JOURNAL OF INFORMATION SCIENCE AND ENGINEERING 36, 279-291 (2020)
DOI: 10.6688/JISE.202003_36(2).0008

Conforming Dynamics in the Metric Spaces

M. LELLIS THIVAGAR[1], ABDULSATTAR ABDULLAH HAMAD[2] AND S. G. AHMED[3]
[1,2]*School of Mathematics*
Madurai Kamaraj University
Madurai, Tamil Nadu, 625021 India
[3]*Department of Computational Mathematics and Numerical Analysis*
Faculty of Engineering
Zagazig University
Zagazig, 44519 Egypt
E-mails: mlthivagar@yahoo.co.in[1]; satar198700@gmail.com[2];
al_kasrage@yahoo.com; sgahmed1331962@outlook.com[3]

This work is devoted to conformal dynamics in metric spaces, it consists of two parts, the first concerning hyperbolic groups, and the second is the iteration of branched coatings in topological spaces. These two parts are linked by D. Sullivan's dictionary. We chose to orient the exhibition taking the conjecture of J. W. Cannon as breadcrumb. The object of this work has been to analyze these different methods to understand to what extent they were different than D. Sullivan [28].

Keywords: conformal dynamics, hyperbolic, topological, conformal geometry, structure

1. INTRODUCTION

Conventional compliant dynamic systems include, but are not limited to, the study of discrete subgroups of $\mathbb{P}$SL2 ($\mathbb{C}$) by homo groups (Kleinean groups) and new version of logical fractions on the sphere of Riemann $\hat{\mathbb{C}}$. In the early eighty, D. Sullivan unifies these two theories by establishing deep connections between they are not only the central objects correspond, but also the statements and the demonstration of many results [24]. The principle of the compliant lift is at the heart of this dictionary. It expresses that the dynamics allows scaling with bounded distortion. He is responsible for the fractal aspect sets and sets of Julia, and allows establishing their properties in quantifying qualitative properties. At the same time, Mr. Groov develops the theory of hyperbolic groups [20]. It concerns "most" finite presentation groups, and relies, among other things, on the Kleinean group theory: these are the first examples of conformal dynamics without underlying differentiable structure. By freeing geometry from its Riemannian structure, and even of its differential structure, we strengthen the links between properties algebraic of these groups, their actions by isometrics on hyperbolic spaces and their actions by question form transformations on fractals. The principle of the compliant lift remains engine in this frame. It is also the key to approaching the dynamic on invertible transformations from a topological point of view and to draw from the interesting properties. The productivity of the compliant dynamics is explained by its interactions with geometry in negative curvature, question form geometry, geometric measurement theory, argotic theory and probabilities, not to mention the algebraic aspects in the case of groups. It is all the more remarkable that its properties seem to be determined by the topology of the studied objects, and that the

Received September 23, 2019; revised September 25, 2019; accepted October 2, 2019.
Communicated by Osamah Ibrahim Khalaf.

methods of discretization are so powerful, plan of the memory. A brief preamble introduces some notions of quasi-conformal analysis and hyperbolic geometry common to both main parts of this memoir, inspired by [5]. The first part is devoted to hyperbolic groups. After a quick overview of these objects, researcher compares the different approaches to the Cannon's conjecture by following [3]. In a second part, properties of the random walk on hyperbolic groups based on Green's distance [2, 6]. The second part summarizes articles [4]: it deals with iteration in metric spaces. Researcher first present the framework of our work and researcher develop the subject by putting it next to the groups. In particularly it is emphasized that quasi-conformal structure is naturally associated with a topological dynamic system dilating. Examples are presented, and researcher show that the framework that researcher develop is relevant to characterize the dynamics of rational fractions. My approach to these problems is to introduce a quasicon formal structure obtained considering a hyperbolic structure adapted to the situation envisaged.

Notations

The citations of articles appear in letters like [7], and the references to my works, whose list appears at the end of the memoir, in numbers like [1].

If a, b are positive valued functions, researcher write $a \leq b$ or $b \geq a$ if there is universal constant $u > 0$ such that $a \leq ub$. Researcher write $a \asymp b$ if $a \leq b$ and $b \geq a$. Researcher will use Polish notation $|x - y|$ to express the distance between two points of a space metric. *diam A*Denotes the diameter of set A Let's start by setting the context of research work, recalling some notions of quasicon formal and hyperbolic geometries in metric spaces.

2. CONCEPTS OF METRIC GEOMETRY

Let (X, d) be a metric space, researcher says that (X, d) is clean if the closed balls (of finite radius) are compact. A *geodesic* curve of X was an application $\gamma : I \to X$ distinct on an interval I so that, aimed at all $s, t \in I$, $d(\gamma(t), \gamma(s)) = |t - s|$. If two any points of X are joined by a geodesic segment, so X is *a space geodesic*.

2.1 Quasi-Conformal Geometry

2.1.1 Quasi-conformal transformations and their avatars

The topological isomorphism's quasi conformal are obtained by softening certain properties of the conforming transformations, researcher thus obtain several variants. We mainly rely on [27].

