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On Some New Statistical Correlation Measures for *T*-spherical Fuzzy Sets and Applications in Soft Computing

ABHISHEK GULERIA AND RAKESH KUMAR BAJAJ+
Department of Mathematics
Jaypee University of Information Technology, Waknaghat
Solan, Himachal Pradesh, 173234 India
+*E-mail: rakesh.bajaj@gmail.com*

Correlation and correlation coefficients are the most utilized statistical tools and measures in the field of engineering, intelligence sciences, data analysis, decision making, biological sciences, *etc*. In the present communication, we have proposed two new measures of correlation coefficients and measures of weighted correlation coefficients of two T-spherical fuzzy sets based on the newly defined information energy measure under the perception of the four parameters of impreciseness - degree of membership, indeterminacy (neutral), non-membership and the refusal. Further, by implementing the principle of maximum correlation coefficient over the proposed correlation coefficients, the methodologies for solving the problems of pattern recognition and medical diagnosis have been provided with the help of an example for each. A comparative analysis in contrast with the existing methodologies has been presented with comparative remarks and additional advantages.

Keywords: spherical fuzzy set, T-spherical fuzzy set, information energy, correlation coefficient, pattern recognition, medical diagnosis

1. INTRODUCTION

The researchers in the field of fuzzy sets and information are well aware that various generalizations of the notion of fuzzy sets [22] and Intuitionistic Fuzzy Sets (IFSs) [1] have taken place to model the uncertainties and the hesitancy inherent in many practical circumstances for a wider coverage of flexibility. Yager [21] revealed that the existing structures of fuzzy set and intuitionistic fuzzy set are not capable enough to depict the human opinion in more practical/broader sense and introduced the notion of Pythagorean fuzzy sets which effectively enlarged the span of information by introducing the new conditional constraint. Mahmood *et al*. [16] introduced the notion of Spherical Fuzzy Set (SFS) and T-spherical Fuzzy Set (TSFS) which give additional strength to the idea of picture fuzzy set by broadening/enlarging the space for the grades of all the four parameters.

Next, Kifayat *et al*. [15] studied the geometrical comparison of fuzzy sets, intuitionistic fuzzy sets, Pythagorean fuzzy sets, picture fuzzy sets along with spherical and T-spherical fuzzy sets in detail. Also, they studied various existing similarity measures for

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+Corresponding author.

intuitionistic fuzzy sets and picture fuzzy sets with their limitations that they could not be applied in the broader setup as of the spherical fuzzy set. Further, they proposed various types of similarity measures for TSFS with their usefulness in various fields. Garg *et al*. [7] presented a new improved interactive aggregation operators for TSFSs with application in decision-making. Guleria and Bajaj [10] introduced the notion of eigen spherical fuzzy sets and devised an algorithm to find the greatest and the least eigen spherical fuzzy sets to solve some of the decision-making problems. Next, Guleria and Bajaj [9] successfully proposed the notion of T-spherical fuzzy soft set and studied some new aggregation operators along with some applications in the field of decision-making.

In the recent past, various researchers have extensively studied different types of information measures in connection with the correlation coefficients, similarity measures, entropy, distance measures, discriminant measures which are available in the literature. The correlation coefficient for intuitionistic fuzzy set was first provided by Gerstenkorn and Manko [8]. For probability spaces, the coefficient of correlation was studied by Hong and Hwang [12]. Next, Mitchell [17] proposed the correlation coefficient between IFSs by interpreting an IFS as an ensemble of fuzzy set. After that, using the concept of centroid, Hung and Wu [13] proposed a method to compute the coefficients of correlation for IFSs. Further, Bustince & Burillo [2] and Hong [11] enhanced the concept of the correlation coefficients of IFSs to interval-valued intuitionistic fuzzy sets in different capacities. Various researchers developed different correlation coefficients for IFSs and interval-valued intuitionistic fuzzy sets with their viewpoint and applied them in the different fields [18].

Huang and Guo [14] proposed a revised and improved correlation coefficient of the intuitionistic fuzzy sets and generalized these correlation coefficients over the interval-valued intuitionistic fuzzy sets. They also discussed some properties of the proposed correlation coefficients and proposed methodology for solving the problem of medical diagnosis & clustering analysis. Garg and Rani [6] studied the correlation and weighted correlation coefficients under the complex intuitionistic fuzzy set environment and also investigated some of their properties. Based on these statistical measures, they also proposed a new approach for multi-criteria decision making problems with the help of two illustrative examples and studied the feasibility of their results with the existing approaches. Thao [20] developed a new correlation coefficient between intuitionistic fuzzy sets based on the variance and covariance of the intuitionistic fuzzy sets. They also generalized these correlation coefficients for the interval-valued intuitionistic fuzzy sets and applied to solve the problem of pattern recognition. Chen [4] introduced a correlation based closeness index for interval-valued Pythagorean fuzzy set and discussed its properties. By utilizing the correlation-based closeness index, an algorithm for solving multi-criteria decision making problem under the interval-valued Pythagorean fuzzy environment has also been provided. The paper also demonstrated the feasibility and effectiveness of the proposed methodology through a comparative analysis in contrast with the well known existing methods. Singh [19] introduced the concept of correlation coefficient for picture fuzzy sets as an extension of the correlation coefficient for IFSs and also proposed the weighted correlation coefficients for the picture fuzzy sets with their application in clustering analysis. We extend the concept and literature by proposing new correlation coefficients for TSFSs and present their applications.

The outline of the present work is as follows. In Section 2, we study some basic preliminaries in reference with the T-spherical fuzzy set and correlation coefficient. We propose correlation coefficients and weighted correlation coefficients of two T-spherical fuzzy sets with respect to two different aspects based on the proposed information energy and correlation function in Section 3. Further, the proposed correlation coefficients have been utilized for proposing new methodologies for solving the computational application problems of pattern recognition and medical diagnosis in Section 4. An illustrative example has also been provided for each of the application. In Section 5, a comparative analysis depicting the advantages and listing some important remarks has been carried out.

2. PRELIMINARIES

In this section, we study some important notions in connection with T-spherical fuzzy set and correlation coefficient, which are available in the basic literature.

