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A Hesitant Fuzzy Set Theory Based Approach for Project Portfolio Selection with Interactions under Uncertainty

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This paper proposes a new practical model for project portfolio selection focusing on two issues; the first issue relates to managing uncertainty provoked by hesitant situations and the second issue relates to the interaction effect of projects on final value of portfolio. Based on the above issues, the hesitant fuzzy weighted averaging (HFWA) operator is applied to aggregate the hesitant fuzzy information corresponding to each project and its interactions. A linear programming model is proposed to optimize project portfolio selection problem considering interactive project sets and its dependency unlike the previously literature. Finally, the effectiveness of the proposed technique is illustrated by means of a practical example.

Keywords: project portfolio selection, project interaction, uncertainty, hesitant fuzzy set, group decision making

1. INTRODUCTION

In today's highly competitive economic environment, selection of right sets of projects or optimizing project portfolio is of paramount importance for any organization's striving to achieve competitive advantages. Generally, such selection should be made based on different criteria regarding limited resources and various scarcities. Hence, the project portfolio selection is usually known as a typical multi-decision-making (MCDM) problem [12-14, 16-18, 23, 24, 37, 38, 43].

One of the challenges facing the issue of portfolio selection is that, in real world situation, DMs may not be sure about the providing value of alternatives because of inaccurate or insufficient information [28]. Therefore, uncertainties are usually involved in project assessment such as return of project, alignment of project with organization's strategies, project interactions, *etc.* Application of fuzzy set theory can be a promising approach to tackle this kind of uncertainty. In recent years, utilization of fuzzy techniques in portfolio selection models is being received more and more attention from re-

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searchers dealing with uncertain parameters. [7] defines cash flows of projects by triangular fuzzy numbers and develops a new method to specify the preference of fuzzy projects. [11] apply a constrained fuzzy AHP model to solve project portfolio selection as a multi-goals and multi-attributes problem. [44] present a fuzzy zero-one integer programming model to distinguish the optimal R&D project portfolio handling both uncertain and flexible parameters. [6] propose a fuzzy mixed-integer programming model for R&D project portfolio selection. [19] provide a fuzzy binomial approach to evaluate a project and develop a suitable method to compute the mean value of fuzzy NPV. [35] investigate the (r, Q) policy to minimize the cost function assuming lead time as a fuzzy variable while demand rate, holding cost and shortage cost are sensitive to imprecise selling price. [53] construct a general project portfolio optimization model assuming the investment capitals and the net cash flows of the projects as fuzzy variables. [8] foster a modular decision support system to optimize investment project portfolio in the presence of uncertainty. [31] propose a two-stage project portfolio selection model, in which the return and cost of projects are characterized by triangular fuzzy number. [4] considers different type of interdependencies and proposes a grey theory based method to cope with the uncertainty. [39] suggest a fuzzy hybrid method applying DEA, TOPSIS and integer programming for selecting an optimal set of projects. [30] present a scenario based mathematical model in accordance with uncertainty of parameters. [27] introduce a multi-objective fuzzy model for project portfolio selection in a public organization. [15] discuss a new fuzzy economic order quantity inventory model allowing backorders which consider human learning over the planning horizon. [33] study a periodic review fuzzy inventory model considering lead time, reorder point, and cycle length as decision variables. Reviewing literature addresses different fuzzy approaches to tackle uncertainty and manage vague and imprecise information as a significant challenging research issue in project portfolio selection. However, in the existing literature of project portfolio selection, when imprecise and vague information comprise of two or more sources of vagueness simultaneously, DMs evaluate projects based on each criterion providing a few different possible values, not by a margin of error, or some possibility distribution on the possible values. To fill this gap, hesitant fuzzy sets (HFSs) are utilized in project portfolio selection. Nevertheless, dealing with imprecise and vague information is limited when two or more sources of vagueness appear simultaneously. Consequently, different extensions of fuzzy sets are introduced such as (i) Type 2 fuzzy sets [9, 26], and type n fuzzy sets [9], that incorporate uncertainty about the membership function in their definition. (ii) [3] intuitionistic fuzzy that extends fuzzy sets by an additional uncertainty degree named non-membership degree, (iii) Fuzzy multi-sets [50] dependent on multi-sets that allow elements repeated in the set. However, in some cases, DMs evaluate alternatives based on each criterion providing a few different possible values, not by a margin of error, or some possibility distribution on the possible values. Owing to this fact, [42] introduce a new extension of fuzzy sets called hesitant fuzzy sets (HFSs) to deal with such cases which allows the membership degree of an element to a set presented as a set of possible values.