Definition 1.1: Quasi-symmetric topological isomorphism

− Let f: $(X, d) \to (X', d')$ be a topological isomorphism between metric spaces. Given a topological isomorphism η: $\mathbb{R}_+ \to \mathbb{R}_+$ we say that f is η-Quasi-symmetric if, for all x, y, z such that $d(x, y) \leq td(x, z)$, we have $\acute{d}(fx, fy) \leq \eta(t)\acute{d}(fx, fz)$.

This condition is strong because it implies bounded distortion, as the theorem Koebe for univalent applications [51]: a topological isomorphism η-Quasi-symmetric preserves the bounded sets and if $A \subset B$ with diam $B < \infty$, then $\frac{1}{2\eta\left(\frac{diam\ B}{diam\ A}\right)} \leq \frac{diam\ f(A)}{diam\ f(B)} \leq \eta\left(2\frac{diam\ B}{diam\ A}\right)$.

If X is a metric space, and k, l, m, n are four distinct points, we define their *birapport* by posing $[k:l:m:n] \overset{\text{def}}{=} \frac{|ak-l|}{|k-m|} \cdot \frac{|m-n|}{|l-n|}$.
J. Vaïsälä introduces the following class, which generalizes the homographies of $\hat{\mathbb{C}}$ [28].

Definition 1.2:
– An Implementation f: $X \to X'$ is η-quasimöbius if there is a topological isomorphism η: $\mathbb{R}_+ \to \mathbb{R}_+$ such that, for all a, b, c, $d \in X$ two to two separate, we have $[f(k): f(l): f(m): f(n)] \leq \eta([k: l: m: n])$.

The topological isomorphism's quasimöbius are locally Quasi-symmetric quantitatively, therefore locally bounded distortion [28].

2.1.2 Curve modules

A principle of L. Ahlfors and A. Beurling expresses that all conforming invariant is a function of the module of a family of well-chosen curves. It is the most powerful tool of geometry quasi-conformal that allows the generalizations of classical theory in a metric framework by making the link between analysis and geometry.

Definition 1.3: Curve family module
– Let (X, μ) be a metric space measured, Γ a group of curves of X and $p \geq 1$ a real. We define the p-module of Γ by $Mod_p\Gamma \overset{\text{def}}{=} \inf \int_x \rho^P d\mu$.

In which the infimum is applied to all the components of the borelian ρ: $X \to [0, \infty]$ such that, for any rectifiable curve $\gamma \in \Gamma$, $\int_\gamma \rho d\mu \geq 1$.

Mod_p Modules define a family of external measures on families of curves. In general, it suffices to restrict oneself to the following families of curves.

Definition 1.4: Capacitors and Capacities
– If X is a topological space, a capacitor is defined by a pair of disjoint continua $\{E, F\}$. It was represented by $\Gamma(E, F)$ thegroup of curves that join E and F. We define the p-capacitance of the capacitor by $cap_p(E,F) = \text{mod}_p(E,F) \overset{\text{def}}{=} \text{mod}_p \Gamma(E,F)$.

2.1.3 Loewner spaces

P. Koskela *et al.* [30] developed a theory of topological isomorphism's quasi conformal *s* in certain metric spaces measured, termed Loewner, which allow to extend local properties in global properties [21]. Following work by M. Bourdon and H. Pajot [11, 12] and M. Bonk and B. Kleiner [9, 10], this property has also become an issue in classification and rigidity issues in hyperbolic geometry, Here we define a somewhat more restrictive class of Loewner spaces (by imposing the condition (2) below). If (E, F) is a capacitor, its relative distance $\Delta(E, F)$ is defined by the formula $\Delta(E,F) \overset{\text{def}}{=} \frac{dist(E,F)}{\min\{diam\ E, diam\ F\}}$, Quasi-invariant quantity by the topological isomorphism's quasimöbius.

Definition 1.5: Loewnesque Space

– A measured metric space (X, d, μ) is a space Loewnesque if $\exists$ a dimension $Q > 1$ such that the two following propertiesbe checked:
1. Loewner's condition. There is a declining function ψ: $\mathbb{R}_+ \to \mathbb{R}_+$ such that for each capacitor (E, F), we have $mod_Q(E, F) \geq \psi((E, F))$.
2. Ahlfors-regularity. (X, d, μ) is Q-Ahlfors-regular, that is to say that for anyball $B(R)$ of radius $R \in]\, 0, diamX]$, we have $\mu(B(R)) \asymp RQ$.

Point (2) gives higher bounds on Q-modules and point (1) imposes lower bounds. The basic example is the Euclidean space $\mathbb{R}^n$ $n \geq 2$.

Note that, when space has good connectivity properties. For example, it is linearly locally connected, that is, here occurs $C > 0$ so that for all $x \in X$ and all $r > 0$, we have the following properties:
(1) Every pair of points in $B(x, r)$ belongs to a continuum contained in $B(x, C_r)$;
(2) Any pair of points in $X \backslash \overline{B(x, r)}$ belongs to a continuum contained in $X \backslash B\overline{(x, (1/C)r)}$.