Definition 1 *A T-spherical fuzzy set S in X is given by*

$$S = \{< x, \mu_S(x), \eta_S(x), \nu_S(x) >| \, x \in X\},$$

where $\mu_S : X \to [0,1]$, $\eta_S : X \to [0,1]$ and $\nu_S : X \to [0,1]$ denotes the degree of membership, degree of neutral membership (abstain) and degree of non-membership respectively and satisfy the condition $\mu_S^n(x) + \eta_S^n(x) + \nu_S^n(x) \leq 1$; $\forall x \in X$. The degree of refusal for any T-spherical fuzzy set S and $x \in X$ is given by $r_S(x) = \sqrt[n]{1 - (\mu_S^n(x) + \eta_S^n(x) + \nu_S^n(x))}$.

Next, we outline the basic preliminaries related to the correlation coefficients in context with intuitionistic fuzzy sets. Some of the important correlation coefficients between two intuitionistic fuzzy sets I_1 and I_2 over $X = \{x_1, x_2, x_3, \ldots, x_m\}$ proposed by various researchers are given below.

- **Gerstenkorn and Manko [8]**:
$$K(I_1, I_2) = \frac{\sum_{i=1}^{m} \left(\mu_{I_1}(x_i)\mu_{I_2}(x_i) + \nu_{I_1}(x_i)\nu_{I_2}(x_i)\right)}{\left[\sum_{i=1}^{m} \left(\mu_{I_1}(x_i) + \nu_{I_1}(x_i)\right)\right]^{1/2} \left[\sum_{i=1}^{m} \left(\mu_{I_2}(x_i) + \nu_{I_2}(x_i)\right)\right]^{1/2}}.$$

- **Szmidt and Kacprzyk [18]**:
$r_{IFS}(I_1, I_2) = \frac{1}{3}\left[r_1(I_1, I_2) + r_2(I_1, I_2) + r_3(I_1, I_2)\right]$; where,
$$r_1(I_1, I_2) = \frac{\sum_{i=1}^{m} \left(\mu_{I_1}(x_i) - \overline{\mu_{I_1}}\right)\left(\mu_{I_2}(x_i) - \overline{\mu_{I_2}}\right)}{\left(\sum_{i=1}^{m} (\mu_{I_1}(x_i) - \overline{\mu_{I_1}})^2\right)^{\frac{1}{2}} \left(\sum_{i=1}^{m} (\mu_{I_2}(x_i) - \overline{\mu_{I_2}})^2\right)^{\frac{1}{2}}},$$
$$r_2(I_1, I_2) = \frac{\sum_{i=1}^{m} \left(\nu_{I_1}(x_i) - \overline{\nu_{I_1}}\right)\left(\nu_{I_2}(x_i) - \overline{\nu_{I_2}}\right)}{\left(\sum_{i=1}^{m} (\nu_{I_1}(x_i) - \overline{\nu_{I_1}})^2\right)^{\frac{1}{2}} \left(\sum_{i=1}^{m} (\nu_{I_2}(x_i) - \overline{\nu_{I_2}})^2\right)^{\frac{1}{2}}},$$
$$r_3(I_1, I_2) = \frac{\sum_{i=1}^{m} \left(\pi_{I_1}(x_i) - \overline{\pi_{I_1}}\right)\left(\pi_{I_2}(x_i) - \overline{\pi_{I_2}}\right)}{\left(\sum_{i=1}^{m} (\pi_{I_1}(x_i) - \overline{\pi_{I_1}})^2\right)^{\frac{1}{2}} \left(\sum_{i=1}^{m} (\pi_{I_2}(x_i) - \overline{\pi_{I_2}})^2\right)^{\frac{1}{2}}}.$$

3. CORRELATION COEFFICIENTS OF T-SFS

Let $R = \{< x_i, \mu_R(x_i), \eta_R(x_i), \nu_R(x_i) >| \, x_i \in X;\ i = 1,2,3\ldots,m\}$ be a T-spherical fuzzy set over X (universe). In order to propose new correlation coefficients for T-spherical fuzzy sets, we first propose the information energy for a T-spherical fuzzy set as

$$I(R) = \sum_{i=1}^{m} \left((\mu_R^n)^2(x_i) + (\eta_R^n)^2(x_i) + (\nu_R^n)^2(x_i) + (r_R^n)^2(x_i) \right); \tag{3.1}$$

and the weighted information energy as

$$I_w(R) = \sum_{i=1}^{m} w_i \left((\mu_R^n)^2(x_i) + (\eta_R^n)^2(x_i) + (\nu_R^n)^2(x_i) + (r_R^n)^2(x_i) \right); \tag{3.2}$$

where w_i's are the weights of x_i's $\in X$ with $w_i \geq 0$ and $\sum_{i=1}^{m} w_i = 1$, $\forall\, i = 1,2,\ldots,m$.
Consider two T-spherical fuzzy sets given by
$R = \{< x_i, \mu_R(x_i), \eta_R(x_i), \nu_R(x_i) >| \, x_i \in X;\ i = 1,2,3\ldots,m\}$ and
$S = \{< x_i, \mu_S(x_i), \eta_S(x_i), \nu_S(x_i) >| \, x_i \in X;\ i = 1,2,3\ldots,m\}$ over X (universe). Then the correlation function between R and S has been proposed as

$$C(R,S) = \sum_{i=1}^{m} \left[\mu_R^n(x_i)\mu_S^n(x_i) + \eta_R^n(x_i)\eta_S^n(x_i) + \nu_R^n(x_i)\nu_S^n(x_i) + r_R^n(x_i)r_S^n(x_i) \right]. \tag{3.3}$$

It may be easily verified that the correlation function given by Eq. (3.3) satisfies:

- $C(R,R) = I(R)$;
- $C(R,S) = C(S,R)$.

