),\ i = 1,2,...,m, \qquad \text{and} \qquad j = 1,2,...,n \tag{1}$$

where r_{ij}^t represents the normalized number of the index attribute value at time t, $0 \le r_{ij}^t \le 1$.

At time t, the following objective function can be established based on the smallest square sum of the weighted generalized Euclidean distance about the risk assessment sample set for all categories.

$$\begin{aligned} J &= \min\left\{\sum_{j=1}^{n}\left(w_j^t\sum_{h=1}^{c}(u_{hj}^t)^2(d_{hj}^t)^2\right)\right\} \\ &= \min\left\{\sum_{j=1}^{n}\left[w_j^t\sum_{h=1}^{c}\left((u_{hj}^t)^2\sum_{i=1}^{m}(w_i^t(r_{ij}^t - s_{ih}^t))^2\right)\right]\right\} \end{aligned} \tag{2}$$

where the generalized Euclidean distance d_{hj}^t is used to represent the difference between sample j and category h. n samples are clustered into c categories according to m indices. u_{hj}^t is the relative membership degree of the sample j belonging to category h at time t, and meets the following conditions:

$$\begin{cases} \sum_{h=1}^{c} u_{hj}^t = 1 \\ 0 \le u_{hj}^t \le 1. \\ \sum_{j=1}^{n} u_{hj}^t > 0 \end{cases} \tag{3}$$

w_i^t is the weight of the ith index at time t, and satisfies the following constraints:

$$\sum_{i=1}^{m} w_i^t = 1, 0 \le w_i^t \le 1. \tag{4}$$

s_{ih}^t is the normalized number of the cluster center for index i in the category h at time t and $0 \le s_{ih}^t \le 1$. w_j^t is the weight value of data j at time t.

According to the Lagrangian function method, the various elements in the data point weight vector $DW^t = (w^t)$, the cluster weight vector $CW^t = (cw_h^t)$, the index weight vector $W^t = (w_i^t)$, the fuzzy membership matrix $U^t = (u_{hj}^t)$, and the fuzzy clustering center matrix $S^t = (s_{ih}^t)$ are obtained when the constraint conditions (3) and (4) are met.

At time t, data j weight is defined as:

$$w_j^t = 2^{-\lambda(t-t_0)}, j = 1,2,...,n \tag{5}$$

where λ is the attenuation factor, t_0 is the time at which the data point arrives.

The weight of the hth cluster is defined as:

$$cw_h^t = \sum_{j=1}^{n} u_{hj}^{t-1} w_j^t, h = 1,2,...,c. \tag{6}$$

The weight of ith index is defined as:

$$w_i^t=\left\{\sum_{k=1}^{m}\frac{\sum_{j=1}^{n}\left(w_j^t\sum_{h=1}^{c}\left[u_{hj}^{t-1}(r_{ij}^t-s_{ih}^{t-1})\right]^2\right)}{\sum_{j=1}^{n}\left(w_j^t\sum_{h=1}^{c}\left[u_{hj}^{t-1}(r_{kj}^t-s_{kh}^{t-1})\right]^2\right)}\right\}^{-1},\quad i=1,\ 2,\ \ldots,\ m. \tag{7}$$

Fuzzy membership of sample j belonging to h cluster is defined as:

$$u_{hj}^t=\left\{w_j^t\sum_{k=1}^{c}\frac{\sum_{i=1}^{m}\left[w_i^t(r_{ij}^t-s_{ih}^{t-1})\right]^2}{\sum_{i=1}^{m}\left[w_i^t(r_{ij}^t-s_{ik}^{t-1})\right]^2}\right\}^{-1},h=1,\ 2,\ \ldots,\ c,\ \text{and}\ j=1,\ 2,\ \ldots,\ n. \tag{8}$$

Fuzzy cluster center of i index for h cluster is defined as:

$$s_{ih}^t=\frac{\sum_{j=1}^{n}(w_j^t)^2(u_{hj}^t)^2(w_i^t)^2r_{ij}^t}{\sum_{j=1}^{n}(u_{hj}^t)^2(w_i^t)^2},\ i=1,\ 2,\ \ldots,\ m,\ \text{and}\ h=1,\ 2,\ \ldots,\ c. \tag{9}$$

Based on cloud models theory, according to the historical data stream of risk assessment index summaries for retail supply chain unconventional emergency, the weighted generalized expectation (first moment) of risk probability reference distribution F_{X^t} for unconventional emergency is calculated by Eq. (10).

$$E[X^t]=\frac{1}{c}\sum_{h=1}^{c}\left\{cw_h^t(x)\left[\frac{1}{n}\sum_{j=1}^{n}u_{hj}^t(x)\right]\right\} \tag{10}$$

The weighted generalized variance (secondary moment) of risk probability reference distribution F_{X^t} for unconventional emergency is calculated by Eq. (11).