Regarding the significant compatibility of hesitant fuzzy set with decision making problems, more and more researchers are being attracted to develop different methods and theories under hesitant fuzzy environment [32, 48, 54]. Several types of operators are proposed to rank alternatives with such types of information [40, 45, 55]. However, there exist a few researches on practical application of HFSs in real world problems.

In project portfolio selection, the decision makers are usually required to evaluate projects and their interactions under different criteria in order to select the best set of projects. However, it is quite difficult to achieve final agreement when the decision makers are hesitant and irresolute for one thing or another. To cope with such cases, we utilized HFSs in project portfolio selection to consider decision maker's hesitancy in providing membership degree of both projects and their interaction effects in terms of different criteria. Then, the hesitant fuzzy information corresponding to each project and project interactions are aggregated employing the hesitant fuzzy weighted averaging (HFWA) operator [46]. Regarding different constraints and scarcities, a stand-alone multi-criteria ranking or scoring operator may not be sufficient to find out the best set of projects. Therefore, the aggregated hesitant fuzzy information is applied in a linear mathematical programming model. Here, the aggregating operator should provide score of each project and not rank them. That is why, we use hesitant fuzzy weighted averaging (HFWA) operator among different aggregating operator.

To be compatible with real world situation, the proposed model considers project interactions in terms of identified "interactive project sets" and their dependencies. Here, an interactive project set is a set of projects having interactive effect with each other when all are selected. It is a common situation that the interactive effect of a project set such as (p_1, p_2, p_3, p_4) be different from its subsidiaries interactive project set such as (p_1, p_2) and dependent of interactive project sets should be noted , unlike the previous literature. Moreover, we formulate the interaction effects as linear terms. Accordingly, the proposed model benefits the easiness of linear programming, while previous models are complicated by quadratic terms [5, 36, 52]. In summery the main motivations of this study are: (1) To capture the uncertainties related to decision maker's hesitancy in providing membership degree of both projects and project interactions applying hesitant fuzzy set theory, (2) To consider the effect of all interactive project sets and their dependencies on portfolio selection, (3) To linearize interaction effects, which are generally formulated in quadratic terms, (4) To demonstrate the efficiency of the proposed model in practice.

The remainder of this study is organized as follows. In Section 2, some basic concepts related to hesitant fuzzy sets are introduced. In next section 3, the project portfolio selection problem is formulated to deal with project interactions with hesitant fuzzy information, which is presented as a 0-1 linear programming. In the proposed model, the values of project interactions and project parameters such as return, strategy alignment and probability of success take the form of hesitant fuzzy numbers, while the information about attribute weights is completely known. Section 4 points out an illustrative example. Section 5 concludes the paper and gives some remarks.

2. PRELIMINARIES

Generally, the decision makers are not certain about a value, but have hesitancy between different possible, when they evaluate the degrees such that an alternative should satisfy an attribute [47]. Accordingly, [41, 42] introduce a new generation of fuzzy sets called hesitant fuzzy set (HFS) to deal with such cases. Some preliminary concepts related to hesitant fuzzy sets are introduced in the following.

Definition 1: [41, 42] Let X be a fixed set. A hesitant fuzzy set (HFS) on X is in terms of a function which returns a subset of [0, 1] when applying to X. To be easily understood, [45] express the HFS by mathematical symbol:

$$A = \{\langle x, h_A(x)\rangle\} \; x \in X\} \tag{1}$$

where $h_A(x)$ is a set of some values in [0, 1], denoting the possible membership degrees of the element $x \in X$ to the set A. [46] denote $h = h_A(x)$ a hesitant fuzzy element (HFE) and Θ the set of all hesitant fuzzy elements (HFEs).