Also, the weighted correlation function between R and S be given by

$$C_w(R,S) = \sum_{i=1}^{m} w_i \left[\mu_R^n(x_i)\mu_S^n(x_i) + \eta_R^n(x_i)\eta_S^n(x_i) + \nu_R^n(x_i)\nu_S^n(x_i) + r_R^n(x_i)r_S^n(x_i) \right]; \tag{3.4}$$

where w_i's are the weights of the elements x_i's $\in X$ with $w_i \geq 0$ and $\sum_{i=1}^{m} w_i = 1$, $\forall\, i = 1,2,\ldots,m$. Next, we propose the following definitions of correlation coefficients for TS-FSs along with its weighted version:

Definition 2 *Let R and S be two T-spherical fuzzy sets over the domain of discourse X given by*
$R = \{< x_i, \mu_R(x_i), \eta_R(x_i), \nu_R(x_i) >| \, x_i \in X\}$ *and* $S = \{< x_i, \mu_S(x_i), \eta_S(x_i), \nu_S(x_i) >| \, x_i \in X\}$. *The correlation coefficient between R and S is defined as* $K_1(R,S) = \frac{C(R,S)}{[I(R) \cdot I(S)]^{1/2}}$

$$= \frac{\sum_{i=1}^{m} \left[\mu_R^n(x_i)\mu_S^n(x_i) + \eta_R^n(x_i)\eta_S^n(x_i) + \nu_R^n(x_i)\nu_S^n(x_i) + r_R^n(x_i)r_S^n(x_i) \right]}{\left[\sum_{i=1}^{m} \begin{pmatrix} (\mu_R^n)^2(x_i) + (\eta_R^n)^2(x_i) \\ +(\nu_R^n)^2(x_i) + (r_R^n)^2(x_i) \end{pmatrix} \right]^{1/2} \cdot \left[\sum_{i=1}^{m} \begin{pmatrix} (\mu_S^n)^2(x_i) + (\eta_S^n)^2(x_i) \\ +(\nu_S^n)^2(x_i) + (r_S^n)^2(x_i) \end{pmatrix} \right]^{1/2}}. \tag{3.5}$$

Theorem 1 *The correlation coefficient of two T-spherical fuzzy sets $K_1(R,S)$, given by equation (3.5), is a valid statistical measure.*

Proof: It may be noted that a proposed correlation coefficient measure must fulfill the following well established axioms[18]:

- **Axiom 1:** $K_1(R,S) = K_1(S,R)$.
- **Axiom 2:** $0 \leq K_1(R,S) \leq 1$.
- **Axiom 3:** $K_1(R,S) = 1$ if and only if $R = S$.

Since $C(R,S) = C(S,R)$, therefore in view of Eq. (3.5) in Definition 2, Axiom 1 is quite obvious, *i.e.*, $K_1(R,S) = K_1(S,R)$. Next, we prove that Axiom 2 is satisfied. By definition, $K_1(R,S)$ is clearly a non-negative quantity. It is now sufficient to show that $K_1(R,S) \leq 1$. We consider

$$
\begin{aligned}
C(R,S) &= \sum_{i=1}^{m} [\mu_R^n(x_i)\mu_S^n(x_i) + \eta_R^n(x_i)\eta_S^n(x_i) + \nu_R^n(x_i)\nu_S^n(x_i) + r_R^n(x_i)r_S^n(x_i)] \\
&= [\mu_R^n(x_1)\mu_S^n(x_1) + \eta_R^n(x_1)\eta_S^n(x_1) + \nu_R^n(x_1)\nu_S^n(x_1) + r_R^n(x_1)r_S^n(x_1)] + \\
&\quad + [\mu_R^n(x_2)\mu_S^n(x_2) + \eta_R^n(x_2)\eta_S^n(x_2) + \nu_R^n(x_2)\nu_S^n(x_2) + r_R^n(x_2)r_S^n(x_2)] + \\
&\quad + \ldots + [\mu_R^n(x_m)\mu_S^n(x_m) + \eta_R^n(x_m)\eta_S^n(x_m) + \nu_R^n(x_m)\nu_S^n(x_m) + r_R^n(x_m)r_S^n(x_m)] .
\end{aligned}
$$

Using the Cauchy-Schwarz inequality, we have

$$
\begin{aligned}
[C(R,S)]^2 \leq \{ & \Big((\mu_R^n)^2(x_1) + (\eta_R^n)^2(x_1) + (\nu_R^n)^2(x_1) + (r_R^n)^2(x_1)\Big) + \\
& + \Big((\mu_R^n)^2(x_2) + (\eta_R^n)^2(x_2) + (\nu_R^n)^2(x_2) + (r_R^n)^2(x_2)\Big) + \\
& + \ldots + \Big((\mu_R^n)^2(x_m) + (\eta_R^n)^2(x_m) + (\nu_R^n)^2(x_m) + (r_R^n)^2(x_m)\Big)\} \\
\times \{ & \Big((\mu_S^n)^2(x_1) + (\eta_S^n)^2(x_1) + (\nu_S^n)^2(x_1) + (r_S^n)^2(x_1)\Big) + \\
& + \Big((\mu_S^n)^2(x_2) + (\eta_S^n)^2(x_2) + (\nu_S^n)^2(x_2) + (r_S^n)^2(x_2)\Big) + \\
& + \ldots + \Big((\mu_S^n)^2(x_m) + (\eta_S^n)^2(x_m) + (\nu_S^n)^2(x_m) + (r_S^n)^2(x_m)\Big)\} \\
= & \sum_{i=1}^{m} \Big[(\mu_R^n)^2(x_i) + (\eta_R^n)^2(x_i) + (\nu_R^n)^2(x_i) + (r_R^n)^2(x_i)\Big] \\
& \times \sum_{i=1}^{m} \Big[(\mu_S^n)^2(x_i) + (\eta_S^n)^2(x_i) + (\nu_S^n)^2(x_i) + (r_S^n)^2(x_i)\Big] .
\end{aligned}
$$

$\Rightarrow$ $[C(R,S)]^2 \leq I(R) \cdot I(S)$. Therefore, in view of Definition 2, it is clear that $K_1(R,S) \leq 1$ which proves the Axiom 2. Further, if $R = S$, *i.e.*, $\mu_R = \mu_S,\ \eta_R = \eta_S$ and $\nu_R = \nu_S\ \forall\, x_i \in X$, then from Eq. (3.5), we get $K_1(R,S) = 1$. It is easy to note that the converse is also true which proves the Axiom 3. This completes the proof of the theorem. We have

• For $n = 2$, the correlation coefficient between T-spherical fuzzy sets Eq. (3.5), becomes the correlation coefficient between the spherical fuzzy sets.

