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A Novel Variable Lie Hypergraph Technique for an Energy Aware Routing Protocol to Improve Infotainment Services in VANETs

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Wireless technology is growing at a breakneck pace. The Vehicular Ad-hoc Network (VANET) is one of the most intriguing Intelligent Transportation System (ITS) area, that comprises of many automobiles travelling on the road. The emergence of such networks transforms the realm of wireless communication. The significant evolution of VANET technology is due to the enormous growth in the number of smart vehicles over time. Since itinerant vehicles rely on battery power, one of the most challenging issues is to send multimedia messages and emergency alerts swiftly and reliably. Toward this, a novel Variable Lie Hypergraph based Energy Aware Routing Protocol (VLH-EARP) is proposed to minimise latency and energy depletion in order to ameliorate infotainment services in VANETs. First, variable hypergraph construction for Lie algebra of upper triangular matrices is introduced which it is employed for clustering of vehicles, and cluster head selection was exploited by variable hypergraph transversal property. Next, the vehicles in hyperedges are transformed to Upper Triangular Matrix (UTM). Finally, the route is identified by Lie commutators of Lie algebra of upper triangular matrices. The performance of the VLH-EARP is assessed by simulating the network with varied sizes, based on the metrics packet delivery ratio, energy factor, end-to-end delay, and throughput. Simulation results exemplify that the proposed VLH-EARP outperforms over the compared routing approaches.

Keywords: VANETs, hypergraph, energy, Lie algebra, routing

1. INTRODUCTION

VANET is a thriving wireless technology that acts as driver assistance to minimize the critical situation in Vehicle to Vehicle (V2V) communication like bumper to bumper jams, random barking, road accidents, alternative ways for emergency vehicles. Also, it is helpful for amenity applications to passengers, *e.g.*, internet connectivity, sharing live audio or video streaming, weather and infotainments during drives. Some of the applications developed by automobile manufacturers and government, are Crash Avoidance Metrics Partnership (CAMP), ASSESS (Assessment of Integrated Vehicle Safety Systems for improved vehicle safety), Advanced Forward-Looking Safety Systems (vFSS), FLEETNET and CARTALK [1].

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VANET faces some issues and challenges, such as multipath fading, energy imbalance during data transmission, road obstacles, traffic congestion, mobility, road topology, traffic diversion model etc. These challenges make it difficult to transmit data in VANETs. However, due to the open communication and dynamic infrastructure, VANETs suffers in selecting reliable path and energy depletion of nodes. So designing an energy-efficient reliable routing protocol is essential. VANET routing protocols may be generally categorised as follows: proactive [2], reactive [3], hybrid [4], and geographic-based routing protocols [5].

The topology-based routing protocol category includes proactive and reactive routing protocols. The proactive routing protocol maintains routes to all nodes located in the entire network, which increases overhead and end-to-end delay. A source node initiates a discovery process in reactive routing to reach only the required destination. The hybrid routing approach is often known as a cluster-based routing (CBR) technique that combines the benefits of both proactive and reactive routing protocols. In this protocol, the vehicles are clustered where each cluster has an elected leader called a cluster heads (CH) which are responsible for data transmission.

CBR protocols are typically used in VANET environments to improve scalability and reduce overhead messages. Although the clustering approach reduces routing control overhead, the repeated cluster formation and CH elections increases the overhead. Furthermore, many clustering techniques are extensively proposed based on theoretical mathematics such as probability theory, graph theory, wheel theory, and game theory. A posthaste emerging theories called Lie [6], and Hypergraph [7] provoked renewed interest in applying it to real-world problems to find an optimal solution. This article introduces a variable hypergraph with the spark from variable set theory [8] and the construction of a variable hypergraph from Lie commutators. To best of our knowledge, this is the first work of its kind to introduce a VANET formulation via a variable hypergraph. Lie algebra is coalesced with variable hypergraph to form a novel Variable Lie Hypergraph based Energy Aware Routing Protocol technique for optimizing the energy efficiency in V2V communication. The key contributions can be summarized as,

1) Variable hypergraph formulation for q-clustering of VANET nodes,
2) CH selection using the minimum variable hypergraph transversal algorithm,
3) Construction of an element of Lie algebra of upper triangular matrices for an efficient routing and
4) the novel routing mechanism using Lie commutators that selects robust relay-node.

The rest of the paper is arranged, so the next section surveys the literature. Section 3 describes the fundamental concepts necessary for our scheme. Section 4 describes the proposed work, and in Section 5 performance evaluation is given. Finally, Section 6 concluded the paper.

2. RELATED WORK

Researchers have developed many technologies recently for wireless communication [9] and IoT [10]. VANET is the wireless technology with highly dynamic in nature so many varied routing protocols for VANETs based on different topology, applications, and problems are proposed. All of these protocols seek to increase throughput and delivery ratio while minimising control overhead. Many routing protocols are designed depending on metrics like speed, density, distance, etc. Some of them are as follows.

In topology based routing, packets are transferred based on the node's link information in the network. Proactive, reactive and hybrid routing falls under topology based routing. Proactive routing always maintains a routing table for packet transfer and relies on shortest path algorithms [11], whereas reactive routing only finds a path when required. A hybrid routing protocol integrates proactive and reactive protocols.