This property is used mainly for the construction of capacitors. Researcher can consult [8] and references therein for more properties of these spaces and topological isomorphism's quasi-conformal.

2.1.4 Conforming gauge

The quasi-conformal geometry is generally interested in the property functions and invariant spaces by quasi-symmetrical topological isomorphism's, for this, we introduce the following notion.

Definition 1.6:
– If (Z, d) is a metric space, the gauge complies $C(Z, d)$ is the set of metrics δ on Z such that Id: $(Z, d) \to (Z, \delta)$ is quasisym metric. Researcher seek to determine two types of properties relating to a gauge.
  (1) Properties that do not depend on the metric chosen in the gauge, such as purely topological properties, completeness, linear local connectivity, the group of quasimöbius transformations, *etc.*
  (2) Properties that are satisfied for at least one metric of the gauge, such as being ahlfors-regular, Loewnesque, to have a countable group of transformations compliant, *etc.*

P. Pansu draws from the compliant gauge a numerical characteristic: the dimension $dim\mathcal{C}(\mathcal{Z})$, which is described as the infimum of the size of Hausdorff $dim\mathcal{H}(\mathcal{Z}, d)$ of (Z, d) when d goes through the gauge of $\mathcal{Z}$ [23]. This quantity is always reduced by the topological dimension of space. In practice, gauges contain metrics Ahlfors-regular, and then researcher is interested in the Ahlfors-regular dimension $dim_{AR}\mathcal{C}(\mathcal{Z})$, that is to say at the infimum of the Hausdorff dimensions among the distances Ahlfors-regular of the gauge of $\mathcal{Z}$. J. Tyson shows that $dim\mathcal{C}(\mathcal{Z}) = dim_{AR}\mathcal{C}(\mathcal{Z}) = dim_{\mathcal{H}}\mathcal{Z}$ if $\mathcal{Z}$ is Loewnesque [26].

2.2 Hyperbolic Spaces

Naturally, metric spaces with a quasi-conformal structure are obtained by considering the infinite edges of hyperbolic spaces in the sense of M. Gromov researcher can consult [19] for details, as well as for the following paragraph. If X is a metric space, we define the Gromov product as well. If $x, y, z \in X$, we pose $(X|y)z\ d \overset{\text{def}}{=} 1/2(|x-z| + |y-z| - |x-y|)$.

Definition 1.7: Hyperbolic Space

– A metric space (X, d) (unbounded) is hyperbolic if there exists a constant δ such that for all $a, b, c, w \in X$, $(a|c)_w \geq \min\{(a|b)_w, (b|c)_w\} - \delta$.

Hyperbolicity is a property of X on a large scale. To detention this info, we define the view of quasi-isometries, a notion introduced in this form by [21] A. L. Edmonds [22] and naturally justified in the dynamic contexts that we interest.

Definition 1.8: Quasigeodesic

– Let X, Y be binary metric spaces, $\lambda \geq 1$, $c \geq 0$ two constants. An application f: $X \rightarrow Y$ is an embedding (λ, c)-quasoisometric if, for all $x, x' \in X$, we have

1. $1/\lambda d_X(x, x') - c \leq d_Y(f(x), f(x')) \leq \lambda d_X(x, x') + c$.

We say that f is a (λ, c)-quasi-isometry if there exists g:$Y \rightarrow X$ which also satisfies (1) and such that for all $x \in X$, $d_X(g(f(x)), x) \leq c$. A quasi-geodesic is the picture of the $\mathbb{R}$ period by a quasi-isometric embed.

Hyperbolic geodesic spaces have many properties, including the following,

(1) The product of Gromov $(x|y)_w$ is comparable to the distance from w to any geodesic segment $[x, y]$.
(2) The *lemma of pursuit* of M. Morse affirms that any quasi-religious of a space hyperbolic is at finite distance from a geodesic. Morse's tracking lemma implies that the stuff of hyperbolicity is invariant by quasi-isometries in the category of geometric metric spaces.

2.2.1 Edge of a hyperbolic space

Let (X, w) be a specific hyperbolic space pointed to. We say that a sequence (x_n) tends to infinity if $\liminf_{mn \to \infty x_m}$, $(x_m|x_n)_w = \infty$ the visual edge ∂X of X is the sequence set that tends toward infinity matrix multiplication of the equivalent relationship $(x_n) \sim (y_n)$ if $(x_m|x_n)_w$ tends to infinity.

In the case of Hadamard varieties that have the property of visibility, we obtain the same compactification as that presented by [18]. Gromov's product extends to infinity so that quasi-ultra metric inequality is still verified. A visual metric view of w and parameter $\varepsilon > 0$ is a distanced d "on ∂X such that d" (a, b) is comparable to e – "$(a, b)w$. There are always metrics visual for $\varepsilon > 0$ small enough.