• For $n = 1$, the correlation coefficient between T-spherical fuzzy sets Eq. (3.5), becomes the correlation coefficient between the picture fuzzy sets [19].

• For $n = 2$ and $r_R = 0$ & $r_S = 0$ (*absentia of degree of refusals*), the correlation coefficient

between T-spherical fuzzy sets Eq. (3.5), becomes the correlation coefficient between the Pythagorean fuzzy sets [5].
• For $n = 1$ and $r_R = 0$ & $r_S = 0$ (*absentia of degree of refusals*), the correlation coefficient between T-spherical fuzzy sets Eq. (3.5), becomes the correlation coefficient between the intuitionistic fuzzy sets [8].

Definition 3 *Let R and S be two T-spherical fuzzy sets over the domain of discourse X given by $R = \{< x_i, \mu_R(x_i), \eta_R(x_i), \nu_R(x_i) >| x_i \in X\}$ and $S = \{< x_i, \mu_S(x_i), \eta_S(x_i), \nu_S(x_i) >| x_i \in X\}$. The correlation coefficient between R and S is defined as $K_2(R,S) = \frac{C(R,S)}{\max[I(R),I(S)]}$*

$$= \frac{\sum_{i=1}^{m}\left[\mu_R^n(x_i)\mu_S^n(x_i) + \eta_R^n(x_i)\eta_S^n(x_i) + \nu_R^n(x_i)\nu_S^n(x_i) + r_R^n(x_i)r_S^n(x_i)\right]}{\max\left[\left(\sum_{i=1}^{m}\begin{pmatrix}(\mu_R^n)^2(x_i) + (\eta_R^n)^2(x_i) \\ +(\nu_R^n)^2(x_i) + (r_R^n)^2(x_i)\end{pmatrix}\right), \left(\sum_{i=1}^{m}\begin{pmatrix}(\mu_S^n)^2(x_i) + (\eta_S^n)^2(x_i) \\ +(\nu_S^n)^2(x_i) + (r_S^n)^2(x_i)\end{pmatrix}\right)\right]}. \tag{3.6}$$

Theorem 2 *The correlation coefficient of two T-spherical fuzzy sets $K_2(R,S)$, given by Eq. (3.6), is a valid statistical measure.*

Proof: It may be noted that a proposed correlation coefficient measure must fulfill the well established axioms which are already mentioned in the proof of Theorem 1. Axiom 1 and Axiom 3 can be verified easily in view of the proposed Definition 3. For proving Axiom 2, it is easy to see that $K_2(R,S) \geq 0$. Therefore, it remains to prove that $K_2(R,S) \leq 1$. Since $[C(R,S)]^2 \leq I(R) \cdot I(S)$, therefore, it simply implies that $C(R,S) \leq \max[I(R),I(S)]$. Hence, the proposed correlation coefficient measure (3.6) is a valid statistical measure.
In view of the real world problems where the weights are a kind of key factors in the computational analysis, we propose the following weighted correlation coefficients between TSFSs by extending the correlation coefficients (3.5) and (3.6):

$$\begin{aligned} K_1^w(R,S) &= \frac{C_w(R,S)}{[I_w(R) \cdot I_w(S)]^{1/2}} \\ &= \frac{\sum_{i=1}^{m} w_i\left(\mu_R^n(x_i)\mu_S^n(x_i) + \eta_R^n(x_i)\eta_S^n(x_i) + \nu_R^n(x_i)\nu_S^n(x_i) + r_R^n(x_i)r_S^n(x_i)\right)}{\left[\sum_{i=1}^{m} w_i\begin{pmatrix}(\mu_R^n)^2(x_i) + (\eta_R^n)^2(x_i) \\ +(\nu_R^n)^2(x_i) + (r_R^n)^2(x_i)\end{pmatrix}\right]^{1/2} \cdot \left[\sum_{i=1}^{m} w_i\begin{pmatrix}(\mu_S^n)^2(x_i) + (\eta_S^n)^2(x_i) \\ +(\nu_S^n)^2(x_i) + (r_S^n)^2(x_i)\end{pmatrix}\right]^{1/2}}; \end{aligned} \tag{3.7}$$

$$\begin{aligned} K_2^w(R,S) &= \frac{C_w(R,S)}{\max[I_w(R), I_w(S)]} \\ &= \frac{\sum_{i=1}^{m} w_i\left(\mu_R^n(x_i)\mu_S^n(x_i) + \eta_R^n(x_i)\eta_S^n(x_i) + \nu_R^n(x_i)\nu_S^n(x_i) + r_R^n(x_i)r_S^n(x_i)\right)}{\max\left[\sum_{i=1}^{m} w_i\begin{pmatrix}(\mu_R^n)^2(x_i) + (\eta_R^n)^2(x_i) \\ +(\nu_R^n)^2(x_i) + (r_R^n)^2(x_i)\end{pmatrix}, \sum_{i=1}^{m} w_i\begin{pmatrix}(\mu_S^n)^2(x_i) + (\eta_S^n)^2(x_i) \\ +(\nu_S^n)^2(x_i) + (r_S^n)^2(x_i)\end{pmatrix}\right]}; \end{aligned} \tag{3.8}$$

where w_i's are the weights of x_i's $\in X$ with $w_i \geq 0$ and $\sum_{i=1}^{m} w_i = 1$, $\forall i = 1,2,\ldots,m$.