The most popular proactive routing protocols are DSDV, GSRP, FSR and OLSR. The Destination-sequenced distance vector (DSDV) [12] depends on Bellman-Ford algorithm to transmit packets. There are two sorts of packets sent in DSDV, namely, full dump and incremental, where the full dump packet contains all routing information, whereas incremental sends only updates. All nodes in the DSDV maintain neighbour information in the table and exchange table information with their neighbours. DSDV is not suitable for large scale networks because of frequent incremental packet transfer. Global state routing protocol (GSRP) [13] is analogous to DSDV; every node in this protocol maintains three tables: the distance table, topology table, next-hop table, and one list called the neighbour list. The distance table maintains the shortest distance between the nodes. Topology table and next-hop table maintains link state information and the next hop for which the message is to be sent, respectively.

In Optimized link state routing (OLSR) [14], each node regularly floods the status of its links. Each node records the link state information it receives from other nodes. When packets are to be transmitted, each node chooses pair of neighbours called multipoint relays (MPR) to transfer it to the destination.

DSR, ADOV, TORA are some of the reactive protocols. AODV [15] is accomplished primarily through two phases: route discovery and route maintenance. RREQ message is disseminated from the source node to neighbours for route discovery until it reaches the destination. If a node does not receive a HELLO message within a particular interval, an RRER message is sent to its neighbours, informing them about link breakage. The route is maintained by notifying the link breakage if a node does not receive a HELLO message within a particular interval. Dynamic Source Routing (DSR) [16] is similar to ADOV, except storing the path in the header; DSR employs source routing instead of looking ahead to the routing table of every intermediate node. In Temporally Ordered Routing Algorithm (TORA) [17] every node builds a directed cyclic graph for route creation and maintenance using height metric, which makes it efficient and scalable.

Zone Based Routing is the most well-known hybrid routing technique (ZBR) [18]. The network area is split into zones; nodes in every zone are within the transmission radius of the zone. The hops volume determines the zone's dimension, which is responsible for the intra and inter-zone routing. Later Dan Lin [19] proposed a Moving Zone (MoZo) based routing for V2V communication. The moving patterns of the vehicles are analyzed to create a moving zone. The head vehicle is chosen from each zone to transfer the packets and manage information.

Many nature inspired algorithms are used for the purpose of VANET routing, like hybrid ant colony optimization, [20], genetic firefly algorithm [21], particle swarm optimization [22] and many more. Recently, Arbelo Lolai [23], proposed reinforcement learning based routing for VANETs. The Q-routing functions for Road Model Segment Selection using higher connectivity and distance, and Intermediate Vehicle Selection using speed, direction, link reliability, packets in the queue and signal fading are utilized for a higher packet delivery ratio and minimized latency. Shaik Shafi [24] proposed trust based energy efficient routing using Cooperative Relay Vehicles (CRVs). CRV is chosen based on energy factor and mobility of the vehicle, and the best path is found using the parameters hop count, congestion and Link Expiration Time (LET) along the path.

Now a days to represent structures more effectively hypergraph is utilized because of its n-ary relations, that is, its hyperedges can cover more than one vertices. Wireless networks are using the hypergraph properties for efficient clustering and routing. Jocelyn [25] proposed a hypergraph clustering for WSN by hypergraph partitioning to optimise energy in sensor nodes by transforming it into a weighted hypergraph where weight is the distance between the nodes. Later for VANETs, Kumar [26] exploited the directed hypergraph for clustering and relative speed, link lifetime and connectivity level are used to select the CH.

3. PRELIMINARIES

This section gives insights into Lie algebra and hypergraph required for our approach.

Definition 1 *A Lie algebra* $\mathbf{g}$ *is a vector space with a second bilinear inner composition law* $([.,.])$ *called the bracket product or Lie bracket, which satisfies* $[\theta,\theta] = 0,\ \ \textit{for all}\ \ \theta \in \mathbf{g}$ *and* $J(\theta,\gamma,\omega) = 0,\ \ \textit{for all}\ \ \theta,\gamma,\omega \in \mathbf{g}$ *where* J *is the Jacobiator defined as,* $J(\theta,\gamma,\omega) = [[\theta,\gamma],\omega] + [[\gamma,\omega],\theta] + [[\omega,\theta],\gamma]$ *known as Jacobi identity.*

Definition 2 *The matrix Lie algebra* $\mathbf{g_d}$ *for a given* $d \in \mathbb{N}$ *consists of all* $d \times d$ *upper triangular matrices with dimension* $\frac{d(d+1)}{2}$. *It is expressed as,*

$$\mathbf{g_d}(y_{r,s}) = \begin{pmatrix} y_{11} & y_{12} & \cdots & y_{1d} \\ 0 & y_{22} & \cdots & y_{2d} \\ \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & y_{dd} \end{pmatrix}, \qquad y_{rs} \in \mathbf{R}. \tag{1}$$

The basis is $\mathcal{B}_d = \{Y_{i,j}\}_{1\le i\le j\le d}$ *of* $\mathbf{g_d}$, *where the* $(i,\ j)$ *position of* $Y_{i,j}$ *is 1 and other entries are zero.*

For distinct $i,\ j$ *and* k *with* $1 \le i < j < k \le d$, *the law defined with respect to* $\mathcal{B}_d$ *is as follows,*

$$[Y_{i,j}, Y_{j,k}] = Y_{i,k}; (Type\ 1); [Y_{i,i}, Y_{i,j}] = Y_{i,j}; (Type\ 2); [Y_{i,j}, Y_{j,j}] = Y_{i,j}. (Type\ 3) \tag{2}$$