Definition 2: Let $E = \{h_1, h_2, \ldots, h_n\}$ be a set of n HFEs, Θ be a function on E, Θ: $[0,1]^N \rightarrow [0,1]$, then

$$\Theta_E = \cup_{\gamma \in \{h_1 \times h_2 \times \ldots \times h_n\}} \{\Theta(\gamma)\}. \tag{2}$$

Based on Definition 2 and the defined operations for HFEs, [46] propose a series of aggregation operators for HFEs which are applied to merge a set of input values into a single representative output.

Definition 3: [45] Let $h_i(i = 1, 2, \ldots, n)$ be a collection of HFEs. A hesitant fuzzy weighted averaging (HFWA) operator is a mapping $H^n \rightarrow H$ such that

$$HFWA(h_1, h_2, \ldots, h_n) = \oplus w_i h_i = \cup_{\gamma_1 \in h_1, \gamma_2 \in h_2, \ldots, \gamma_n \in h_n} \{1 - \prod_{i=1}^{n} (1-\gamma)^{w_i}\} \tag{3}$$

where $w = (w_1, w_2, \ldots, w_n)^r$ be the weight vector of $h_i(i = 1, 2, \ldots, n)$, and $w_j > 0$, $\sum_{j=1}^{n} w_j = 1$.

Definition 4: [45] For an HFEh, $s(h) = \frac{1}{l_h} \sum_{\gamma \in h} \gamma$ is called the score of h, where l_h is the number of the elements in h. For two HFEs h_1 and h_2, if $s(h_1) > s(h_2)$, then h_1 is superior to h_2 and denoted by $h_1 > h_2$; If $s(h_1) = s(h_2)$, then $h_1 = h_2$.

3. PROJECT PORTFOLIO MODELING UNDER HESITANT FUZZY SETS

The problem of project portfolio selection and optimization arises when the proposed projects or investment opportunities to the organization are greater than its available funds and resources. In this condition, realization of the goals and mission of the organization be contingent upon the proper and limited selection of proposed projects. The decision-makers, therefore, always face two options, *i.e.*, whether to select a project or not. So, one of the better methods for modeling these kinds of problems is 0-1 programming model. Utilizing 0-1 programming model, we consider N projects for evaluation and selection based on several criteria, and decision variable x_i denotes whether proposed project p_i (i = 1, 2, ..., I) is included in the portfolio (x_i = 1) or not (x_i = 0). Hence, a project portfolio can be indicated by the value of $x = (x_1, x_2, \ldots, x_I)$. Due to hesitation in project evaluations on different criteria, DMs may provide different values for the project p_i based on the criteria c_j (j = 1, 2, …, J). These values are considered as a hesitant fuzzy element HFE, h_{ij}. Where the same value represented by different DMs emerges only once in h_{ij}. It should be noted that all criteria are of the same type. Otherwise, the criteria values of cost type should be normalized as follow:

$$\acute{h}_{ij} = \cup_{\gamma_{ij}\in h_{ij}}\{1-\gamma_{ij}\} \tag{4}$$

A schematic diagram of project portfolio selection process with interactions under hesitant fuzzy environment is showed in the following flowchart (Fig. 1)