Theorem 3 *The weighted correlation coefficient of two T-spherical fuzzy sets $K_1^w(R,S)$, given by Eq. (3.7), is a valid statistical measure.*

Proof: Axiom 1 and Axiom 3 can be verified easily. However, for proving Axiom 2, it may be easily noted that $K_1^w(R,S) \geq 0$. It remains to prove that $K_1^w(R,S) \leq 1$. By equation (3.4), we write

$$\begin{aligned}C_w(R,S) = \sum_{i=1}^{m} w_i\,[\mu_R^n(x_i)\mu_S^n(x_i) + \eta_R^n(x_i)\eta_S^n(x_i) + \nu_R^n(x_i)\nu_S^n(x_i) + r_R^n(x_i)r_S^n(x_i)]\\ = w_1\,[\mu_R^n(x_1)\mu_S^n(x_1) + \eta_R^n(x_1)\eta_S^n(x_1) + \nu_R^n(x_1)\nu_S^n(x_1) + r_R^n(x_1)r_S^n(x_1)] +\\ + w_2\,[\mu_R^n(x_2)\mu_S^n(x_2) + \eta_R^n(x_2)\eta_S^n(x_2) + \nu_R^n(x_2)\nu_S^n(x_2) + r_R^n(x_2)r_S^n(x_2)] +\\ + \ldots + w_m\,[\mu_R^n(x_m)\mu_S^n(x_m) + \eta_R^n(x_m)\eta_S^n(x_m) + \nu_R^n(x_m)\nu_S^n(x_m) + r_R^n(x_m)r_S^n(x_m)]\end{aligned}$$

$$\begin{aligned}= [\sqrt{w_1}\mu_R^n(x_1)\sqrt{w_1}\mu_S^n(x_1) + \sqrt{w_1}\eta_R^n(x_1)\sqrt{w_1}\eta_S^n(x_1) + \sqrt{w_1}\nu_R^n(x_1)\sqrt{w_1}\nu_S^n(x_1)\\ + \sqrt{w_1}r_R^n(x_1)\sqrt{w_1}r_S^n(x_1)] + [\sqrt{w_2}\mu_R^n(x_2)\sqrt{w_2}\mu_S^n(x_2) + \sqrt{w_2}\eta_R^n(x_2)\sqrt{w_2}\eta_S^n(x_2)\\ + \sqrt{w_2}\nu_R^n(x_2)\sqrt{w_2}\nu_S^n(x_2) + \sqrt{w_2}r_R^n(x_2)\sqrt{w_2}r_S^n(x_2)]\\ + \ldots + [\sqrt{w_m}\mu_R^n(x_m)\sqrt{w_m}\mu_S^n(x_m) + \sqrt{w_m}\eta_R^n(x_m)\sqrt{w_m}\eta_S^n(x_m)\\ + \sqrt{w_m}\nu_R^n(x_m)\sqrt{w_m}\nu_S^n(x_m) + \sqrt{w_m}r_R^n(x_m)\sqrt{w_m}r_S^n(x_m)].\end{aligned}$$

By Cauchy-Schwarz inequality, we have,

$$\begin{aligned}[C_w(R,S)]^2 \leq \{w_1\left((\mu_R^n)^2(x_1) + (\eta_R^n)^2(x_1) + (\nu_R^n)^2(x_1) + (r_R^n)^2(x_1)\right)\\ + w_2\left((\mu_R^n)^2(x_2) + (\eta_R^n)^2(x_2) + (\nu_R^n)^2(x_2) + (r_R^n)^2(x_2)\right) + \ldots\\ + w_m\left((\mu_R^n)^2(x_m) + (\eta_R^n)^2(x_m) + (\nu_R^n)^2(x_m) + (r_R^n)^2(x_m)\right)\}\\ \times \{w_1\left((\mu_S^n)^2(x_1) + (\eta_S^n)^2(x_1) + (\nu_S^n)^2(x_1) + (r_S^n)^2(x_1)\right)\\ + w_2\left((\mu_S^n)^2(x_2) + (\eta_S^n)^2(x_2) + (\nu_S^n)^2(x_2) + (r_S^n)^2(x_2)\right) + \ldots\\ + w_m\left((\mu_S^n)^2(x_m) + (\eta_S^n)^2(x_m) + (\nu_S^n)^2(x_m) + (r_S^n)^2(x_m)\right)\}\\ = \sum_{i=1}^{m} w_i\left[(\mu_R^n)^2(x_i) + (\eta_R^n)^2(x_i) + (\nu_R^n)^2(x_i) + (r_R^n)^2(x_i)\right]\\ \times \sum_{i=1}^{m} w_i\left[(\mu_S^n)^2(x_i) + (\eta_S^n)^2(x_i) + (\nu_S^n)^2(x_i) + (r_S^n)^2(x_i)\right].\end{aligned}$$

$\Rightarrow [C_w(R,S)]^2 \leq I_w(R) \cdot I_w(S)$. Therefore, in view of the equation (3.7), it is clear that $K_1^w(R,S) \leq 1$, which proves the theorem.

Theorem 4 *The weighted correlation coefficient of two T-spherical fuzzy sets $K_2^w(R,S)$, given by equation (3.8), is a valid statistical measure.*

Proof: The proof is on the similar lines with the proof of Theorem 2.

4. COMPUTATIONAL APPLICATIONS

In this section, computational application problems of pattern recognition and medical diagnosis have been taken into consideration.

4.1 Pattern Recognition

We first present the "*Principle of Maximum Correlation Coefficient[5]*" in the light of spherical fuzzy sets as follows:

Principle of Maximum Correlation Coefficient: Let us consider 'm' different classes of patterns in which there are many members in each class. Here, we represent each member of each class by a T-spherical fuzzy set, say, A_α where, $\alpha = 1,2,\ldots,m$ in X (universe discourse). If we have an unknown pattern Q (as another T-spherical fuzzy set) with us which is to be recognized in terms of its possible belongingness to one of the 'm' classes, then the degree of closeness index between A_α and Q is given by

$$\alpha^* = arg\max_{\alpha}\{K(A_\alpha,Q)\}; \tag{4.1}$$

where $K(A_\alpha,Q)$ will be computed based on the proposed correlation coefficients. More the value of α^*, more will be the belongingness of the pattern Q in the α^{th} class. For simplicity of the calculation in the following illustrative example of pattern recognition, we take the value $n = 2$ and $m = 3$: Let us take three representative patterns A_1, A_2 and A_3 from three different classes C_1, C_2 and C_3 under consideration respectively, which are being described by T-spherical fuzzy sets over the domain of discourse $X = \{x_1,x_2,x_3\}$:

$$A_1 = \{(x_1,0.5,0.4,0.2),(x_2,0.4,0.3,0.4),(x_3,0.4,0.5,0.1)\};$$
$$A_2 = \{(x_1,0.6,0.5,0.1),(x_2,0.5,0.1,0.3),(x_3,0.5,0.5,0.1)\};$$
$$A_3 = \{(x_1,0.4,0.4,0.2),(x_2,0.4,0.5,0.2),(x_3,0.3,0.3,0.4)\}.$$

Consider an unknown sample pattern Q which is given by
$Q = \{(x_1,0.4,0.4,0.2),(x_2,0.5,0.6,0.1),(x_3,0.3,0.4,0.4)\}$.