3.1 Associating Combinatorial Structures with Lie Algebras

Given a d-dimensional Lie algebra $\mathbf{g}$ with basis $\mathcal{B} = \{v_i\}_{i=1}^{d}$, recall the method introduced in [27] for associating a combinatorial structure with $\mathbf{g}$. If $[c_x, c_y] = \sum_{z=1}^{d} f_{x,y}^{z} c_z$, a combinatorial structure can be associated with $\mathbf{g}$ as follows:

a) A vertex v_x is taken, for each $c_x \in \mathcal{B}$.

b) Full triangle is drawn for any three vertices $v_x < v_y < v_z$ if and only if $(f^z_{x,y}, f^x_{y,z}, f^y_{x,z}) \neq (0,0,0)$, where $f^z_{x,y},\ f^x_{y,z}$ and $f^y_{x,z}$ represents the weights of the edges $v_xv_y,\ v_yv_z$ and v_xv_z respectively.

b1) Edges are drawn with dotted line if its weight is zero.
b2) Let $v_xv_yv_z$ and $v_xv_yv_l$ be two triangles satisfying $f^z_{x,y} = f^l_{x,y}$ then only one edge is drawn between vertices v_x and v_y shared by the two triangles.

c) If two vertices v_x and v_y satisfies $v_x < v_y$, a directed edge from v_y to v_x is drawn if $f^x_{x,y} \neq 0$ or a directed edge from v_x to v_y if $f^y_{x,y} \neq 0$.

For upper triangular matrix Lie algebra $\mathbf{g}_d$, Ceballos in [28], have defined an order for associating each vertex with a vector from the basis $\mathcal{B}_d$ of $\mathbf{g}_d$. More concretely, the order is the one of the elements of each row of matrix $g_d(y_{r,s})$ in Eq. (1) is given as, $\{Y_{1,1}, Y_{1,2}, \dots, Y_{1,d}\}$ with $\{v_1, v_2, \dots, v_d\}$, $\{Y_{2,2}, Y_{2,3}, \dots, Y_{2,d}\}$ with $\{v_{d+1}, v_{d+2}, \dots, v_{2d-1}\}, \dots \{Y_{d,d}\}$ with $\{v_{\frac{d(d+1)}{2}}\}$.

Definition 3 *A set $\mathcal{V_S}$ is variable set that contains variables that vary over time τ or with some parameters. As time interval varies cardinality of $\mathcal{V_S}$ fluctuates, depending on this, we have following types of $\mathcal{V_S}$ for any instances of time τ_1 and τ_2 with $\tau_1 \neq \tau_2$, 1) $\mathcal{V_S}$ is said to be increasing variable set when $\tau_1 < \tau_2$ then $|\mathcal{V_S}(\tau_1)| < |\mathcal{V_S}(\tau_2)|$, 2) $\mathcal{V_S}$ is said to be decreasing variable set when $\tau_1 < \tau_2$ then $|\mathcal{V_S}(\tau_1)| > |\mathcal{V_S}(\tau_2)|$.*

Definition 4 *A hypergraph on $\mathcal{X} = \{v_1, v_2, \dots, v_p\}$ is a family, $\mathcal{H} = (\mathcal{HE}_1, \mathcal{HE}_2, \dots, \mathcal{HE}_q)$ of subsets of X such that $\mathcal{HE}_i \neq \emptyset$, $i = 1, 2, \dots, q$ and $\bigcup_{i=1}^{q} \mathcal{HE}_i = X$. The elements $v_1, v_2, \dots, v_p$, of X are called vertices, and the sets $\mathcal{HE}_1, \mathcal{HE}_2, \dots, \mathcal{HE}_q$ are the hyperedges of the hypergraph.*

Definition 5 *A set $\mathcal{HT} \subseteq \mathcal{X}$ is called a transversal of a hypergraph $\mathcal{H} = (\mathcal{X}, \mathcal{HE})$ if $\mathcal{HT} \cap \mathcal{HE}_i \neq \emptyset, \forall\ \mathcal{HE}_i \in \mathcal{HE}$, and it is said to be minimal if no proper subset$\mathcal{HT}'$ of $\mathcal{HT}$ is a transversal of $\mathcal{H}$.*

4. PROPOSED WORK

This section outlines our proposed Variable Lie hypergraph approach for VANET routing. The proposed protocol employs commutators of Lie algebra of upper triangular matrices to determine the efficient intermediate forwarder vehicle and the optimal path between source and destination. At the beginning, the association of Lie algebra of upper triangular matrices with a variable n-dimensional combinatorial structure (hypergraph) is is introduced as in [29]. The proposed system operates on three predominant phases: (1) clustering, which includes variable hypergraph construction and minimum variable hypergraph transversal algorithm for CH selection; (2) the upper triangular matrix construction for an optimal path selection; (3) Lie commutators based routing algorithm.