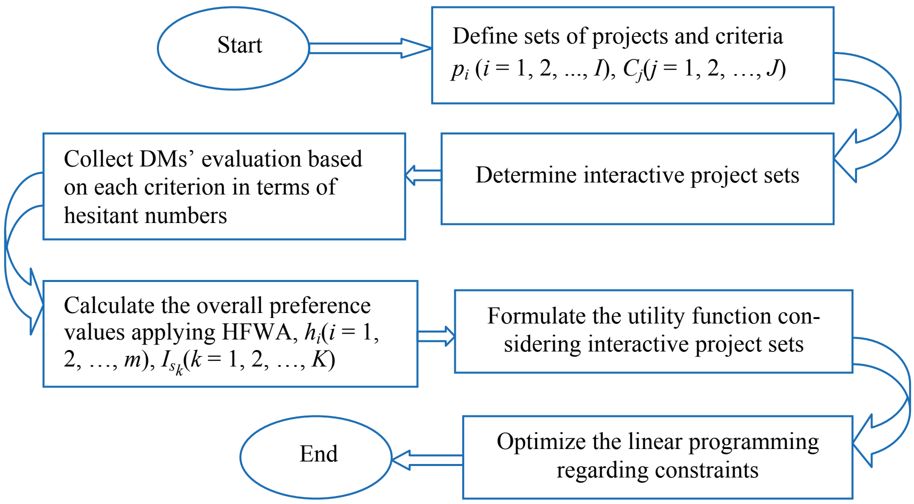

Fig. 1. Schematic diagram of the model.

3.1 Algorithm with Discussion of the Model

The algorithm of project portfolio modeling applying hesitant fuzzy sets is formulated as follows:

Step 1: Calculate the overall preference values, h_i (i = 1, 2, …, m) of the project p_i can be calculated based on the hesitant fuzzy weighted averaging (HFWA) operator.

Step 2: Compute the score value $S(h_i)$(i = 1, 2, …, m), of project p_i (i = 1, 2, …, m) based on definition 4.

Step 3: Formulate the utility function of project portfolio selection problem can be formulated as the following objective:

$$Max\ U = \sum_{i=1}^{m} S(h_i)x_i \tag{5}$$

where U demonstrates the total utility values of the selected project portfolio when projects are assumed independent. In practice, there may be some interactions among projects based on different criteria. Accordingly, decision making considering no interaction may leads to an undesirable outcome. However, literature refers some articles which consider synergies or interaction effects among projects in project portfolio selection models by adding a quadratic terms [5, 52] which intensify the complexity of the model.

Recently, [36] propose a linear formula to apply interaction effects on portfolio selection. They identify different sets of projects with interaction S_k and define a set of binary variable AS_k to consider interaction effects of projects belonging to the set S_k which is equal to 1 only when all members of the set S_k are selected. In spite of the ability to consider interaction effect of more than two projects, the weakness of the proposed model appears when at least one defined set of project, for instance S_1, be subset of another set such as S_2, and the selected portfolio includes all members of S_2. In such cases, both binary variables referring to S_1 and S_2 are equal to 1 and project interactions of both sets are added. While S_1 is a subset of S_2, interaction effect of the members with each other and the other members of S_2 is considered as project interaction of set S_2. Regarding the weakness of former model, we formulate the interaction effects as a new linear programming model. Let $\boldsymbol{I}_{s_k j}$ be the interactive effect of projects belong to set S_k on the criterion j, and S_k (k = 1, 2, …, K) be the set of projects that simultaneous selection of which will lead to interaction effects. Here, $\boldsymbol{I}_{s_k j}$ is considered as hesitant fuzzy element.

Step 4: Apply the HFWA operator (Definition 3) to calculate the score of the overall hesitant fuzzy preference values $S(I_{s_k})$ of interactive effect related to set S_k. The interactive effects are occurred only when all projects belonging to set S_k are selected.
Step 5: Use a binary variable M_{s_k} to formulate the interactive effects as a linear term, which will be equal to 1 only when all projects belonging to set S_k are selected.
Step 6: Optimize the following linear programming for project portfolio selection considering interactive effects.

$$Max\ V = \sum_{i=1}^{m} S(h_i)x_i + \sum_{k=1}^{K} S(I_{s_k})M_{s_k} \tag{6}$$

such that $M_{s_k} \le \min(x_i)\ \forall i \in S_k$, $k = (1, 2, \ldots, K)$; $M_{s_k} + M_{s_{k'}} \le \min(x_i)\ \forall S_{k'} \subset S_k$, $k, k' = (1, 2, \ldots, K)$, $k \ne k'$; $M_{s_k} = \{0, 1\}$.