Fuzzy membership of sample j belonging to h cluster is defined as:

$$D[X^t]=\frac{1}{c-1}\sum_{h=1}^{c}\left\{cw_h^t(x)\left[\frac{1}{n-1}\sum_{j=1}^{n}\left(u_{hj}^t(x)-E_h(X^t)\right)^2\right]\right\} \tag{11}$$

where $E_h[X^t]$ is the weighted generalized expectation of historical data stream for h cluster.

According to the current data stream of risk assessment index summaries for retail supply chain unconventional emergency, the weighted generalized expectation (first moment) of risk probability real distribution F_{Y^t} for unconventional emergency is calculated by Eq. (12).

$$E[Y^t]=\frac{1}{c}\sum_{h=1}^{c}\left\{cw_h^t(y)\left[\frac{1}{n}\sum_{j=1}^{n}u_{hj}^t(y)\right]\right\} \tag{12}$$

The weighted generalized variance (secondary moment) of risk probability real distribution F_{Y^t} for unconventional emergency is calculated by Eq. (13).

$$D[Y^t] = \frac{1}{c-1}\sum_{h=1}^{c}\left\{cw_h^t(y)\left[\frac{1}{n-1}\sum_{j=1}^{n}\left(u_{hj}^t(y) - E_h(Y^t)\right)^2\right]\right\} \tag{13}$$

where $E_h[Y^t]$ is the weighted generalized expectation of current data stream for h cluster.

3. DIFFERENCE MEASURE MODEL OF RISK PROBABILITY DISTRIBUTION BASED ON MOMENT GENERATING FUNCTION

3.1 Moment Generating Function of Risk Random Variable for Unconventional Emergency

(A) Definition of moment generating function for risk random variable

Definition 1: A hypothesis is that X^t is the risk random variable of retail supply chain unconventional emergency at time t, e^{aX^t} is a function of real number a. The moment generating function of the risk random variable X^t for the unconventional emergency is defined as:

$$M_{X^t}(a) = E[e^{aX^t}] = \begin{cases} \int_{-\infty}^{+\infty} e^{ax} f^t(x)dx & \text{if } X^t \text{ is countinous risk random variable, the probability densisty is } f^t(x). \\ \sum_i e^{ax_i^t} p_i^t & \text{if } X^t \text{ is discrete risk random variable, the distribution law is } P^t(X^t = x_i^t) = p_i^t. \end{cases} \tag{14}$$

where the expectation exists (Namely, the integral or the series converges absolutely).

(B) Property of moment generating function for risk random variable

Property 1: If the moment generating function $M_{X^t}(a)$ of the risk random variable X^t for retail supply chain unconventional emergency is defined in an open interval of a which contains 0, $M_{X^t}(a)$ has an arbitrary order derivative, which is calculated by Eq. (15).

$$M_{X^t}^{(k)}(a) = E\left[(X^t)^k e^{aX^t}\right] \tag{15}$$

The special case is as follows.

$$M_{X^t}^{(k)}(0) = E\left[(X^t)^k\right] \tag{16}$$

where k = 0, 1, ….

Property 2:

$$M_{bX^t+c}(a) = e^{ca} M_{X^t}(ba). \tag{17}$$

Property 3: If X^t and Y^t are independent risk random variables of retail supply chain unconventional emergency at time t, Eq. (18) is met on the interval that the moment generating function $M_{X^t}(a)$ and $M_{Y^t}(a)$ are established.

$$M_{X^t+Y^t}(a) = M_{X^t}(a) \bullet M_{Y^t}(a) \tag{18}$$

Namely, the moment generating function of the sum of two independent risk random variables is equal to the product of the moment generating functions of the two risk random variables for retail supply chain unconventional emergency.

Property 4: (uniqueness theorem) If the moment generating function $M_{X^t}(a)$ is defined in an open interval of a containing 0, $M_{X^t}(a)$ uniquely determines a risk probability distribution function of retail supply chain unconventional emergency.