2.2.2 Quasi-conform structure to infinity

If d_1 and d_2 are visual distances based in w_1 and w_2 and of parameter ε_1 and ε_2, then Id: $(\partial_X, d_1) \rightarrow (\partial_X, d_2)$ is Quasisymmetric. We have more generally:

Theorem 1.9:

– A (λ, c)-quasi-isometry ϕ: $X \rightarrow Y$ between δ-hyperbolic spacesgeodesics is continually extended into a topological isomorphism ϕ: $\partial X \rightarrow \partial Y$ and, if d_X and d_v visual metrics, there are exists η: $\mathbb{R}_+ \rightarrow \mathbb{R}_+$ which depends only on $(\delta, \lambda, c, \varepsilon X/\varepsilon Y)$ such that φ is η-quasimöbius.

The veracity of this theorem arises from the reality that it is in a hyperbolic space, the logarithm of birapport $[a: b: c: d]$ in essence, the range between the geodesics amounts from four points to infinity $[a, b]$ and $[c, d]$ to the sign and to an additive constant. We have a partial reciprocal, due to F. Paulin [Pau], in the context of hyperbolic spaces Geodetic. To a triplet $\{a, b, c\} \in \partial^3 X$, we associate an ideal triangle of vertices $\{a, b, c\}$ the hyperbolicity of X allows us to consider a center of the triangle, that is to say a point x whose distance to the three sides is minimal. This defines an application $p\ X$: $\partial^3 X \to X$. Let's assume that the geodesic hyperbolic space X is quasi-enveloped if there exists a constant D such that every point of X is at most at a distance D from $p\ X\,(\partial^3 X)$. Such a situation is comparable to the fact that X being quasi-starred (there exists $w \in X$ such that all x is at distance at most D' from a geodesic radius from w). We then

Theorem 1.10:

– Let X, Y be two hyperbolic geodesic spaces. Any transformation quasimöbius φ: $\partial X \to \partial Y$ is prolonged into a quasi-isometry: $X \to Y$.

Theorem 2.0: Hyberolic Groups

Definition 2.1 (Spatial Intervention): Group G works geometrically on a tidy metric space X if

(1) Every component works by isometry;

(2) The motion is nicely deterministic, *i.e.*, to all modular components K and L of X, the datatype $g \in G$ of the group is such that $g(K) \cap L \neq \theta$ is finite;

(3) The action is co-compact.

For instance, if G is a finite form and S is a determinate and symmetric group of producers of G, we may recognize the Cayley $\mathcal{G}$ graph connected by $\mathcal{S}$: the vertexes are the group components, and the couple $(g, g') \in G \times G$ determines the edge is $1/g\ g' \in \mathcal{S}$. By offering $\mathcal{G}$ with the length metric that helps make every edge isometric toward the section $[0, 1]$, we have the term metric connected with $\mathcal{G}$. $\mathcal{G}$ is a geodesic and tidy space, and the deed of G on its own by conversions on the left stimulates a geometrical action on $\mathcal{G}$.

It later turned out that group only accepts basically one geometric intervention

Lemma 2.2:

– Let X be a geodesic and tidy space, and let G be a group that works geometrically on X. After which G is of a finite form, and X is quasi-isometric to every regionally beaked G Cayley graph.

By expansion, we can conclude that space is quasi-isometric to a finite type group if it is quasi-isometric to one of its locally finished Cayley graphs.

Definition 2.3: Hyperbolic group

– A group is hyperbolic if it operates geometrically on a hyperbolic, clean and geodesic space. Hyperbolic group is said to have been basic when it is discrete or quasi-isometric at $\mathbb{Z}$. On resolve always assume non-elementary groups.

2.3 Hyperbolic Spaces Quasi-Regulated

Most of the interesting properties of hyperbolic spaces and groups are established in the context of geodetic (and clean) spaces; however, the concept of quasi-isometry does not preserve this type of properties. It has been important in my work on random walks to be able to work with non-geodesic hyperbolic spaces. The concept of Quasirègle structure emerged [6]. She is from an in-depth reading of Morse's pursuit lemma. Indeed, not only this one asserts that any quasi-logical is at a limited distance from a true geodesic, it also establishes a property of alignment of the points: if q: $\mathbb{R} \to X$ is $a(\lambda, c)$-quasigeodesic in a geodesic δ-hyperbolic space, there's a variable $\tau = \tau(\delta, \lambda, c)$ such that, for all $s < t < u$. We have $(q(s)|q(u))_{q(t)} \leq \tau$. This gives us to the following definition.

Definition 2.4: Quasirules, quasiréglées structures, quasiréglées spaces and quasi-isometriesquasiréglantes

– Let (X, d) be a metric space. $A(\lambda, c, \tau)$-Quasirègle is $a(\lambda, c)$-quasigeodetic q: $I \to X$ such that, for all $s < t < u$ in I, $(q(s)|q(u))_{q(t)} \leq \tau$.

A quasi-settled structure $\mathcal{G}$ on X is a set of (λ, c, τ)-quasirules so that any pair of X is connected to the element $\mathcal{G}$, where (λ, c, τ) are constants. The metric space X will be said to be quasi-regulated if there are constants (λ, c, τ) such that X is (λ, c)-quasigeodésic and all (λ, c)-quasigéodésique is a (λ, c, τ)-quasirègle. A quasi-isometric embedding f: $X \to Y$ of a geodesic space is quasi-coherentif the geodesic image defines a quasi-fixed structure on $f(X)$. We state two results that justify this notion and that generalize the framework geodesic [19].