There are usually different constraints that restrict project portfolio selection. The most important constraints are resource constraints such as budgetary, human resource and equipment constraint. These kinds of restrictions can be modeled as follow:

$$\sum_{i=1}^{n} r_{ij} x_i \le R_j\ (j = 1, 2, \ldots, r)$$

where r_{ij} denotes requirement amount of resource j for project i and R_j denotes total available resource j. Moreover, there might be some mandatory projects which should be included in portfolio based on certain considerations. Following constraint refer to such considerations: $x_i = 1$, $\forall X_i \in S_m$ where S_m be the set of mandatory projects. Additionally, regarding logical constraint, there might be some complementary or mutually exclusive projects. For instance, projects 1 must be selected if portfolio includes project 4, or project 2 and 3 are mutually exclusive. These constraints have been modeled as follows: $x_4 - x_1 \le 0$; $x_2 + x_3 \le 1$. The project portfolio selection model presented above is a general model which considers DMs hesitancy in project evaluation. However, there might be other constraints in different situations that can be added when it requires.

4. NUMERICAL EXAMPLE

In this section, a numerical example is presented to illustrate how the decision maker's hesitancy could be applied on project evaluation and portfolio selection. The example is based on a real context and relates to project portfolio selection of a project-oriented organization which wants to select the best set of project from 20 candidates' projects. However, real data have not been applied for reasons of secrecy. Selection should be done based on three maximization type criteria: expected return of projects, project alignment with organization's strategies and the organization's ability to successfully implementation of projects which is considered as probability of success. Three decision makers are asked to evaluate projects in terms of proposed criteria. In order to avoid impress each other, the decision makers are desired to appraise projects in anonymity. The decision makers' opinions are combined and hesitant fuzzy decision matrix is expressed in Table 1.

Table 1. The evaluation values of projects and project interactions on criteria given by three decision makers.

	Return	Strategy alignment	Probability of success
p_1	(0.63 0.18)	(0.76 0.69 0.13)	(0.30 0.51)
p_2	(0.09 0.73 0.04)	(0.92 0.77)	(0.78 0.87)
p_3	(0.55 0.03 0.90)	(0.49 0.72 0.38)	(0.78 0.80 0.27)
p_4	(0.39 0.24)	(0.23 0.94 0.79)	(0.53 0.37 0.58)
p_5	(0.6 0.89)	(0.28 0.92)	(0.68 0.49)
p_6	(0.37 0.96 0.47)	(0.88 0.73)	(0.93 0.83 0.67)
p_7	(0.06 0.56 0.66)	(0.22 0.23 0.20)	(0.59 0.55 0.13)
p_8	(0.58 0.18 0.96)	(0.77 0.72)	(0.14 0.51 0.92)
p_9	(0.76 0.75)	(0.94 0.98 0.60)	(0.79 0.68)
p_{10}	(0.73 0.97)	(0.65 0.06 0.30)	(0.95 0.32 0.58)
p_{11}	(0.99 0.29)	(0.24 0.58 0.06)	(0.24 0.84)
p_{12}	(0.59 0.35 0.65)	(0.68 0.24)	(0.63 0.53 0.89)
p_{13}	(0.78 0.28 0.74)	(0.40 0.42 0.85)	(0.63 0.16)
p_{14}	(0.88 0.53 0.88)	(0.78 0.6)	(0.37 0.56)
p_{15}	(0.35 0.84 0.14)	(0.75 0.62 0.91)	(0.64 0.06 0.91)
p_{16}	(0.31 0.08 0.12)	(0.56 0.27 0.89)	(0.98 0.91)
p_{17}	(0.05 0.36 0.50)	(0.16 0.61)	(0.90 0.59 0.71)
p_{18}	(0.51 0.07 0.39)	(0.12 0.38)	(0.77 0.73 0.45)
p_{19}	(0.73 0.43 0.69)	(0.29 0.93)	(0.62 0.07)
p_{20}	(0.37 0.8)	(0.28 0.65 0.39)	(0.04 0.32)

In addition to project evaluation, decision makers introduce different sets of interactive projects and investigate their interaction effects on mentioned criteria listed as hesitant fuzzy decision matrix in Table 2.