Property 5: (convergence theorem) If a column of the moment generating function $M_n^t(a)$ converges to a certain moment generating function in an open interval of a containing 0, that is:

$$\lim_{n\to\infty} M_n^t(a) = M^t(a). \tag{19}$$

It is true for all a in the open interval containing 0. Then the risk probability distribution $F_n^t(x)$, corresponding to the column of the moment generating function $M_n^t(a)$, converges to the risk probability distribution $F^t(x)$, corresponding to the limit moment generating function $M^t(a)$ in the retail supply chain unconventional emergency, that is:

$$\lim_{n\to\infty} F_n^t(x) = F^t(x). \tag{20}$$

3.2 Relationship between Moment Generating Function and Moments of Risk Probability Distribution for Unconventional Emergency

The moment generating function is generated by the moments of the risk probability distribution for retail supply chain unconventional emergency. If the certain constraint conditions are met, the moment generating function can determine the risk probability distribution of retail supply chain unconventional emergency.

The moment generating function can be expanded into a power series within the symmetry maximum open interval about $a = 0$ contained in its domain.

$$M_{X^t}(a) = \sum_{n=0}^{\infty} \frac{E[(X^t)^n]}{n!} a^n. \tag{21}$$

The coefficients can be used as a point in the real vector space $S = \{(x_1, x_2, \ldots, x_n, \ldots) | x_i \in R\}$ of infinite dimension moments, denoted as point x, that is:

$$x = \left(1, E[X^t], \frac{E[(X^t)^2]}{2!}, \ldots, \frac{E[(X^t)^n]}{n!}, \ldots\right). \tag{22}$$

It may be desirable to associate the moment generating function with the real vector of an infinite dimension moments, namely corresponding to the moments of the risk probability distribution for retail supply chain unconventional emergency.

3.3 Difference Moment Measure Model of Risk Probability Distribution for Unconventional Emergency

The moment generating function has good properties such as uniqueness theorem and convergence theorem. This study uses the difference of the moment generating functions to measure the difference degree of the risk probability distributions in retail supply chain unconventional emergency. If the probability distribution functions of risk reference random variable X^t and real random variable Y^t for retail supply chain unconventional emergency meet the conditions of the uniqueness theorem for moment generating function or their moments meet the conditions of the Karman's theorem, based on the moment generating function theory and functional theory, it is possible to measure moment generating function difference of retail supply chain unconventional emergency risk through the distance functions in real vector space of infinite dimensional moments. The difference degree of the risk probability distributions is measured based on the moments in retail supply chain unconventional emergency.

Proposition 1: If there is a real number $R > 0$, when $|a| < R$, both the moment generating functions $M_{X^t}(a)$ and $M_{Y^t}(a)$ of the risk reference random variable X^t and real random variable Y^t converge in retail supply chain unconventional emergency, or the moments of X^t and Y^t exist and meet with:

$$\sum_{n=1}^{\infty}\frac{1}{\{E[(X^t)^{2n}]\}^{\frac{1}{2n}}}=\infty, \tag{23}$$

and

$$\sum_{n=1}^{\infty}\frac{1}{\{E[(Y^t)^{2n}]\}^{\frac{1}{2n}}}=\infty. \tag{24}$$

At time t, the difference of the distribution functions F_{X^t} and F_{Y^t} of the risk reference random variable X^t and the real random variable Y^t for retail supply chain unconventional emergency can be measured as follow:

$$D(F_{X^t}\,\|\,F_{Y^t})=\left\{\sum_{n=1}^{\infty}\frac{[E[(X^t)^n]-E[(Y^t)^n]]^2}{n!}\right\}^{\frac{1}{2}}. \tag{25}$$

Obviously, if all moment differences of the two risk probability distributions are bounded, that is $|E[(X^t)^n] - E[(Y^t)^n]| \le M$, where M is a positive constant, then it will be:

$$D(F_{X^t}\,\|\,F_{Y^t})\le M\sqrt{e-1}. \tag{26}$$

So that the series converges. This measure makes sense.

The measure $D(F_{X^t}\|F_{Y^t})$ does not depend on the specific form of the risk probability distribution for retail supply chain unconventional emergency. It is represented only by the moments of the risk probability distribution, and it has the following characteristics:

(1) Formally, the measure $D(F_{X^t}\|F_{Y^t})$ is the square root of the weighted squares sum (with a weight of $\frac{1}{n!}$) of the origin moment differences of the two risk probability distributions. And as the order increases, the weights of the moment differences of each order decrease. At the same time, the measure $D(F_{X^t}\|F_{Y^t})$ is the increasing function of the origin moment differences for the two risk probability distributions.
(2) The measure $D(F_{X^t}\|F_{Y^t})$ is a distance function that meets non-negativity, symmetry, triangle inequality.
(3) The measure $D(F_{X^t}\|F_{Y^t})$ has good astringency.

The measure $D(F_{X^t}\|F_{Y^t})$ is sensitive to translational and scale transformations of random variable, for which it is necessary to standardize processing risk reference random variable X^t and real random variable Y^t of retail supply chain unconventional emergency in this study.