Now, the main objective of the problem is to find out the class to which Q belongs. Based on some prior information about the weights, we chose the weights of x_1, x_2 and x_3 as 0.5, 0.2 and 0.3 for the calculation of the weighted correlation coefficients. As per the principle of maximum correlation coefficient stated above, the computed values of the correlation coefficients and the weighted correlation coefficients with respect to the proposed ones, *i.e.*, Eqs. (3.5), (3.6), (3.7) and (3.8), are tabulated in Table 1 which shows that the unknown pattern Q belongs to the class C_3.

Table 1. Computed values of correlation coefficients.

Classes	$K_1(A_\alpha,Q)$	$K_2(A_\alpha,Q)$	$K_1^w(A_\alpha,Q)$	$K_2^w(A_\alpha,Q)$
A_1	0.9239	0.8701	0.6767	0.6417
A_2	0.8411	0.7826	0.5852	0.5452
A_3	**0.9692**	**0.9521**	**0.7087**	**0.7055**

4.2 Medical Diagnosis

Next, we consider a standard medical diagnosis problem where it needs to diagnose a patient P under a given diagnoses set $D = \{$*"Viral fever, Malaria, Typhoid, Stomach problem, Chest problem"*$\}$ with symptoms set $S = \{$*"Temperature, Headache, Stomach pain, Cough, Chest pain"*$\}$. The symptom's information for the diagnoses and the patient's symptoms are provided in Tables 2 and 3 respectively.

Each component of the each table is being represented as a T-spherical fuzzy set whose entries correspond to the membership (yes), neutral (abstain) and non-membership (no) values respectively, *i.e.*, in Table 2, $(\mu,\eta,\nu) = (0.4,0.3,0.4)$ describes the symptom of the temperature for viral fever in different capacities. In view of the "*Principle of Maximum Correlation Coefficient*" with respect to the problem under consideration, *i.e.*,

Table 2. Symptoms characteristic for the diagnosis.

	Viral Fever	Malaria	Typhoid	Stomach Problem	Chest Problem
Temperature	$(0.4, 0.3, 0.4)$	$(0.7, 0.1, 0.2)$	$(0.4, 0.2, 0.7)$	$(0.0, 0.0, 1)$	$(0.7, 0.1, 0.4)$
Headache	$(0.5, 0.5, 0.3)$	$(0.0, 0.0, 0.5)$	$(0.2, 0.0, 0.8)$	$(0.45, 0.4, 0.0)$	$(0.41, 0.3, 0.6)$
Stomach Pain	$(0.6, 0.0, 0.1)$	$(0.5, 0.1, 0.6)$	$(0.5, 0.2, 0.4)$	$(0.9, 0.0, 0.1)$	$(0.5, 0.5, 0.5)$
Cough	$(0.4, 0.3, 0.7)$	$(0.4, 0.6, 0.6)$	$(1, 0.0, 0.0)$	$(0.1, 0.6, 0.7)$	$(0.76, 0.3, 0.2)$
Chest Pain	$(0.3, 0.3, 0.6)$	$(0.1, 0.3, 0.7)$	$(0.1, 0.4, 0.6)$	$(0.7, 0.2, 0.1)$	$(0.2, 0.0, 0.8)$

Table 3. Symptoms for the diagnose under consideration.

	Temperature	Headache	Stomach Pain	Cough	Chest Pain
P	$(0.7, 0.1, 0.4)$	$(0.5, 0.1, 0.8)$	$(0.4, 0.6, 0.5)$	$(0.8, 0.0, 0.4)$	$(0.0, 0.8, 0.5)$

to have a proper diagnose, we compute all the proposed correction coefficients $K(P, d_\alpha)$ between the patient's symptoms and the set of symptoms that are characteristic for each diagnose $d_\alpha \in D$, with $\alpha = \{1, ..., 5\}$. Similar to the equation (4.1), the proper diagnose d_α for the patient P may be based on the value of α^*, given by $\alpha^* = arg \max_{\alpha}(K(P, d_\alpha)$. Based on the personal perception and experience of the medical professional, if some weights are being assigned to diagnose: viral fever, malaria, typhoid, stomach problem, chest problem as 0.25, 0.2, 0.15, 0.2 and 0.2 respectively. Based on the defined weights and other input values, all the computed values for proposed correlation coefficients are being tabulated in Table 4 which clearly depicts that the patient P is suffering chest problem.

Table 4. Computed values of correlation coefficients.

Diagnoses	$K_1(d_\alpha, P)$	$K_2(d_\alpha, P)$	$K_1^w(d_\alpha, P)$	$K_2^w(d_\alpha, P)$
Viral Fever	0.4184	0.3756	0.4224	0.3803
Malaria	0.5500	0.5381	0.5638	0.5567
Typoid	0.8165	0.7976	**0.8247**	0.8048
Stomach problem	0.4058	0.3771	0.4274	0.4141
Chest Problem	**0.8276**	**0.8268**	0.8227	**0.8166**

5. COMPARATIVE ANALYSIS AND ADVANTAGES

In this section, we carry out a comparative analysis to validate the performance of the proposed correlation coefficients of TSFSs with some of the existing approaches.