Regarding the weight vector of each criteria which is assumed to $\omega = (0.5, 0.2, 0.3)$ and based on Eq. (3), the overall preference value $h_i(i = 1, 2, \dots, 20)$ of projects $p_i(i = 1, 2, \dots, 20)$ and the overall preference value of project interactions $I_{s_k}(k = 1, 2, \dots, 10)$ are derived. For example, the overall preference value of p_1 is obtained as follow:

Table 2. The effect of different project sets on return, strategy alignment and probability of success.

Project set	Return	Strategy alignment	Probability of success
$S_1\{p_1, p_2\}$	(0.17 0.13 0.71)	(0.96 0.92 0.91)	(0.82 0.77 0.86)
$S_2\{p_3, p_5\}$	(0.04 0.71 0.22)	(0.45 0.10 0.60)	(0.03 0.26 0.81)
$S_3\{p_2, p_3, p_5\}$	(0.52 0.65 0.94)	(0.24 0.65 0.07)	(0.46 0.83 0.20)
$S_4\{p_1, p_3, p_4\}$	(0.62 0.88 0.76)	(0.77 0.97 0.71)	(0.46 0.41 0.14)
$S_5\{p_7, p_{10}, p_{15}\}$	(0.30 0.51)	(0.28 0.37 0.25)	(0.29)
$S_6\{p_{17}, p_{18}\}$	(0.97 0.58)	(0.32 0.07 0.39)	(0.61 0.57 0.09)
$S_7\{p_{14}, p_{10}, p_2\}$	(0.93)	(0.12 0.46)	(0.12 0.51 0.87)
$S_8\{p_{12}, p_{20}, p_{14}, p_2\}$	(0.55 0.68)	(0.86 0.64 0.05)	(0.55 0.57 0.80)
$S_9\{p_{14}, p_{12}\}$	(0.95 0.58)	(0.82 0.16 0.93)	(0.49 0.65)
$S_{10}\{p_{16}, p_2\}$	(0.81 0.11 0.68)	(0.09 0.34)	(0.92 0.16 0.52)

$$
\begin{aligned}
h_1 &= HFWA(h_1, h_2, \ldots, h_{20}) = HFWA\{(0.63, 0.18), (0.76, 0.69, 0.13), (0.3, 0.51)\} \\
&= \oplus w_i h_i \\
&= \cup_{\gamma_1\in h_{11}, \gamma_2\in h_{12}, \ldots, \gamma_n\in h_{1n}}\{1-\textstyle\prod_{i=1}^{3}(1-\gamma_{1i})^{w_i}\} \\
&= \cup_{\gamma_1\in h_{11}, \gamma_2\in h_{12}, \ldots, \gamma_{20}\in h_{120}}\{1-(1-\gamma_{11})^{0.5}(1-\gamma_{12})^{0.2}(1-\gamma_{13})^{0.3}\} \\
&= \{0.42, 0.52, 0.61, 0.57, 0.41, 0.52, 0.47, 0.32, 0.45, 0.39, 0.29, 0.36, 0.12, 0.29, \\
&\quad 0.21, 0.63, 0.59\}
\end{aligned}
$$