This study considers the following weighted moment sequences as:

$$\left\{\frac{E[X^t]}{\sqrt{D[X^t]}},\frac{1}{2!}\frac{E[X^t-E[X^t]]^2}{D[X^t]},\frac{1}{3!}E\left[\frac{X^t-E[X^t]}{\sqrt{D[X^t]}}\right]^3,\ldots,\frac{1}{n!}E\left[\frac{X^t-E[X^t]}{\sqrt{D[X^t]}}\right]^n,\ldots\right\} \tag{27}$$

$$\left\{\frac{E[Y^t]}{\sqrt{D[X^t]}},\frac{1}{2!}\frac{E[Y^t-E[Y^t]]^2}{D[X^t]},\frac{1}{3!}E\left[\frac{Y^t-E[Y^t]}{\sqrt{D[Y^t]}}\right]^3,\ldots,\frac{1}{n!}E\left[\frac{Y^t-E[Y^t]}{\sqrt{D[Y^t]}}\right]^n,\ldots\right\} \tag{28}$$

where the expectation and variance are scaled with respect to the variance of X^t. The third-order and more than third-order central moments are scaled with respect to their variance. In this study, the distribution difference of risk reference random variable X^t and real random variable Y^t of retail supply chain unconventional emergency is measured by the distance of the above two infinite dimensional real number sequences, shown as proposition 2.

Proposition 2: If there is a real number $R > 0$, when $|a| < R$, both the moment generating functions $M_{X^t}(a)$ and $M_{Y^t}(a)$ of the risk reference random variable X^t and the real random variable Y^t of retail supply chain unconventional emergency converge, or the moments of X^t and Y^t exist and meet with:

$$\sum_{n=1}^{\infty}\frac{1}{\{E[(X^t)^{2n}]\}^{\frac{1}{2n}}}=\infty,$$

and

$$\sum_{n=1}^{\infty}\frac{1}{\{E[(Y^t)^{2n}]\}^{\frac{1}{2n}}}=\infty.$$

At time t, the difference of the probability distribution functions F_{X^t} and F_{Y^t} for the risk reference random variable X^t and the real random variable Y^t in retail supply chain unconventional emergency can be measured as follow:

$$MD(F_{Y^t}\|F_{X^t})=\frac{\{E[X^t]-E[Y^t]\}^2}{D[X^t]}+\frac{1}{2}\left[1-\frac{D[Y^t]}{D[X^t]}\right]^2 + \sum_{n=3}^{\infty}\frac{1}{n!}\left\{E\left[\frac{X^t-E[X^t]}{\sqrt{D[X^t]}}\right]^n-E\left[\frac{Y^t-E[Y^t]}{\sqrt{D[Y^t]}}\right]^n\right\}^2. \tag{29}$$

$MD(F_{Y^t}\|F_{X^t})$ is called as the difference moment measure of the risk probability distribution for retail supply chain unconventional emergency.

In the risk early warning decision system of retail supply chain unconventional emergency, this study uses the first two terms of Eq. (29), and calculates the difference moment measure value of the risk probability distributions for retail supply chain unconventional emergency based on Eqs. (10)-(13).

$$\begin{aligned}MD(F_{Y^t}\|F_{X^t})&=\frac{\{E[X^t]-E[Y^t]\}^2}{D[X^t]}+\frac{1}{2}\left\{1-\frac{D[Y^t]}{D[X^t]}\right\}^2\\&=\frac{\left\{\frac{1}{c}\sum_{h=1}^{c}\left[cw_h^t(x)\left(\frac{1}{n}\sum_{j=1}^{n}u_{hj}^t(x)\right)\right]-\frac{1}{c}\sum_{h=1}^{c}\left[cw_h^t(y)\left(\frac{1}{n}\sum_{j=1}^{n}u_{hj}^t(y)\right)\right]\right\}^2}{\frac{1}{c-1}\sum_{h=1}^{c}\left[cw_h^t(x)\left(\frac{1}{n-1}\sum_{j=1}^{n}(u_{hj}^t(x)-E_h(X^t))^2\right)\right]}\\&+\frac{1}{2}\left\{1-\frac{\frac{1}{c-1}\sum_{h-1}^{c}\left[cw_h^t(y)\left(\frac{1}{n-1}\sum_{j=1}^{n}(u_{hj}^t(y)-E_h(Y^t))^2\right)\right]}{\frac{1}{c-1}\sum_{h-1}^{c}\left[cw_h^t(x)\left(\frac{1}{n-1}\sum_{j=1}^{n}(u_{hj}^t(x)-E_h(X^t))^2\right)\right]}\right\}^2\end{aligned} \tag{30}$$

4. THE EXAMPLE ANALYSIS

This study takes the MN chain retail supply chain as the research object. The simulation experiment platform is MATLAB R2012b under the Windows 7 operating system. The experimental environment is CPU Intel 2.5GHz 4G memory.