5.1 Correlation Cofficients in Pattern Recognition

In order to validate the performance of the proposed correlation coefficients in pattern recognition, we consider an example where there are three representatives centrally located patterns A_1, A_2 and A_3 from three different classes C_1, C_2 and C_3 respectively. It may be noted that the patterns described by the spherical fuzzy sets have significantly wide coverage than intuitionistic fuzzy set or picture fuzzy set, *i.e.*, a decision maker is not strictly bounded in forwarding its opinion. We present the following computations for the patterns under consideration with the domain of discourse as $X = \{x_1, x_2, x_3\}$:

• **Correlation Coefficient by [8]:** Suppose the representative patterns are given in the form of intuitionistic fuzzy set as follows:

“$A_1 = \{(x_1, 0.4, 0.5), (x_2, 0.7, 0.1), (x_3, 0.3, 0.3)\}$”;
“$A_2 = \{(x_1, 0.5, 0.4), (x_2, 0.7, 0.2), (x_3, 0.4, 0.3)\}$”;
“$A_3 = \{(x_1, 0.4, 0.5), (x_2, 0.7, 0.1), (x_3, 0.4, 0.3)\}$”.

Consider an unknown sample pattern Q in the form of intuitionistic fuzzy set which is given by “$Q = \{(x_1, 0.1, 0.1), (x_2, 1.0, 0.0), (x_3, 0.0, 1.0)\}$”. Now, the main objective of the problem is to find out the class to which Q belongs. Based on the correlation coefficient proposed by [8], we obtain the following values: $K_{IFS}(A_1, Q) = K_{IFS}(A_2, Q) = K_{IFS}(A_3, Q) = 0.6292$.

• **Correlation Coefficient by [19]:** Suppose the representative patterns are given in the form of picture fuzzy set as follows:
“$A_1 = \{(x_1, 0.4, 0.5, 0.1), (x_2, 0.7, 0.1, 0.1), (x_3, 0.3, 0.3, 0.2)\}$”;
“$A_2 = \{(x_1, 0.5, 0.4, 0.1), (x_2, 0.7, 0.2, 0.1), (x_3, 0.4, 0.3, 0.1)\}$”;
“$A_3 = \{(x_1, 0.4, 0.5, 0.1), (x_2, 0.7, 0.1, 0.1), (x_3, 0.4, 0.3, 0.2)\}$”.

Consider an unknown sample pattern Q in the form of picture fuzzy set which is given by “$Q = \{(x_1, 0.1, 0.1, 0.4), (x_2, 1.0, 0.0, 0.0), (x_3, 0.0, 1.0, 0.0)\}$”. Based on the correlation coefficient proposed by [19], we obtain $K_{P_1}(A_1, Q) = 0.7937, K_{P_1}(A_2, Q) = 0.7746, K_{P_1}(A_3, Q) = 0.7721$.

• **Proposed Correlation Coefficients:** Suppose the representative patterns are given in further translated form of spherical fuzzy set ($n = 2$) as follows:
“$A_1 = \{(x_1, 0.4, 0.5, 0.2), (x_2, 0.7, 0.1, 0.3), (x_3, 0.3, 0.3, 0.5)\}$”;
“$A_2 = \{(x_1, 0.5, 0.4, 0.3), (x_2, 0.7, 0.2, 0.2), (x_3, 0.4, 0.3, 0.3)\}$”;
“$A_3 = \{(x_1, 0.4, 0.5, 0.4), (x_2, 0.7, 0.1, 0.4), (x_3, 0.4, 0.3, 0.4)\}$”.

Similarly, if we consider an unknown sample pattern Q in the form of spherical fuzzy set which is given by “$Q = \{(x_1, 0.1, 0.1, 0.9), (x_2, 1.0, 0.0, 0.0), (x_3, 0.0, 1.0, 0.0)\}$”, then using the proposed correlation coefficients, we obtain the following values:
$K_1(A_1, Q) = 0.393535, K_1(A_2, Q) = 0.4047, K_1(A_3, Q) = 0.460449$;
$K_2(A_1, Q) = 0.264412, K_2(A_2, Q) = 0.27633, K_2(A_3, Q) = 0.293014$.

Comparative Remarks:

• All the values the correlation coefficients $K_{IFS}(A_i, Q)$; $\forall i$ obtained by [8] are found to be identical which shows a kind of limitation/drawback as an unknown pattern could not belong to all the classes simultaneously.

• However, on the other hand, the values the correlation coefficients $K_{P_1}(A_i, Q)$; $\forall i$ obtained by [19] are different but the difference is not that much prominent as desired.

• Based on the results obtained using the proposed correlation coefficients, we clearly assert that the values for classification are significantly differentiable in both the cases. Certainly, the proposed correlation coefficients of spherical fuzzy sets have an added advantage of allowing the decision maker to give their opinion freely without any restriction.

5.2 Correlation Cofficients in Medical Diagnosis

Here, we validate the performance of the proposed correlation coefficients for the medical diagnosis problem by taking an example where it needs to diagnose a patient P under a diagnoses set D = {“*Viral fever, Malaria, Typhoid, Stomach problem, Chest problem*”} and a symptom’s S = {“*Temperature, Headache, Stomach pain, Cough, Chest pain*”}.

• **Correlation Coefficient by [5]:** The characteristic symptoms for the diagnoses and the symptoms for patient are provided in the respective tables in the form of intuitionistic fuzzy sets/Pythagorean fuzzy sets respectively. Based on the personal perception and experience of the medical professionals, if some weights are being assigned to diagnose: "viral fever, malaria, typhoid, stomach problem, chest problem" as 0.15, 0.25, 0.20, 0.20 and 0.15 respectively. On the basis of the computed values of the correlation coefficients proposed by [5], the patient is most probably suffering from Malaria.

Table 5. Computed values of correlation coefficients [5].

Diagnoses	$K_1(d_\alpha,P)$	$K_2(d_\alpha,P)$	$K_3(d_\alpha,P)$	$K_4(d_\alpha,P)$
Viral Fever	0.8622	0.8328	0.8768	0.8594
Malaria	**0.9047**	**0.8895**	**0.8907**	**0.8745**
Typoid	0.7808	0.7485	0.7972	0.7627
Stomach problem	0.6233	0.6229	0.6647	0.6595
Chest Problem	0.5080	0.5075	0.5175	0.5163

• **Proposed Correlation Coefficients:** In case the information received from the experts is more extensive and flexible, we suppose the characteristic symptoms for the diagnoses and the symptoms for patient to be in the form of spherical fuzzy sets ($n = 2$) given in Tables 6 and 7 respectively.