Thus, the scores of the overall hesitant fuzzy preference values of projects are equal to $s(h_1) = 0.64$, $s(h_2) = 0.67$, $s(h_3) = 0.6$, $s(h_4) = 0.5$, $s(h_5) = 0.72$, $s(h_6) = 0.76$, $s(h_7) = 0.41$, $s(h_8) = 0.67$, $s(h_9) = 0.8$, $s(h_{10}) = 0.78$, $s(h_{11}) = 0.67$, $s(h_{12}) = 0.6$, $s(h_{13}) = 0.58$, $s(h_{14}) = 0.67$, $s(h_{15}) = 0.62$, $s(h_{16}) = 0.71$, $s(h_{17}) = 0.52$, $s(h_{18}) = 0.45$, $s(h_{19}) = 0.6$, $s(h_{20}) = 0.48$. And, the scores of the overall hesitant fuzzy preference values of interactive project sets are $s(I_{s_1}) = 0.75$, $s(I_{s_2}) = 0.4$, $s(I_{s_3}) = 0.67$, $s(I_{s_4}) = 0.72$, $s(I_{s_5}) = 0.36$, $s(I_{s_6}) = 0.68$, $s(I_{s_7}) = 0.81$, $s(I_{s_8}) = 0.63$, $s(I_{s_9}) = 0.75$, $s(I_{s_{10}}) = 0.54$. Now, the obtained scores should be utilized to find out the optimum project portfolio. Regarding Table 2, interactive project sets S_2 and S_9 are subsets of S_3 and S_8, respectively. Therefore, to appraise the effect of the proposed model on the final portfolio of projects, we consider two cases. Case 1 is related to the proposed approach, which models the situation that at least one defined set of interactive projects be subset of another set as new constraints.

$$
\begin{aligned}
\text{Max } V &= 0.64x_1 + 0.67x_2 + 0.6x_3 + \ldots + 0.48x_{20} \\
&\quad + 0.75M_{s_1} + 0.4M_{s_2} + 0.67M_{s_3} + \ldots + 0.54M_{s_{10}}
\end{aligned} \tag{7}
$$

such that $M_{s_1} \le \min(x_1, x_2)$; $M_{s_2} \le \min(x_3, x_5)$; …; $M_{s_8} \le \min(x_{12}, x_{14}, x_2, x_{20})$; $M_{s_9} \le \min(x_{14}, x_{12})$; $M_{s_{10}} \le \min(x_{16}, x_2)$; $M_{s_2} + M_{s_3} \le 1$; $M_{s_9} + M_{s_8} \le 1$; $M_{s_k} = 0, 1$ ($k = 1, 2, \ldots, 10$). As interactive project sets, S_2 and S_9, are subsets of S_3 and S_8, respectively. The constraints $(M_{s_2} + M_{s_3}) \le 1$ and $(M_{s_9} + M_{s_8}) \le 1$ are considered to prevent adding surplus interactive effects on utility value of selected portfolio. In case 2, the interactive effects on utility value of selected portfolio is formulated based on Shaksi-Niaei *et al.* [36] by the following constraints, which determine the value of binary variable M_{s_k}: $M_{s_k} \le x_i\, \forall k$, $(i) i\in S_k$); $M_{s_k} \ge \sum_{i\in s_k} x_i - \text{len}(S_k)$ where $\text{len}(S_k)$ indicates the length of set S_k. Therefore, the project portfolio would be formulated as follow:

$$\text{Max } V = 0.64x_1 + 0.67x_2 + 0.6x_3 + \ldots + 0.48x_{20} + 0.75M_{s_1} + 0.4M_{s_2} + 0.67M_{s_3} + \ldots + 0.54M_{s_{10}} \quad (8)$$
$$M_{s_1} \le x_j,\ \forall j \in S_1;\ M_{s_1} \ge \textstyle\sum_{j\in s_1} x_j - 3;\ M_{s_2} \le x_j,\ \forall j \in S_2;\ M_{s_2} \ge \textstyle\sum_{j\in s_2} x_j - 3;$$
$$M_{s_3} \le x_j,\ \forall j \in S_3;\ M_{s_3} \ge \textstyle\sum_{j\in s_3} x_j - 4;\ M_{s_{10}} \le x_j,\ \forall j \in S_{10};\ M_{s_{10}} \ge \textstyle\sum_{j\in s_{10}} x_j - 4;$$

In addition, there exist three kinds of resource constraints that restrict project portfolio selection. Accordingly, it is necessary to consider resource consumption of projects. Total available resources and resource consumption of each project are presented in Table 3. Both cases are solved by using Lingo package and results are listed in the Table 4.

Table 3. Resource constraints.