4.1 Parameters Used in the Example

The risk assessment indices and current values of retail supply chain unconventional emergency are shown in Table 1.

4.2 Databases of the Example

The multi-data fusion-based cloud model summary data generation algorithm is used to extract the three-dimensional feature values, namely m = 3 index feature values, and summary center cloud drips r_{ij}^t of the risk assessment index system for retail supply chain unconventional emergency as shown in Table 2.

Table 1. Risk assessment monitoring indices and current values.

Risk dimensions	Monitoring indices	Expectation (E_x)	Entropy (E_n)	Hyper entropy (H_e)
Supply risk dimension	Supplier order satisfaction rate	0.6594	0.5971	0.4124
	Supplier delivery failure rate	0.6594	0.5971	0.4124
	Supplier production flexibility	0.6414	0.3974	0.4851
	Supplier product qualification rate	0.6785	0.3478	0.6471
	Supply chain node enterprise owned rate	0.7921	0.6051	0.5529
	Supply chain core enterprise owned rate	0.7157	0.6133	0.5628
	Production flexibility	0.6982	0.5322	0.3977
	Fraction defective of product	0.8105	0.5199	0.4098
	Core enterprise innovation ability	0.8703	0.5366	0.5492
	Core enterprise overall coordination and control ability	0.6692	0.7022	0.5920
	Profit distribution fair degree	0.5987	0.4901	0.3920
	Contract trust	0.6871	0.5257	0.4688
	Contract execution rate	0.6138	0.3510	0.3188
Demand risk dimension	Final product price level	0.6474	0.3195	0.3368
	Demand fluctuations level	0.7021	0.5089	0.5239
	Key customer churning rate	0.7322	0.6698	0.6240
	Customer satisfaction	0.6912	0.4321	0.4103
Logistics risk dimension	Delaying in delivery rate	0.6171	0.5698	0.3398
	Product damage rate	0.7367	0.5364	0.3912
	Information sharing	0.7684	0.4098	0.4985
	Strength of the bullwhip effect	0.7841	0.6912	0.4681
	Data conversion failure degree	0.6781	0.6681	0.4952
	Information technology and major equipment failure rate	0.7156	0.3098	0.3521
	Cash flow blocking degree	0.7354	0.7098	0.6057
	Return on equity	0.8015	0.6951	0.6685
	Asset-liability ratio	0.8912	0.4523	0.4494
	Quick action ratio of main node enterprises	0.7864	0.3856	0.3691

Table 2. Three-dimensional feature values and summary center cloud drips.

Risk dimensions	Expectation (E_x)	Entropy (E_n)	Hyperentropy (H_e)	Summary center cloud drip (r_{ij}^t)
Supply risk dimension	0.7187	0.5105	0.5012	0.6553
Demand risk dimension	0.6815	0.4578	0.4748	0.4875
Logistics risk dimension	0.7323	0.5432	0.4478	0.5276

For the current data stream of the risk probability real distribution assessment index summaries for retail supply chain unconventional emergency in 2016, according to the k-means algorithm, the feature values of summary center cloud drips for the three risk dimensions at each day are clustered in each month to get c centroids with weights, and set c = 8. After repeating this process by q times (namely, window length), $n = c \times q$ weighted centroids are obtained, which is the number of the summary data samples of risk probability real distribution assessment index system for retail supply chain unconventional emergency, taking q = 12 months. $n = c \times q$ centroid data with weight values are shown in Table 3.

The fuzzy clustering center S_{ih}^t of risk probability real distribution assessment index attribute i for category h in 2016 is shown in Table 4 in retail supply chain unconventional emergency.

The updated fuzzy membership degree u^t of the risk probability real distribution sample j belonging to category h in 2016 is shown in Table 5 in retail supply chain unconventional emergency.

Table 3. Centroid data with weight value.