Algorithm 1: Minimum variable hypergraph transversal

Input: $\mathcal{V_H} = (\mathcal{V_X}, \mathcal{V_{HE}})$
Output: $\mathcal{V_{HT}}$
for $i = 2, \ldots, q$ **do**
  Find $\mathcal{V_{HT}}(\mathcal{V}_{\mathcal{H}i-1})$
  Compute $\mathcal{V_H}\mathcal{T}(\mathcal{H}_i) = Min(\mathcal{V_{HT}}(\mathcal{V}_{\mathcal{H}i-1}) \vee \{\{v\},\ v \in \mathcal{V}_{\mathcal{HE}i}\})$

Definition 6 *A variable hypergraph is a tripe $\mathcal{V_H} = (\mathcal{V_X}, \mathcal{V_{HE}}, \tau_i)$ with both $\mathcal{V_X}$ and $\mathcal{V_{HE}}$ are variable sets with respect to τ_i, where $\mathcal{V_X} = \{\vartheta_i \mid i = 1, 2, \ldots, p\}$ and $\mathcal{V_{HE}} = \{\mathcal{V}_{\mathcal{HE}i} \mid i = 1, 2, \ldots, q\}$, $\mathcal{V}_{\mathcal{HE}i} \neq \emptyset$ and $\bigcup_{i=1}^{q} \mathcal{V}_{\mathcal{HE}i} = \mathcal{V_X}$. The elements $\vartheta_1, \vartheta_2, \ldots, \vartheta_p$ of $\mathcal{V_X}$ are called vertices, and the sets $\mathcal{V}_{\mathcal{HE}1}, \mathcal{V}_{\mathcal{HE}2}, \ldots, \mathcal{V}_{\mathcal{HE}q}$ are the variable hyperedges.*

Definition 7 *If $\mathcal{V_H} = (\mathcal{V_X}, \mathcal{V_{HE}}, \tau_i)$ is a variable hypergraph. A set $\mathcal{V_{HT}} \subseteq \mathcal{V_X}$ is called a variable transversal of $\mathcal{V_H}$ if $\mathcal{V_{HT}} \cap \mathcal{V}_{\mathcal{HE}i} \neq \emptyset, \forall\ \mathcal{V}_{\mathcal{HE}i} \in \mathcal{V_{HE}}$. That is, variable hypergraph transversal is a variable set which changes with respect τ_i and has non-empty intersection with every variable hyperedge of $\mathcal{V_{HE}}$.*

The minimum variable hypergraph transversal is given in Algorithm 1 in which $\mathcal{V}_{\mathcal{H}i}$ denote the variable hypergraph with variable hyperedges $\mathcal{V}_{\mathcal{HE}1}, \mathcal{V}_{\mathcal{HE}2}, \ldots, \mathcal{V}_{\mathcal{HE}i}$.

4.1 Construction of Variable Hypergraph

A variable hypergraph $\mathcal{V_H}$ construction for Lie algebra $\mathbf{g}_d$ for arbitrary $d \in \mathbb{N}$ is introduced here with the Lie commutators of Type 2 defined in Eq. (2), and defining a new commutator (Type 4) by combining Types 1 and 2 commutators with respect to the basis $\mathcal{B}_d$ for distinct i, j and k with $1 \leq i < j < k \leq d$.

$$[Y_{i,i}, Y_{i,j}] = Y_{i,j}\ (Type\ 2)\ \ and\ [[Y_{i,i}, Y_{i,j}], Y_{j,j}] = Y_{i,j}\ (Type\ 4). \tag{3}$$

The construction of $\mathcal{V_H}$ for $\mathbf{g}_d$ is as follows:

a) Construct a variable hyperedge $\mathcal{V}_{\mathcal{HE}i}$ for the vertices corresponding to the diagonal elements, by Type 4 by which each diagonal element is connected with rest of the diagonal elements.

b) Given three vertices $i,\ j$ and k make a hyperedge $\mathcal{V}_{\mathcal{HE}i}$ containing $i,\ j$ and k if and only if corresponding basis elements persuade Types 2 or 4.

4.2 Clustering Protocol

The clustering protocol includes two phases, the construction of a variable hypergraph and CH selection.

4.2.1 Variable hypergraph formation for VANETs

Variable hyperedges of a $\mathcal{V_H}$ is constructed by considering vehicles of VANETs as vertices. Initially, assuming the cardinality of each $\mathcal{V}_{\mathcal{HE}i}$ is less than or equal to $\left\lfloor \frac{p}{q} \right\rfloor$

where p and q $(p/8 \leq q < p/3)$ are the number of vertices and hyperedges, respectively. Denoting $\mathcal{V} = S \cup T$ where S and T are set of marked and unmarked vertices, respectively. At the initialization phase, $\mathcal{V} = T$ and $S = \phi$ and variable hyperedges are generated by combining three factors of the vehicle namely 1) Speed difference and 2) Direction of the vehicle and 3) Inter-vehicle distance.