Project	Resource1	Resource 2	Resource 3
p_1	150.00	14.00	8.00
p_2	600.00	11.00	1.00
p_3	350.00	8.00	5.00
p_4	700.00	4.00	13.00
p_5	900.00	4.00	4.00
p_6	470.00	10.00	13.00
p_7	240.00	14.00	4.00
p_8	390.00	7.00	3.00
p_9	900.00	3.00	15.00
p_{10}	550.00	3.00	1.00
p_{11}	380.00	1.00	10.00
p_{12}	280.00	5.00	7.00
p_{13}	410.00	2.00	12.00
p_{14}	780.00	10.00	5.00
p_{15}	580.00	9.00	7.00
p_{16}	360.00	11.00	2.00
p_{17}	950.00	6.00	2.00
p_{18}	260.00	9.00	2.00
p_{19}	600.00	4.00	8.00
p_{20}	370.00	1.00	3.00
Total available resources	5000	60	75

As shown in Table 4, the utility value of case 2 is more than case 1, *i.e.*, project portfolio of case 2 is more desirable. Regarding the objective function of both models, utility value is obtained based on the score of selected projects and incorporated interactive project sets. However, it should be noted that the incorporated interactive project sets in optional project portfolio of case 2 are $\{S_2, S_3, S_4, S_7, S_8, S_9, S_{10}\}$; while S_2 and S_9 are respectively a subset of S_3 and S_8, (see Table 2), and interaction effect of the members with each other and the other members of S_3 and S_8 is considered as project interaction of sets S_3 and S_8, respectively. Therefore, according to the score of S_2 (0.4) and S_9 (0.75), the utility value of case 2 would be equal to 9.27 not 10.28; while the objective value of the proposed model, utility value of case 1, is equal to 9.55 with no subset in the incorporated interactive project sets.

Table 4. Utility values of cases 1 & 2.

Project	Case 1		Case 2	
	Optional portfolio	incorporated interactive project sets	Optional portfolio	incorporated interactive project sets
p_1	*			
p_2	*		*	
p_3	*		*	
p_4	*			
p_5	*		*	
p_6				
p_7				
p_8				
p_9				
p_{10}	*	S_1, S_3, S_4	*	S_2, S_3, S_7
p_{11}	*	S_7, S_9	*	S_8, S_9, S_{10}
p_{12}	*		*	
p_{13}			*	
p_{14}	*		*	
p_{15}				
p_{16}			*	
p_{17}				
p_{18}				
p_{19}				
p_{20}			*	
Portfolio value		9.55		10.42

5. CONCLUSIONS

Recently, project portfolio selection is recognized as a critical issue which has considerably impact on competitive advantage of organizations. As selection process is made on the evaluation of project groups, project interactions can significantly change the results of evaluation that intensify the complexity of decision making. Generally speaking, due to the existence of uncertainty, consideration of precise values or even distribution details for decision making is impractical. In this paper project portfolio optimization is modeled in a situation that two or more sources of vagueness appear simultaneously and the decision makers may not be sure about the providing values and are hesitant and irresolute for a few different possible values, not by a margin of error, or some possibility distribution on the possible values. So the evaluation of projects and their interaction effects are given in terms of hesitant fuzzy numbers. Efficient project portfolio selection is evidently critical for any organization striving to achieve competitive advantages and corporate strategies. Regarding inaccurate or insufficient information of real-world situations, the DMs may not be sure about the providing values. Thereby, expressing project assessment as an exact and precise value is not realistic. Hence, utilization of fuzzy techniques is proper approach in such situation. Here, the hesitant fuzzy information corresponding to each project and interactive project sets are aggregated based on the hesitant fuzzy weighted averaging (HFWA) operator. Subsequently, as a stand-alone multi-criteria scoring operator may not be sufficient to find out the best set of projects without violating constraints, the obtained score are utilized in the proposed linear programming model to find out the optimum project portfolio. To be compatible with real world situation, the proposed model considers project interactions in terms of identified "interactive project sets". In addition to linearity, the strength of the proposed model is its ability in considering the dependency of interactive project sets, unlike the existing previous literature. This proposed project portfolio selection with interactions under uncertainty may be studied in future applying granular computing techniques [1, 2, 10, 20-22, 25, 29, 34, 49, 51].

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