Numbe rof past months	First dimension	Second dimension	Third dimension
1	0.7254	0.5731	0.6421
	0.6631	0.5127	0.6314
	0.7632	0.4351	0.6421
	0.5731	0.4671	0.7314
	0.6371	0.4276	0.4376
	0.7861	0.6412	0.5314
	0.7112	0.6721	0.4111
	0.5214	0.3147	0.7421
⋮	⋮	⋮	⋮
12	0.0123	0.0657	0.1308
	0.0268	0.0384	0.2014
	0.0364	0.0958	0.0749
	0.0168	0.1035	0.0374
	0.0069	0.0358	0.0981
	0.0514	0.0235	0.1684
	0.0351	0.0098	0.1098
	0.0169	0.0369	0.0951

Table 4. Fuzzy clustering center value.

center	Supply risk dimension	Demand risk dimension	Logistics risk dimension
S_1	0.3854	0.2489	0.1224
S_2	0.2125	0.2089	0.1865
S_3	0.1389	0.1984	0.1668
S_4	0.2089	0.1974	0.1751
S_5	0.3068	0.5174	0.6118
S_6	0.4951	0.3877	0.3106
S_7	0.1415	0.1001	0.0958
S_8	0.0231	0.0148	0.0078

Table 5. Updated fuzzy membership degree matrix U.

CC^1 \ SD^2	r_1	r_2	r_3	r_4	r_5	r_6	r_7	r_8	r_9	r_{10}	…	r_{96}
S_1	0.1548	0.1809	0.0977	0.1404	0.0246	0.0991	0.3301	0.4672	0.2761	0.0329	…	0.0022
S_2	0.5098	0.1483	0.0696	0.2237	0.9035	0.5351	0.1678	0.1004	0.0354	0.0136	…	0.0011
S_3	0.0454	0.0801	0.0981	0.0770	0.0084	0.0406	0.0684	0.0312	0.0679	0.4925	…	0.0162
S_4	0.0860	0.2366	0.1471	0.1138	0.0168	0.0701	0.1263	0.0996	0.2491	0.1187	…	0.0047
S_5	0.1115	0.2306	0.4100	0.2753	0.0306	0.1711	0.1667	0.2503	0.2776	0.0341	…	0.0027
S_6	0.0342	0.0461	0.0544	0.0559	0.0058	0.0289	0.0526	0.0193	0.0355	0.1381	…	0.0134
S_7	0.0303	0.0431	0.0821	0.0673	0.0057	0.0314	0.0460	0.0178	0.0343	0.0956	…	0.9493
S_8	0.0280	0.0342	0.0411	0.0465	0.0047	0.0237	0.0421	0.0143	0.0239	0.0745	…	0.0104

CC^1: clustering center SD^2: sample data

The cluster weight $cw_h^t(h = 1, 2, \ldots, c)$ of the weight vector DW^t for risk severity assessment cluster in 2016 is shown in Table 6, which reflects the importance of risk probability grade for retail supply chain unconventional emergency. The cluster weight value of risk probability grade 3 is the highest, which shows that the unconventional emergency in retail supply chain is small probability event that is not easy to occur, but once it occurs, the degree of harm is very significant.

Table 6. The cluster weight value.

Cluster / Numerical value	cw_1	cw_2	cw_3	cw_4	cw_5	cw_6	cw_7	cw_8
Cluster weight value	5.5571	7.2858	10.6585	5.027	5.4678	8.046	6.9475	3.0103

4.3 Difference Moment Measure Values of Risk Probability Distributions

In this study, the historical data stream of risk assessment monitoring index summaries for the long-term operation is used as the summary data stream of risk probability reference distribution for retail supply chain unconventional emergency in 2016. Combined with the summary data stream of risk probability real distribution, the difference moment measure value of the risk probability distributions for retail supply chain unconventional emergency is calculated by Eq. (30) in 2016.

$$MD(F_{Y^t} \parallel F_{X^t}) = \frac{\left\{\frac{1}{c}\sum_{h=1}^{c}\left[cw_h^t(x)\left(\frac{1}{n}\sum_{j=1}^{n}u_{hj}^t(x)\right)\right] - \frac{1}{c}\sum_{h=1}^{c}\left[cw_h^t(y)\left(\frac{1}{n}\sum_{j=1}^{n}u_{hj}^t(y)\right)\right]\right\}^2}{\frac{1}{c-1}\sum_{h=1}^{c}\left[cw_h^t(x)\left(\frac{1}{n-1}\sum_{j=1}^{n}(u_{hj}^t(x)-E_h(X^t))^2\right)\right]}$$
$$+\frac{1}{2}\left\{1-\frac{\frac{1}{c-1}\sum_{h=1}^{c}\left[cw_h^t(y)(\frac{1}{n-1})\sum_{j=1}^{n}(u_{hj}^t(x)-E_h(X^t))^2\right]}{\frac{1}{c-1}\sum_{h=1}^{c}\left[cw_h^t(x)(\frac{1}{n-1})\sum_{j=1}^{n}(u_{hj}^t(x)-E_h(X^t))^2\right]}\right\}^2$$
$$= 0.817 \ (h = 1, \ldots, 8;\ j = 1, \ldots, 96).$$

It can be seen from the simulation result that the difference moment measure value of the probability distribution functions F_{Y^t} and F_{X^t} of the risk real random variable Y^t and the risk reference random variable X^t is larger in 2016. The risk probability real distribution has a wider range of change relative to the risk probability reference distribution in robust multi criteria early warning decision model for retail supply chain unconventional emergency.