Vehicle Speed Difference Every vehicle travels at a different speed, since vehicle speed affects the performance of the VANET, it is a significant element impacting the stability of the clusters generated. Denoting $\tilde{\gamma}_i$ and $\tilde{\gamma}_j$ are the current speed of the vehicles ϑ_i and ϑ_j respectively, and $\tilde{\gamma}_{ij}$ is the speed difference between the vehicle, and is calculated using below equation.

$$\tilde{\gamma}_{ij} = \sqrt{(\tilde{\gamma}_i - \tilde{\gamma}_j)^2}$$

Vehicle Direction The vehicle can travel in the same direction or in the opposite direction. The vehicle direction is an important factor when selecting the most stable vehicles that can remain in clusters for an extended time, below equation calculates the direction of moving vehicle with respect to the angle α.

$$\alpha = \cos^{-1}\left(\frac{(\hat{px}_i - px_i)(\hat{px}_j - px_j) + (\hat{py}_i - py_i)(\hat{py}_j - py_j)}{\sqrt{(\hat{px}_i - px_i)^2 + (\hat{py}_i - py_i)^2}\sqrt{(\hat{px}_j - px_j)^2 - (\hat{py}_j - py_j)^2}}\right)/\pi$$

where the $(px_i,\ py_i)$ and $(px_j,\ py_j)$ represents the coordinates of vehicle ϑ_i and ϑ_j at time τ respectively and $(\hat{px}_i,\ \hat{py}_i)$ and $(\hat{px}_j,\ \hat{py}_j)$ represents the coordinates of vehicle ϑ_i and ϑ_j at time $\tau + \Delta\tau$ respectively and α_i represents the direction of the vehicle ϑ_i.

Variable hyperedges $\mathcal{V}_{\mathcal{HE}i}$ are generated by randomly selecting a vehicle ϑ_i and adding every node ϑ_j whose inter-node distance with ϑ_i is less than transmission range and it falls under any one of the following combinations; (i) $\alpha_i = \alpha_j$ and $\tilde{\gamma_{ij}} = 0$; (ii) $\alpha_i \neq \alpha_j$ and $\tilde{\gamma_{ij}} = 0$; (iii) $\alpha_i = \alpha_j$ and $\tilde{\gamma_{ij}} \neq 0$; and (iv) $\alpha_i \neq \alpha_j$ and $\tilde{\gamma_{ij}} \neq 0$. Each $\mathcal{V}_{\mathcal{HE}i}$ is represented as a cluster. Pseudo code for variable hypergraph formation is presented in Algorithm 2.

Algorithm 2: Variable hypergraph formation

```
INPUT: V = S ∪ T set of n vertices and an integer q ≥ 2
OUTPUT: Hyperedges V_HE1 V_HE2, V_HE3, ..., V_HEq
for i = 1, ..., q do
    Select a random vertex ϑ_i from T.
    Calculate the inter-node distance between ϑ_i and other vertices in V_X and put them in a list L_i if it is less
      than radio range.
    Select the first ⌊p/q⌋ elements of Temp and add their corresponding vertices to hyperedge V_HEi with the
      vertex ϑ_i
    Update T and S.
```

4.2.2 CH selection by minimum variable hypergraph transversal

The next step is to select a node in a cluster termed Cluster Head which coordinates the cluster and plays an integral role in the routing process. The proposed work elects the CHs by minimum variable hypergraph transversal algorithm. After the construction of variable hyperedges, using Algorithm 1 a minimal transversal set for a hypergraph is generated that satisfy Eq. (4) which is our required CH.

$$CH = max\{CC(\vartheta_i) \mid \vartheta_i \in \mathcal{V}_{\mathcal{HE}j}\} \tag{4}$$

$CC(\vartheta_i) = \frac{1}{\sum\limits_{\vartheta_j \in \mathcal{V}_{\mathcal{HE}i}} \mathcal{D}(\vartheta_i, \vartheta_j)}$, where $\mathcal{D}(\vartheta_i, \vartheta_j)$ is an Euclidean distance between ϑ_i and ϑ_j and $CC(\vartheta_i)$ is the closeness centrality of the vehicle ϑ_i.

4.3 Construction of Upper Triangular Matrix and Routing Algorithm

An Upper Triangular Matrix (UTM) is designed to evaluate each feasible path by quickly identifying a more reliable relay node. Many researchers, in general, use numerous parameters [30] to build the routing matrix of size $p \times p$, where p is the number of vehicles. In the proposed method, two significant factors are combined to place the node in the UTM: energy factor and distance to the neighbour cluster head.

4.3.1 Energy factor

In vehicle-to-vehicle communication, the mobile nodes are battery-powered, which uses more energy during a data packet exchange. To overcome this impediment, selecting a vehicle with high battery life is one of the significant challenges in VANET, therefore, it is regarded as a critical parameter in deciding a co-operative relay node to overcome link breakages. Energy-factor of a ϑ_i can be determined by the following equation and it always lies between 0 to 1.

$$Ene_{fac} = \frac{\mathcal{RE}}{\mathcal{IE}_{t=0}} \tag{5}$$

where $\mathcal{RE} = \mathcal{IE}_{t=0} - \mathcal{EC}_t$, is a node's residual energy, $\mathcal{IE}_{t=0}$ denotes initial energy of the node and $\mathcal{EC}_t$ indicates node's energy consumption at the end of simulation for networking a packet.