Theorem 2.5:

– [7] Let (X, w) be a δ-hyperbolic space and let $k \geq 0$.

(i) If $|X| \leq 2^k + 2$, then here is a metric tree finite & pointed T and an application φ: $X \to T$ such that

$\to \forall x \in X$, $|\varphi(x) - \varphi(w)| = |x - w|$,

$\to \forall x, y \in X$, $|x - y| - 2k\delta \leq |\varphi(x) - \varphi(y)| \leq |x - y|$.

(ii) If $(X_i, w_i)_{1 \leq i \leq n}$ are rays (λ, c, τ)-quasir adjusted with $n \leq 2^k$, such that $X = \cup X_i$, then there exists a real tree pointed T and an application φ: $X \to T$ so that

$\to \forall x \in X$, $|\varphi(x) - \varphi(w)| = |x - w|$,

$\to \forall x, y \in X$, $|x - y| - 2(k+1)\delta - 4c - 2\tau \leq |\varphi(x) - \varphi(y)| \leq |x - y|$,

where $c = \max\{|w - w_i|\}$

2.4 Dynamic to Infinity

Let G be a hyperbolic group operating geometrically on a clean space quasi-regulated (X, d). According to Švarc-Milnor's lemma, the edge topology ∂X depends only on G, and we can therefore *speak of the edge* $\mathcal{C}(G)$ of the group G, which is well defined to topological isomorphism's near. Moreover, Theorem 1.9 indicates that all visual metrics that can be obtain by varying X are quasisymmetrically equivalent to each other. As a result, the conforming gauge $\mathcal{C}(G)$ of the group G, defined as the gauge of ∂X provided with a visual distance only depends on the group G quasi-isometry class. Moreover, by the virtue of the same theorem, the action of G extends to the edge by topological isomorphism squasimöbius. Topologically Θ, we are dealing with a convergence group: either the set Θ of triplets of distinct points of ∂X; then Θ operates properly discontinuously on the action on is even compact, and so dedicates a group of uniform convergence. B. Bowditch has established a reciprocal [13].

Theorem 2.8: If G is a uniform convergence group operating on a space perfect compact metric Z, then Z is hyperbolic and its edge is homeomorphic to Z, in other words, the hyperbolicity of a group is completely characterized by its action topological on its edge. In addition, considering metrics in the compliant gauge of a visual metric, this convergence group is also a group uniformly quasimöbius (Theorem 1.9). Theorem 2.8 implies that the structure quasicon formal is included in the purely topological concept of convergence group uniform: the conformal gauge and all its derivatives are topological invariants of the action uniform convergence of the group. These remarks are the starting point of the work of M. Bonk and B. Kleiner [9] and suggest an approach inspired by conformal dynamics: the definition of convergence group uniform implies that there exists a constant m> 0 such that, whatever $\{x, y, z\} \in \partial^3 X = (\partial X)$, here happens $g \in G$ so that $\min\{d_\varepsilon(gx, gy), d_\varepsilon(gx, gz), d_\varepsilon(gz, gy)\} \geq m$. We are naturally led to the following formulation of the principle of the elevatorcompliant [5].

Proposition 2.9: Principle of the compliant lift

– Let G be a convergence group uniformly quasimöbius operating on a Z compact metric space. $r_0 > 0$ and a distortion function η such that, for all $z \in Z$, for any radius $\in]\, 0$, die $Z/2]$, there exists $g \in G$ such that $g(B(z, r)) \supset B(g(z), r_0)$ and $g \mid B(z, r)$ is η-quasi symmetric.

We easily obtain the following corollaries [5]:

Corollary 2.10:

– Let G be a constant conjunction group uniformly quasimöbius operating on a Z compact metric space.

(1) If an exposed is homeomorphic to a ball of $\mathbb{R}^n$, $n \geq 1$, then Z is homeomorphic $\mathbb{S}^n$.
(2) Space Z is doubling, that is, there is a range N so that each ball could be enclosed by at most N half-radius spheres (Ahlfors-regular space is doubling).
(3) Every tangent space is quasimöbius equivalent to spindle Z.

The Z space is linearly locally connected, the points (3) and (4) are due to Monk and B. K. Leiner.

3. CHARACTERIZATION OF KLEINIAN GROUPS COCOMPACTS OPERATING ON THE HYPERBOLIC SPACE

Theorem 1.10 showing the quasi-isometric family of a hyperbolic group is entirely explained by the topology and quasicon formal structure of its edge, the question of whether there is a better action on a better space arises naturally and also if in some cases the class of quasi-isometric of the group could to be determined solely by the topology of its edge [25]. One of delicate points is to provide some varieties with a hyperbolic structure. In particular, his geometric conjecture, demonstrated by G. Perelman, implies that the group fundamental of a compact variety without edge is hyperbolic and non-elemental if and only if the variety can be provided with a complete hyperbolic structure. At the intersection of these issues, J. W. Cannon proposes the following conjecture in [14].