Table 6. Symptoms characteristic for the diagnosis.

	Viral Fever	Malaria	Typhoid	Stomach Problem	Chest Problem
Temperature	$(0.4,0.0,0.5)$	$(0.7,0.0,0.3)$	$(0.3,0.3,0.5)$	$(0.1,0.7,0.3)$	$(0.1,0.8,0.2)$
Headache	$(0.3,0.5,0.2)$	$(0.2,0.6,0.1)$	$(0.6,0.1,0.4)$	$(0.2,0.4,0.5)$	$(0.0,0.8,0.1)$
Stomach Pain	$(0.1,0.7,0.3)$	$(0.0,0.9,0.2)$	$(0.2,0.7,0.2)$	$(0.8,0.0,0.3)$	$(0.2,0.8,0.3)$
Cough	$(0.4,0.3.0.4)$	$(0.7,0.0,0.3)$	$(0.2,0.6,0.4)$	$(0.2,0.7,0.1)$	$(0.2,0.8,0.1)$
Chest Pain	$(0.1,0.7,0.2)$	$(0.1,0.8,0.2)$	$(0.1,0.9,0.3)$	$(0.2,0.7,0.3)$	$(0.8,0.1,0.2)$

Table 7. Patient symptoms under consideration.

	Temperature	Headache	Stomach Pain	Cough	Chest Pain
P	$(0.8,0.1,0.2)$	$(0.6,0.1,0.4)$	$(0.2,0.8,0.1)$	$(0.6,0.1,0.3)$	$(0.1,0.6,0.4)$

Based on the personal perception and experience of the medical professional, suppose that some weights are being assigned to diagnose: "viral fever, malaria, typhoid, stomach problem, chest problem" as 0.15, 0.25, 0.20, 0.20 and 0.15 respectively. Using the proposed correlation coefficients of spherical fuzzy sets, we compute and tabulate the values of the correlation coefficients in Table 8.

Comparative Remarks:

• Incorporating all the correlation coefficients proposed by [5], the values of $K_i(d_\alpha,P)$; $i = 1,2,3,4$ in view of Table 5 indicate that the patient is suffering from Malaria. Here, it may be noted that the input information is either in the form of intuitionistic fuzzy sets or Pythagorean fuzzy sets which is unable to span the full imprecise information in all the aspects.

• However, utilizing the proposed correlation coefficients of spherical fuzzy sets for the further translated data given in Tables 6 and 7, we obtained the revised results. The obtained results have been presented in Table 8 which depicts that the patient is either suffering from Malaria or viral fever. A further precise diagnosis is needed for a better

Table 8. Computed values of correlation coefficients.

Diagnoses	$K_1(d_\alpha,P)$	$K_2(d_\alpha,P)$	$K_1^w(d_\alpha,P)$	$K_2^w(d_\alpha,P)$
Viral Fever	0.8522	**0.8470**	0.8632	**0.8561**
Malaria	**0.8947**	0.8359	**0.8790**	0.8071
Typoid	0.7628	0.7569	0.7821	0.7552
Stomach problem	0.5549	0.5486	0.5956	0.5922
Chest Problem	0.4744	0.4423	0.4826	0.4447

treatment. This approach provides a kind of more reliability and dependability as the decision maker had a more variability in his prediction.

5.3 Advantages of the Proposed Work

In view of the above detailed analysis, the proposed correlation coefficients of T-spherical fuzzy sets and spherical fuzzy sets are found to be worthy enough in contrast with the existing related literatures. The following are the major advantages of the proposed work:

• The additional exponents of the degrees of membership, neutral membership, non-membership and degree of refusal in case of the spherical fuzzy sets certainly provide a wider coverage and wider geometrical span.
• The proposed correlation coefficients have capabilities to address the related dependability on the imprecise information which has a degree of refusal with more reliablity.
• The drawback in the existing statistical measures for intuitionistic fuzzy sets and picture fuzzy sets is that the condition does not allow the decision makers to allocate the membership values of their own choice. Somehow, this makes the decision makers bounded for providing their input in a particular domain. However, the proposed correlation coefficients of spherical fuzzy sets provide a generalization which make a strong impact.
• The discussion on the results obtained in case of pattern recognition and medical diagnosis in the above subsections shows that the proposed work handled the generalized framework in an effective and consistent way.

6. CONCLUSIONS AND SCOPE FOR FUTURE WORK

The correlation coefficients for T-spherical fuzzy sets with their weighted versions have been well introduced along with their validity proof. These correlation coefficients cover an add-on reliability and are able to capture more flexibility of the imprecise information. In view of the revised principle of maximum correlation coefficients, the proposed correlation coefficients have been implemented in solving the problems of pattern recognition and medical diagnosis. Some important comparative remarks and advantages of the proposed methodology have been listed. It has also been concluded that the results obtained are found to be consistent and methodologies outlined above can be extended for larger dimensional problems. In future, the proposed correlation coefficients can also be comprehensively used in the cluster analysis when the information data is to be taken in the form of SFSs. In addition to this, the application may further be projected in the field of bidirectional approximate reasoning [3, 19].

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Abhishek Guleria is currently working as a research scholar in the Department of Mathematics, Jaypee University of Information Technology, Waknaghat, Solan, HP, India. He has received his M.Sc. and M.Phil. from HP University, Shimla, India in 2015 and 2017 respectively. His research interests areas are fuzzy information measures, decision making, pattern recognition, soft computing techniques and applications.

Rakesh Kumar Bajaj has received his M.Sc. from IIT, Kanpur in 2002 and Ph.D. in Mathematics from JUIT, Waknaghat in 2009. He is working as an Associate Professor of Mathematics in JUIT, Waknaghat, Solan, HP, India since 2003. His interests include fuzzy information measures, pattern recognition, fuzzy clustering, and fuzzy mathematics.