Similarly, according to the summary data streams of the risk assessment monitoring indices for retail supply chain unconventional emergencies in 2014 and 2015, the difference moment measure values of the probability distributions of unconventional emergencies are calculated as shown in Table 7.

Table 7. Difference moment measure values of risk probability distributions.

Parameter / Year	cw_1	cw_2	cw_3	cw_4	cw_5	cw_6	cw_7	cw_8	MD^1
2014	2.7392	4.9064	7.3851	2.1593	2.6438	6.3820	5.0631	1.9532	0.3527
2015	4.9058	6.2750	9.3852	4.1580	4.6384	7.3814	5.7312	2.1649	0.6841
2016	5.5571	7.2858	10.6585	5.0270	5.4678	8.0460	6.9475	3.0103	0.8179

MD^1: $MD(F_{Y^t}||F_{X^t})$

As shown in Table 7, with the change of summary data stream parameter values of unconventional emergency risk assessment indices in retail supply chain, the cluster weight value $cw_n^t(h = 1, 2, \ldots, c)$ of the weight vector DW^t of unconventional emergency risk severity assessment cluster increases continuously, that is to say that the cluster weight value of risk probability grade increases continuously indicating increasing probability of the retail supply chain unconventional emergency.

Because of the increasing value of the difference moment measure $MD(F_{Y^t}||F_{X^t})$ of the risk probability distributions for retail supply chain unconventional emergency from 2014 to 2016, the Knight uncertainties faced by decision makers are increasing. The more the real distribution of risk probability deviates from the reference distribution of risk probability corresponding with the normal operation of retail supply chain, the higher the early warning grade of unconventional emergencies risk in retail supply chain will be.

4.4 Theoretical Comparison Analysis between Difference Moment Measure of Risk Probability Distributions and Common Indices

The common indices for measuring the differences of probability distributions are Csiszar-f metrics and Bregman metrics. The common indices are established by the specific density function or distribution law of the probability distribution. In this study, the relative entropy is taken as an example to achieve theoretical comparison analysis between difference moment measure $MD(F_{Y^t}||F_{X^t})$ of risk probability distributions and common indices in the retail supply chain unconventional emergency.

Definition 2: The relative entropy of the random variable X (distribution column: $P(X = x_i) = p_i$, $i = 1, 2, \ldots, n_D$) relative to Y (distribution column: $P(Y = y_i) = q_i$, $i = 1, 2, \ldots, n_D$) is defined as:

$$D(X \,||\, Y) = \sum_{i=1}^{n_D} p_i \ln \frac{p_i}{q_i}.$$

The relative entropy of the probability density function $f(x)$ relative to $g(x)$ (also known as Kullback-Leibler information) is defined as:

$$D(f \,||\, g) = E_f\left[\ln \frac{f(x)}{g(x)}\right] = \int f(x) \ln \frac{f(x)}{g(x)} dx.$$

The relative entropy $D(f||g)$ metric measures the difference between probability distributions from the perspective of information degradation, and measures the information lost when the prior distribution $g(x)$ is used to replace the unknown real distribution $f(x)$. The difference between the two probability distributions can be measured to some extent. The larger the $D(f||g)$ is, the greater $f(x)$ deviates from $g(x)$. Relative entropy has asymmetry and does not satisfy the triangle inequality, so it is not a true distance metric.

The difference moment measure $MD(F_{Y^t}||F_{X^t})$ of risk probability distributions for retail supply chain unconventional emergency measures the information lost when the reference distribution F_{X^t} is used to replace the unknown real distribution F_{Y^t}, and can measure the difference between the two probability distributions from the deviation of the first two moments of the probability distribution to some extent. Compared with relative entropy, $MD(F_{Y^t}||F_{X^t})$ has the following advantages in applications:

(1) Relative entropy needs to be calculated using the density function or distribution law of the probability distribution. If the specific form of the probability distribution for random variable can be obtained, the relative entropy calculation of the probability distribution often involves complex integral operation or summation operation, which is very cumbersome and even impossible to obtain analytical solutions. In Knight uncertain environment, the specific probability distribution form of retail supply chain unconventional emergency risk is often difficult to obtain, while only the moments of the risk probability distribution can be obtained. The relative entropy can not be calculated.