3.1 Conjecture 3.1 [13]

A hyperbolic edge group homeomorphic to $\mathbb{S}^2$ operates geometrically on the hyperbolic space $\mathbb{H}^3$. This conjecture is true in dimension 2 [17], and false in dimension strictly greater than 3. This stems from constructions by M. Gromov and W. Thurston of compact manifolds of curvature pinch between (−1) and $(-1 - \varepsilon)$ which do not admit of metric of constant curvature and this, for all $\varepsilon > 0$ and in any dimension $n \geq 4$. Note that P. Pansu shows that the curvature of deformations of varieties locally. Non-real symmetric cannot become arbitrarily pinched [23], so the varieties of M. Gromov and W. Thurstonare also not equivalent to other varieties locally symmetrical. The approaches of J. W. Cannon *et al.* [15, 16] and M. Bonk and B. Kleiner [9, 10] consist in recognizing the conformal structure of the Riemann sphere atfrom the group gauge, using discrete versions of the family modules of curves. This allows them to reduce themselves to a uniformly quasimöbiusaction on$\hat{\mathbb{C}}$and conclude with a correction theorem of D. Sullivan. The first approach is more combinatorial, the second more analytical [3].

3.2 Discrete Modules

The advantage of discrete modules is that they are generally easier to analyze, and are invariants by topological isomorphism: they can therefore be considered in any topological space. Both approaches use slightly different versions of discrete modules. Is X a topological space and S a finite overlap of X. We denote by $\mathcal{M}_Q(S)$ the set of applications ρ: $S \to \mathbb{R}_+$ such that $0 < P\ \rho(s)\ Q < \infty$ so-called *admissible metrics.* J. W. Cannon's definition is [12].

Let $K \subset X$; we denote $S(K)$ the set of $s \in S$ that intersect K. The ρ-length of K is by definition $l_\rho(K) \overset{\text{def}}{=} \sum_{S(K)} \rho(s)$.

If Γ is a group of curves in X and $\rho \in \mathcal{M}_Q(S)$, we describe its Q-module combinatory by

$$\operatorname{mod}_Q(\Gamma, \mathcal{S}) \overset{\text{def}}{=} \inf_{\rho \in \mathcal{M}_Q(\mathcal{S})} \frac{\sum_{S \in K} \rho(S)^Q}{(\inf_{\gamma \in \Gamma} \iota_p(\gamma))^Q}.$$

M. Bonk and B. Kleiner work only on the $\mathcal{N} = \mathcal{N}(S)$ nerve of the overlay. If (E, F) is a capacitor of X, we define

$$\widehat{\operatorname{mod}}_Q(E, F, S) \overset{\text{def}}{=} \inf_{\rho \in \mathcal{M}_Q(\mathcal{S})} \frac{\sum_{S \in K} \rho(S)^Q}{(\inf_{\gamma \in \Gamma} \iota_p(\gamma))^Q}.$$

Where $\hat{\Gamma}$ is the set of curves in $\mathcal{N}$ that connect $S(E)$ and $S(F)$, and here the distance of a curve $\hat{\gamma} \in \hat{\Gamma}$ is calculated by summing ρ on the vertices of N crossed by $\hat{\gamma}$. When X and S are sufficiently regular, then these two notions of modules are in equivalents [3].

3.3 Standardization Theorems

We can now state the standardization theorems that make it possible to recognize the Riemann sphere.

We say that a series of finite recoveries (S_n) of the sphere S^2 whose mesh tends towards zero is $K-conform$ to the Cannon sense if

(1) For every ring A, here occurs $m > 0$ and n_0 so that, for $n \geq n_0$ we have $\text{mod}_{sup}(A, S_n)$, $\text{mod}_{inf}(A, S_n) \in [m/K, K_m]$;

(2) For all $x \in X$, every neighborhood V of x and all $m > 0$, there exists a ring $A \subset V$ it was separates x from $X \setminus V$ and n_0 such that, for $n \geq n_0$, $\text{mod}_{sup}(A, S_n) \geq m$.

We state a first result of combinatorial nature which corresponds to a version Simplified Riemann combinatorial theorem of J. W. Cannon [14], we will say that recovery is K-adapted if its nerve is of valence at most K and if it is equivalent to a triangulation of the sphere.

Theorem 3.5 [34]:
– Let (S_n) be a sequence of K-adapted finite overlays and K-conformal of a topological 2-sphere X whose mesh tends to 0. There is a topological isomorphism φ: $X \to \hat{\mathbb{C}}$ such that $\text{mod}_{2\varphi}(A) \asymp \text{mod}_{inf}(A, S_n) \asymp \text{mod}_{inf}(A, S_n)$ for any ring A of X and all n big enough.Here is the analytic version.