4.3.2 Distance to the neighbour CH

In addition, to energy factor we consider the minimum distance between cluster members of one cluster to neighbour CHs to elect the relay node. This parameter ensures the minimum delay time with minimum energy consumption while data dissemination. It is calculated as,

$$\mathcal{D}_{NCH,\vartheta_i} = min\ \{\mathcal{D}^{\vartheta_i}_{NCH} \mid \vartheta_i \in \mathcal{V}_{\mathcal{HE}j}\} \tag{6}$$

where $\mathcal{D}^{\vartheta_i}_{NCH} = \sqrt{(NCH_{(x)} - \vartheta_{i_{(x)}})^2 + (NCH_{(y)} - \vartheta_{i_{(y)}})^2}$,with$(NCH_{(x)}, NCH_{(y)})$ and $(\vartheta_{i_{(x)}}, \vartheta_{i_{(y)}})$ representing the $(x,\ y)$ coordinates of the NCH and ϑ_i respectively. It aims to fix the vehicle ϑ_i inside an upper triangular matrix (UTM) position say (i, j). In general, it is required to access p^2 elements in the routing matrix of dimension $p \times p$, but our approach requires only $d \times d$ dimension that stores $\frac{d(d+1)}{2}$ elements (vehicles) within it. Also, it is apparent that d is much smaller than p. Theorem 1 calculates the dimension d of an element of Lie algebra of upper triangular matrix.

Theorem 1 *Let η_1 vehicles in c_1 clusters, η_2 vehicles in c_2 clusters, η_3 vehicles in c_3 clusters, η_4 vehicles in c_4 clusters and so on with $\eta_1 > \eta_2 > \eta_3 > \ldots$ then dimension of UTM is,*

$$d = \begin{cases} \eta_1 + [c_2 - (\eta_1 - \eta_2)] + [c_3 - (\eta_2 - \eta_3)] + \ldots & \textit{if } c_1 = 1 \\ [\eta_1 + [c_2 - (\eta_1 - \eta_2)] + \ldots] + (c_1 - 1) & \textit{otherwise} \end{cases}$$

The vehicles in each $\mathcal{V}_{\mathcal{HE}i}$ (clusters) where $i \in \{1, 2, \ldots, q\}$ are placed inside the UTM and it is presented in Algorithm 3.

Algorithm 3: Construction of UTM

INPUT: $\mathcal{V}_{\mathcal{HE}}$
OUTPUT: UTM
Create UTM of dimension $d \times d$, using Theorem 1.
Arrange the variable hyperedges according to its cardinality in descending order say $\{\mathcal{V}_{\mathcal{HE}1}, \mathcal{V}_{\mathcal{HE}2}, \ldots, \mathcal{V}_{\mathcal{HE}q}\}$
for $i = 1, \ldots, q$ **do**
  Find an vehicle say ϑ_a such that, $\vartheta_a = \mathcal{V}_{\mathcal{HE}i} \cap \mathcal{V}_{\mathcal{HT}}$.
  Place the ϑ_a in $(i,\ i)$ position of UTM.
for $i = 1, \ldots, q$ **do**
  Find $\vartheta_b \in \mathcal{V}_{\mathcal{HE}i}$ with maximum $\mathcal{RE}$ from Eq. (5) and minimum $\mathcal{D}$ from Eq. (6).
  Assume that ϑ_b's CH is positioned in $(i,\ i)$ and NCH is positioned in $(j,\ j)$.
  if $i < j$ **then**
    Place the ϑ_b in $(i,\ j)$ position.
  else
    Place the ϑ_b in $(j,\ i)$ position.

The constructed upper triangular matrix is as follows,

$$UTM = \begin{bmatrix} \vartheta_a & \vartheta_b & \ldots & \vartheta_c \\ 0 & \vartheta_d & \ldots & \vartheta_e \\ \vdots & & \ddots & \\ 0 & 0 & \ldots & \vartheta_f \end{bmatrix}$$

where each row represents the cluster and $\mathcal{V}_{\mathcal{HT}} = \{\vartheta_a,\ \vartheta_d, \ldots, \vartheta_f\}$ which are positioned in diagonal and v_b is positioned in $(1,\ 2)$ because v_b has maximum energy and minimum distance to NCH say ϑ_d in the position $(2,\ 2)$ and its CH ϑ_a in the position $(1,\ 1)$.

4.4 Novel Routing Algorithm using Lie Commutator

Several routing techniques are proposed in the literature for diverse pursuits, which are critically associated with the architectural model because of its highly dynamic environment. In this paper, we presented an Variable Lie Hypergraph based Energy Aware Routing Protocol (VLH - EARP) based on Lie commutators of Lie algebra of upper triangular matrices. Whenever source vehicle ϑ_i desires to forward the packets to the destination node ϑ_j, ϑ_i initially checks in the UTM for its position and neighbours. Our approach selects the optimal relay node from UTM based on the Lie commutators explained in Section 4.1 for minimized energy consumption and data transmission latency.

If ϑ_i and ϑ_j are in the same $\mathcal{V}_{\mathcal{HE}k}$ then intra cluster routing takes place by the Type 2 commutator $[(i,\ j) \to (i,\ i)]$ or $[(i,\ i) \to (i,\ j)]$ in Eq. (3). If ϑ_i and ϑ_j are in different