The difference moment measure $MD(F_{Y^t}||F_{X^t})$ of risk probability distributions for retail supply chain unconventional emergency only depends on the first two moments of the risk probability distribution, which can be obtained according to the big data statistical method based on fuzzy data stream clustering algorithm. It is a nonparametric index that can be easily calculated, which greatly facilitates its application.

(2) In Knight uncertain environment, using a single risk probability real distribution for early warning decision brings certain risks in the retail supply chain unconventional emergency. Based on the historical data and other information, a maximum entropy decision model is established to get the risk probability reference distribution in this study. The neighborhood of the risk probability reference distribution is used to describe the uncertainties of probability distribution for the risk real random variable of retail supply chain unconventional emergency. Due to the difficulty of obtaining the specific probability distribution form of risk real random variable, it is barely possible to calculate the common probability distribution difference measures such as relative entropy.

The difference moment measure $MD(F_{Y^t}||F_{X^t})$ of risk probability distributions for retail supply chain unconventional emergency eliminates the dependency on the specific probability distribution form. The deviation of the first two moments for the real distribution F_{Y^t} and the reference distribution F_{X^t} is calculated by big data statistic method, which approximately describes the degree of deviation between the real distribution and the reference distribution of the risk probability for unconventional emergency. Furthermore, the uncertainties of the probability distribution for risk real random variable of retail supply chain unconventional emergency is approximately described by a neighborhood $\{F_{Y^t}|MD(F_{Y^t}||F_{X^t}) \leq \eta\}$ of the reference distribution based on the risk probability distribution difference moment measure. It is helpful to improve the accuracy of the robust multi criteria early warning decision for unconventional emergency risk in the retail supply chain.

Retail supply chain unconventional emergency risk has the characteristics of coupling, derivative, sudden, destructive, persistent and significant impact [32, 33]. Under Knight uncertain environment, combined with the summary data stream parameter values of the risk assessment indices for the MN retail supply chain unconventional emergency, this study uses the fuzzy data stream clustering algorithm to obtain the updated fuzzy membership degree u_{hj}^t of the risk assessment sample j belonging to category h and the cluster weight cw_h^t for risk severity assessment cluster in retail supply chain unconventional emergency. The difference moment measure value $MD(F_{Y^t}\|F_{X^t})$ of the risk probability real distribution and the reference distribution is calculated depending only upon the moments for the risk probability distribution in retail supply chain unconventional emergency, which verifies the feasibility of the difference moment measure method for unconventional emergency risk probability distribution based on moment generating function and fuzzy data stream clustering proposed in this study.

5. CONCLUSIONS

The research of the robust multi-criteria early warning decision involves the uncertainty of the probability distribution for risk random variable of retail supply chain unconventional emergency. In Knight uncertain environment, using the single risk probability real distribution of unconventional emergency for early warning decision brings certain risks. Therefore, under the uncertain environment of early warning decision model, the moments of the risk probability reference distribution are determined based on summary historical data stream of long-term operational risk assessment monitoring indices, while the moments of the risk probability real distribution are determined based on summary current data stream of risk assessment monitoring indices in retail supply chain unconventional emergency. The difference between the moment generating functions of retail supply chain unconventional emergency risk is measured by the distance function in the real vector space of infinite dimension moments. The deviation range between the real distribution and the reference distribution of the risk probability for unconventional emergency is further evaluated based on the moments. The uncertainty of the robust multi-criteria early warning decision model is described by multiple risk probability real distributions for unconventional emergency based on the deviation range. This can avoid the uncertain risk of the robust multi-criteria early warning decision-making and make the robust multi-criteria early warning decision-making optimal within a certain setting error range of probability distribution model.

The early warning decision model of retail supply chain unconventional emergency risk is established base on the robust multi-criteria early warning decision model for maximum and minimum expected utility. The uncertainty influence of the robust multi-criteria early warning decision model shows as the uncertainty influence of the probability distribution for risk real random variable of retail supply chain unconventional emergency. In the Knight uncertain environment, the moments of the probability distribution rather than the specific probability distribution form of unconventional emergency risk can be obtained. The main contribution of this study is the proposal of a new difference moment measure method of risk probability distribution for retail supply chain unconventional emergency based on the fuzzy statistical method of big data with cloud model theory [34, 35], moment generating function theory [36, 37] and functional theory [38, 39] in Knight uncertain and big date environment, which can overcome the drawbacks of the difference measure methods for probability distributions presented in [5-7]. In the future, we will apply the difference moment measure method of risk probability distributions to solve the robust multi-criteria decision-making problems of risk early warning in retail supply chain unconventional emergency in Knight uncertain and big data environment.

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