Theorem 3.6 [9]:
– Let X be a metric space homeomorphic to $\mathbb{S}^2$. Yes
(1) X is 2-regular and linearly nearby linked, or
(2) X is a Q-Loewnesque space, $Q \geq 2$ then X is Quasisymmetric to $\hat{\mathbb{C}}$

We propose in [3] a unified demonstration of this results, the idea for Theorem 3.5 is to associate with each other S_n a standardized stack of circles. In on $\hat{\mathbb{C}}$ and build an injection φ_n: $X_n(\subset X) \to \hat{\mathbb{C}}$ which is a base point of $s \in S_n$ associates the centre of the corresponding circle. Estimates a priori on the modules.

4. DYNAMIC GROSSIÈRE COMFORME

4.1 Branched Coatings

There have been different definitions of branched coatings which, for the most part, coincide in the context of varieties. Our definition, close to that of A. Edmonds [18], generalizes quite obviously the topological properties of the rational fractions, and behaves well in dynamic problems.

Let X, Y be locally compact spaces, and let f: $X \to Y$ be an application continuous to finished fibers. For $x \in X$, the local degree of f at x is

$$deg(f; x) = {}_{u}^{\inf} sup \{\# f^{1}(\{\mathcal{Z}\}) \cap U : \mathcal{Z} \in f(U)\}$$

where U goes through the neighborhoods of x.

Definition 4.1: Branched coating

− Continuous application f: $X \to Y$ to fibers finished is a branched coating of degree $d \geq 1$ if

(i) For all $y \in Y$, we have

$$\sum_{x \in f^{-1}(y)} \deg(f; x) = d;$$

(ii) For all $x_0 \in X$, there will be modular neighborhoods and V of x_0 and $f(x_0)$ correspondingly, such as

$$\sum_{x \in f(x)=y} \deg(f; x) = d(f; x_0).$$

For everything $y \in V$.

When X and Y are separated and nearby compact, a branched coating f: $X \to Y$ of degree d is open, closed, clean and subjective. In addition, the connection points $B_f = \{deg(f; x) > 1\}$ and branching $= f(B_f)$ are closed and nowhere dense. Many dynamic arguments use open antecedents and their restrictions related components. For this, we impose on X and Y to be also locally related. Under these conditions, if $V \subset Y$ is open and linked to, and $U \subset X$ is a component linked with $f^1(V)$, then $f|_U$: $U \to V$ is also a separated coating of finite degree. In another way, if $y \in Y$ and $f^1(\{y\}) = \{x_1, x_2, \ldots, x_n\}$, then there is a related neighborhood arbitrarily small V of y such that $f^1(V) = U_1 \sqcup U_1 \sqcup \ldots \sqcup U_n$ is a disjoint union of related neighborhoods U_i of x_i such that $f|_{U_i}$: $U \to V$ is a branched coating of degree $deg(f; x_i)$, $i = 1, 2, \ldots, n$.

5. CONCLUSIONS

The approach presented here makes it possible to distinguish two different problems: the first is to provide the surface area with a complex structure compatible with the combinatorial modules, the second is to compare the metric structure thus obtained with the original (when fixed). The work of J. W. Cannon *et al.* essentially address the first problem, and those of M. Bonk and B. Kleiner to both, but without dissociating them. The existence of a Loewner metric in the gauge of a hyperbolic group is a very delicate problem. It seems that there are no known criteria that allow for ensure existence, but we know that it is not automatic [13]. Indeed, the only known examples of geometric actions on explicit spaces: spaces non compact symmetries of rank 1 and Fuchsian buildings [5] and references are there for more details Loewner's condition has many consequences.

REFERENCES

1. P. Haïssinsky, "Applications of holomorphic surgery to dynamic systems, especially at parabolic points," Ph.D. Thesis, Department of Mathematics, University of Paris South, Orsay, 1998.
2. P. Haïssinsky, "Stacking of circles and combinatorial modules," *Annales L'Institut Fourier*, Vol. 59, 2009, pp. 2175-2222.
3. P. Haïssinsky and K. M. Pilgrim, *Coarse Expanding Conformal Dynamics* (*Asterisk*), Amer Mathematical Society, 2010.