Algorithm 4: Routing algorithm

INPUT: Source ϑ_i and destination ϑ_j
OUTPUT: Path
if ϑ_i and $\vartheta_j \in \mathcal{V}_{\mathcal{HE}k}$ **then**
  if ϑ_i *in UTM is* $(i,\ i)$ *and* ϑ_j *in UTM is* $(i,\ j)$ **then**
    Path is $(i,\ i) \rightarrow (i,\ j)$.
  if ϑ_i *in UTM is* $(i,\ j)$ *and* ϑ_j *in UTM is* $(i,\ i)$ **then**
    Path is $(i,\ j) \rightarrow (i,\ i)$.
  if ϑ_i *in UTM is* $(i,\ j)$ *and* ϑ_j *in UTM is* $(i,\ k)$ **then**
    Path is $(i,\ j) \rightarrow (i,\ i) \rightarrow (i,\ k)$
else
  The sequence of energy efficient relay nodes are selected using Type 4 commutator.
  if ϑ_i *in UTM is* $(i,\ i)$ *and* ϑ_j *in UTM is* $(j,\ j)$. **then**
    if ϑ_i *is a neighbour of* ϑ_j **then**
      Path is $(i,\ i) \rightarrow (i,\ j) \rightarrow (j,\ j)$.
    else
      Path is $(i,\ i) \rightarrow (i,\ a) \rightarrow (a,\ a) \rightarrow \ldots \rightarrow (j,\ j)$ where the nodes in the position $(i,\ a), \ldots$ are best relay nodes.
  if ϑ_i *in UTM is* $(i,\ j)$ *and* ϑ_j *in UTM is* $(k,\ k)$. **then**
    if ϑ_i *is a neighbour of* ϑ_j **then**
      Path is $(i,\ j) \rightarrow (i,\ i) \rightarrow (i,\ k) \rightarrow (k,\ k)$.
    else
      Path is $(i,\ j) \rightarrow (i,\ i) \rightarrow (i,\ a) \rightarrow (a,\ a) \rightarrow \ldots \rightarrow (k,\ k)$ where the nodes in the position $(i,\ a), \ldots$ are best relay nodes.
  if ϑ_i *in UTM is* $(i,\ i)$ *and* ϑ_j *in UTM is* $(k,\ l)$. **then**
    if ϑ_i *is a neighbour of* ϑ_j **then**
      Path is $(i,\ i) \rightarrow (i,\ k) \rightarrow (k,\ k) \rightarrow (k,\ l)$.
    else
      Path is $(i,\ i) \rightarrow (i,\ a) \rightarrow (a,\ a) \rightarrow \ldots \rightarrow (k,\ k)$ where the nodes in the position $(i,\ a), \ldots$ are best relay nodes.
  if ϑ_i *in UTM is* $(i,\ j)$ *and* ϑ_j *in UTM is* $(k,\ l)$. **then**
    if ϑ_i *is a neighbour of* ϑ_j **then**
      Path is $(i,\ j) \rightarrow (i,\ i) \rightarrow (i,\ k) \rightarrow (k,\ k) \rightarrow (k,\ l)$.
    else
      Path is $(i,\ j) \rightarrow (i,\ i) \rightarrow (i,\ a) \rightarrow (a,\ a) \rightarrow \ldots \rightarrow (k,\ l)$ where the nodes in the position $(i,\ a), \ldots$ are best relay nodes.

variable hyperedges say $\mathcal{V}_{\mathcal{HE}k}$ and $\mathcal{V}_{\mathcal{HE}l}$ respectively. The inter cluster routing is carried out by the commutator of Type 4 $[(i,\ i) \rightarrow (i,\ j) \rightarrow (j,\ j)]$ with the intermediate node in the position (i, j). Algorithm 4 presents a routing scheme for a VANET V2V communication.

5. PERFORMANCE ANALYSIS

5.1 Environmental Setup

The simulation environment area covers 10004×1000 m^2 grid, in which the communication range of the vehicle is set as 75 m. The streets have two lanes in each direction. The source and destination of data packets are selected randomly. All the vehicles have capability of wireless communications [9]. The environmental setup is given in Fig. 1.

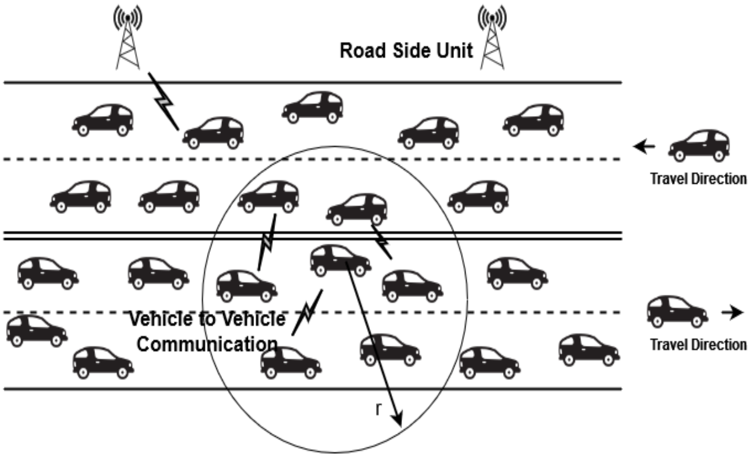


Fig. 1. Simulation environment.

The vehicle's speed is considered non uniform vary from 20 m/s to 120 m/s. The environment is varied with different density of vehicles from 30 to 70 vehicles and the initial energy of the vehicles is set as 1 Joule with V2V communication model. The source and destination are selected randomly for data transmission with the packet size of 64bytes.