4. P. Haïssinsky, "Quasicon form geometry, edge analysis of metric spaces hyperbolic and rigidities (according to Mostow, Pansu, Bourdon, Pajot, Bonk, Kleiner ...)," *Bourbaki Seminar*, Vol. 2007/2008, 2008, No. 993.
5. S. Blachère, P. Haïssinsky, and P. Mathieu, "Harmonic measures versus quasicon formal measures for hyperbolic groups," Hal-00290127, arXiv: math. PR/0806.3915, 2008.
6. A. Ancona, "Theory of potential on graphs and varieties," Summer School of Probability of Saint-Flour XVIII-1988, *Lecture Notes in Mathematics*, Vol. 1427, 1990, pp. 1-112.
7. M. Bonk and B. Kleiner, "Quasisymmetric parameterizations of two dimensional metric spheres," *Inventiones Mathematics*, Vol. 150, 2002, pp. 127-183.
8. M. Bonk and B. Kleiner, "Conformal dimension and Gromov hyperbolic groups with 2-sphere boundary," *Geometry Topology*, Vol. 9, 2005, pp. 219-246.
9. M. Bourdon and H. Pajot, "Poincaré inequalities and quasi-conformal structure on the boundary of some hyperbolic buildings," in *Proceedings of the American Mathematical Society*, Vol. 127, 1999, pp. 2315-2324.
10. M. Bourdon and H. Pajot, "Rigidity of quasi-isometries for some hyperbolic buildings," *Commentarii Mathematici Helvetici*, Vol. 75, 2000, pp. 701-736.
11. M. Bourdon and H. Pajot, "Cohomology ℓ_p and Besov spaces," *Journal für die reine und angewandte Mathematik (Crelles Journal)*, Vol. 558, 2003, pp. 85-108.
12. J. W. Cannon, "The theory of negatively curved spaces and groups," in *Ergodic Theory, Symbolic Dynamics, and Hyperbolic Spaces*, Oxford University Press, NY, 1991, pp. 315-369.
13. J. W. Cannon, "The combinatorial Riemann mapping theorem," *Acta Mathematica*, Vol. 173, 1994, pp. 155-234.
14. J. W. Cannon, W. J. Floyd, and W. R. Parry, "Sufficiently rich families of planar rings," *Annales Academiæ Scientiarum Fennicæ Mathematica*, Vol. 24, 1999, pp. 265-304.
15. C. S. Connell, "Minimal Lyapunov exponents, quasi-conformal structures, and rigidity of non-positively curved manifolds," *Ergodic Theory Dynamical Systems*, Vol. 23, 2003, pp. 429-446.
16. M. Coornaert, "Patterson-Sullivan measurements on the edge of a hyperbolic space in the sense of Gromov," *Pacific Journal of Mathematics*, Vol. 159, 1993, pp. 241-270.
17. M. Coornaert and A. Papadopoulos, *Symbolic Dynamics and Hyperbolic Groups*, Springer, Berlin, LNM, Vol. 1539, 1993.
18. P. Eberlein and B. O'Neill, "Visibility manifolds," *Pacific Journal of Mathematics*, Vol. 46, 1973, pp. 45-109.
19. A. Douady and J. Hubbard, "A proof of Thurston's topological characterization of rational functions," *Acta Mathematica*, Vol. 171, 1993, pp. 263-297.
20. E. Dynkin, "The boundary theory of Markov processes (discrete case)," *Uspehi Matematicheskih Nauk*, Vol. 24, 1969, pp. 3-42.
21. A. L. Edmonds, "Branched coverings and orbit maps," *Michigan Mathematical Journal*, Vol. 23, 1977, pp. 289-301.
22. É. Ghys and P. de la Harpe, ed., "On hyperbolic groups according to Mikhael Gromov," *Progress in Mathematics*, Birkhäuser Boston Inc., Boston, MA, Vol. 83, 1990.
23. M. Gromov, "Hyperbolic groups," in *Essays in Group Theory*, Springer, NY, 1987, pp. 75-263.
24. M. Gromov and W. Thurston, "Constant pinching for hyperbolic manifolds," *Inven-*

tiones, *Mathematica*, Vol. 89, 1987, pp. 1-12.
25. T. Iwaniec and G. J. Martin, "Quasiregular semi groups," *Annales Academia Scientirrum Fennia Mathematica*, Vol. 21, 1996, pp. 241-254.
26. V. Mayer, "Uniformly Quasiregular mappings of Lattès type," *Conformal Geometry and Dynamics*, Vol. 1, 1997, pp. 104-111.
27. F. Paulin, "A hyperbolic group is determined by its edge," *Journal London Mathematical Society*, Vol. 54, 1996, pp. 50-74.
28. D. Sullivan, "Seminar on hyperbolic geometry and conformal dynamical systems," Preprint *IHES*, 1982.
29. M. L. Thivagar and C. Richard, "On Nano forms of weakly open sets," *International Journal Mathematics and Statistics Invention*, Vol. 1, 2013, pp. 31-37.
30. M. L. Thivagar and C. Richard, "On Nano continuity in a strong form," *International Journal of Pure and Applied Mathematics*, Vol. 101, 2015, pp. 893-904.
31. M. L. Thivagar and V. S. Devi, "Computing technique for recruitment process via Nano topology," *Sohag Journal of Mathematics*, Vol. 3, 2016, pp. 37-45.

M. Lellis Thivagar born in the year 1961. He received M.Sc., Ph.D., in Mathematics, and he is presently working as Professor & Head, School of Mathematics, Madurai Kamaraj University, Madurai.

Abdulsattar Abdullah Hamad was born in October 9, 1987, in Slah al Deen, Iraq. He is a Ph.D. scholar in School of Mathematics, Madurai Kamaraj University, and completed Master degree in India, April 2018, and a Bachelor of Mathematics in Iraq, 2014.

S. G. Ahmed was born in March 13, 1962 in Sharkia, Egypt. Since 2006 he is a Professor of Computational Mathematics and Numerical Analysis, Department of Engineering Mathematics and Mathematics, Zagazig University, Zagazig, Egypt. He received his Ph.D. in Computational Mathematics and Numerical Analysis from the University of Portsmouth and Zagazig University through channel system between UK and Egypt in 1996.