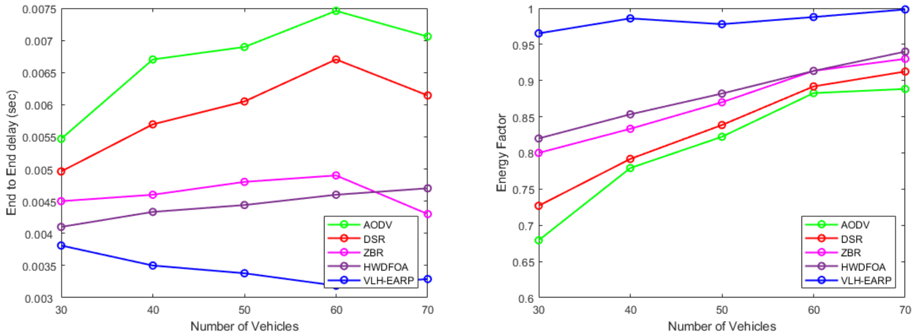
The proposed routing protocol was simulated using Matrix Laboratory (MATLAB) 2020b on a system of Windows 10 operating system with an Intel Core i5 processor and 6GB RAM. The simulation settings are varied under specified conditions to get the results. VLH-EARP is compared to various routing methods such as AODV, DSR, ZBR and HWDFOA[31]. The simulation results indicate that the VLH-EARP does an excellent job of maximising the performance of the VANET by determining the optimum path between the source and destination. The VLH-EARP considers four quality of service metrics (1) Average Delay; (2) Packet Delivery Ratio; (3) Energy Factor; and (4) Throughput to accentuate the performance over compared routing protocols.

5.2 Average Delay

We assessed the VLH-EARP performance in this simulation scenario by calculating the average latency of each benchmark procedure. The average delay of a complete vehicular network is calculated by taking the delay of each node in the network and then averaging these delays. From Fig. 2 (a), it is apparent that VLH-EARP outperforms even when node density increases, which is due to the hypergraph formation and Lie commutators. Further, end to end delay in VLH-EARP is 50% less over AODV, 44% less over DSR, 30% less over ZBR, and 16% less over HWDFOA, at higher mobility scenario.

5.3 Energy Factor

Fig. 2 (b) depicts the energy factor of both VLH-EARP and compares routing protocols for various vehicles for a packet size of 64 bytes. When VLH-EARP is compared to other protocols, the ratio of energy disbursed to initial energy through relay nodes increases, inferring that the VLH-EARP consumes less energy and is about 19%, 17%,



(a) (b)

Fig. 2. Impact of Average Delay and Energy factor over vehicle density changes.

12%, and 10% higher than AODV, DSR, ZBR and HWDFOA respectively. This is due to the preference for relay vehicles and reliable links based on an assortment of parameters, including energy factor, mobility, and Lie commutators for minimized hop count in the destination direction.

5.4 Packet Delivery Ratio

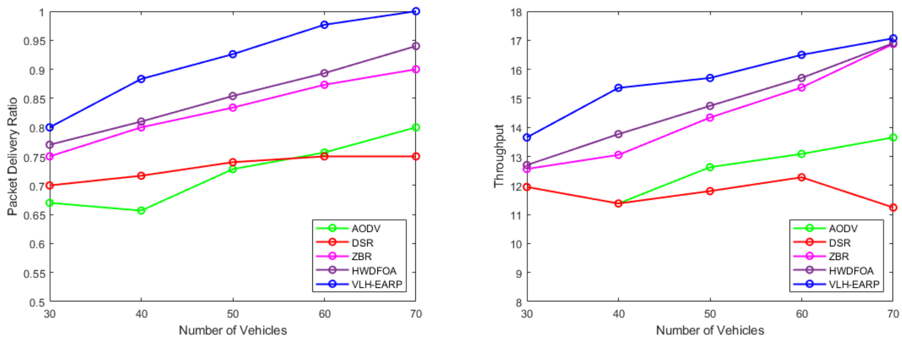
PDR measures the proportion of packets that are successfully transmitted to all packets sent. Fig. 3 (a) displays improved packet delivery ratio in the proposed VLH-EARP when compared with AODV, DSV and ZBR. The packet drop is reduced due to the Lie commutators used in VLH-EARP during the identification of a path from source to destination, thus leading to fewer path breakages as network density increases. From the simulated data, the VLH-EARP packet delivery ratio is 30.9%, 22.6%, 10.5% and 8% higher than AODV, DSR, ZBR and HWDOA respectively.

5.5 Average Throughput

Throughput is the total number of bits disseminated from source to destination in a certain amount of time. The outcomes in Fig. 3 (b) show that the suggested routing protocol effectively provides greater throughput than other comparable routing strategies. As expected throughput increases by 33%, 21.8%, 9.27% and 6.29% over AODV, DSR, ZBR and HWDOA respectively.

6. CONCLUSION

The tremendous number of vehicles in recent years has increased the necessity for communication when a vehicle is in motion, that makes ongoing research of the routing protocol for VANETs. In this paper, the Variable Lie Hypergraph based Energy Aware Routing Protocol (VLH-EARP) is proposed for efficient data transmission between V2V communication. VANET is formalized as a variable hypergraph and routing process is obtained by Lie commutators of Lie algebra of upper triangular matrices. The performance of VLH-EARP has been evaluated by comprehensive simulation and it is carried



(a) (b)

Fig. 3. Impact of packet delivery ratio and throughput over vehicle density changes.

out in MATLAB 2020b for different vehicle density. The results shows that VLH-EARP outperforms in terms of PDR, Throughput, Energy Factor and Delay when comparing with AODV, DSV, ZBR and HWDFOA routing protocols. As future work, this approach can be embedded for fast and secure data transmission.

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