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Neutrosophic Soft Set and Clinical Application

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Neutrosophic soft set is one of the generalizations of classical set theory with parameters. We have introduced weighted similarity measure using the normalized orthogonal distance between two single valued neutrosophic soft sets and their characteristics. Further, a decision-making framework is proposed through an algorithm for multi attribute decision making neutrosophic soft scenario. We also apply the proposed weighted similarity measure to the clinical application; identify the best type of radiotherapy treatment for tumor of moving organs such as lungs or chest walls by evaluating certain medical parameters and computation of mathematical ranking model, which are then compared with other existing similarity measures to illustrate the feasibility of the same.

Keywords: neutrosophic set, soft set, neutrosophic soft set, normalized orthogonal distance, similarity measure, weighted similarity measure, ideal attribute, decision making

1. INTRODUCTION

In 1995, as a generalization of fuzzy set [25] and intuitionistic fuzzy set [1], the neutrosophic set was defined with three different types of membership values by Florentin Smarandache [20]. Neutrosophic set is a powerful tool and the appropriate frame work for dealing with incomplete, indeterminate and inconsistent information in real world practical problems. Kandasamy and Smarandache initially presented basic algebraic neutrosophic structures and their application to advanced neutrosophic models. Consolidating neutrosophic set hypothesis with algebraic structures is a rising pattern in the region of mathematical research. One of the key developments in the neutrosophic set theory is the hybridization of neutrosophic set with various potential algebraic structures such as bipolar set, soft set, hesitant fuzzy set, *etc*. [13, 19, 23].

In 1999, Molodtsov [12] introduced soft set theory as a general mathematical tool for dealing with uncertain, fuzzy, not clearly defined objects. The algebraic structure of soft set theory dealing with uncertainty has been studied by some authors. Maji *et al*. [11] defined the algebraic operations of soft sets for theoretical study. An emerging trend in mathematical research is the convergence of neutrosophic set theory with soft set algebra. Neutrosophic soft algebraic structures and its properties give us a strong mathematical background to explain applied mathematical concepts in engineering, data mining

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and economics. From a scientific or engineering perspective, a decision-making model with parametrization tools plays a key role in real-world problems. Maji [10] introduced a new mathematical model 'Neutrosophic Soft Set' by combining neutrosophic set with soft set. Peng [14] proposed three algorithms by evaluation based on distance from average solution (EDAS), similarity measure and level soft set to solve single-valued neutrosophic soft decision making problems. We propose a new parametrization method for decision making using weighted similarity calculation of the orthogonal distance of the neutrosophic soft set. The significance of this work is subjective and objective assessment of alternatives and criteria through the analytical hierarchy process [2, 3, 6, 18]. The proposed weighted similarity measure is applied to the medical science decision problem and a mathematical model is developed to demonstrate the viability of the similarity measure proposed in a neutrosophic soft context.

The paper is organized as follows. Section 2 briefs about necessary preliminary definitions and results which are essential for a better and clear comprehension of the upcoming sections. Section 3 describes the normalized orthogonal distance between two neutrosophic soft sets and the measure of similarity which act as the pivot element of this work. The decision-making framework and methodology in neutrosophic soft environment is explained in detail, and an algorithm for decision making is proposed in section 4. Section 5 describes a decision-making problem of radiotherapy treatment in oncology with experimental setup, calculations and inference. A valid summary, relevance of this work, future study of this work is described in Section 6, towards the end of the paper.

2. PRELIMINARIES

In this section we present some of the preliminary concepts and findings that are important for a good and consistent understanding of the coming sections.

Definition 2.1. *[21] A neutrosophic set P of the universal set X is defined as* $P = \{(\eta, t_P(\eta), i_P(\eta), f_P(\eta)) : \eta \in X\}$ *where* $t_P, i_P, f_P : X \to (^-0, 1^+)$. *The three components* t_P, i_P *and* f_P *represent membership value (Percentage of truth), indeterminacy (Percentage of indeterminacy) and non membership value (Percentage of falsity) respectively. These components are functions of non standard unit interval* $(^-0, 1^+)$ *[15].*

Remark 2.1. *[4]*

1. *If* $t_P, i_P, f_P : X \to [0, 1]$, *then P is known as single valued neutrosophic set (SVNS). For simplicity SVNS will be called neutrosophic set.*
2. U^X *denotes the set of all neutrosophic subsets of X or neutrosophic power set of X.*

Definition 2.2. *[22] For any neutrosophic subset* $P = \{(\eta, t_P(\eta), i_P(\eta), f_P(\eta)) : \eta \in X\}$, *the support* P^* *of the neutrosophic set P can be defined as* $P^* = \{\eta \in X, t_P(\eta) > 0, i_P(\eta) > 0, f_P(\eta) < 1\}$.

Definition 2.3. *[12] Let X be the universal set of of objects and E be the set of parameters in connection to objects in X. A soft set over X and* $A \subseteq E$ *is a pair* $\langle F, A \rangle$ *where F is a function defined by* $F : A \to P(X)$, *and* $P(X)$ *is the power set of X . For any* *parameter $\rho \in A$, $F(\rho) \subseteq X$ may be considered as the set of ρ - approximate elements of the soft set $\langle F,A\rangle$ and it is represented as*

$$\langle F,A\rangle = \{(\rho, F(\rho)) : \rho \in E,\ F(\rho) = \phi\ if\ \rho \in E - A\}.$$

Example 2.1. *[17] Let $X = \{\eta_1, \eta_2, \eta_3, \eta_4\}$ be the universal set which contains 4 houses under consideration by an agent and $E = \{\rho_1 = cottage, \rho_2 = mansion, \rho_3 = terraced\}$. A customer who wants to select a house from the agent, can construct a soft set F_A that describes the characteristics of the houses according to his own requirements. $F(\rho_1) = \{\eta_1, \eta_2\}, F(\rho_2) = \{\eta_3\}, F(e_3) = \{\eta_3, \eta_4\}$. Then the soft set $\langle F,A\rangle$ is represented as follows;*

$$\langle F,A\rangle = \{(\rho_1, \{\eta_1, \eta_2\}), (\rho_2, \{\eta_3\}), (\rho_3, \{\eta_3, \eta_4\})\}.$$

Definition 2.4. *[17, 7] A pair $\langle F,A\rangle$ is called a neutrosophic soft set over the universal set X is a pair $\langle F,A\rangle$ where F is a function given by $F : A \to NS(X)$ and $NS(X)$ denotes the set of all neutrosophic sets of X .*
A neutrosophic soft set $\langle F,E\rangle = \{(\rho, F(\rho)) : \rho \in E,\ F_A(\rho) \in NS(X)\}$ where $F(\rho)$is a neutrosophic set on X which is characterized by

$$F(\rho) = \{\eta, t_{F(\rho)}(\eta), i_{F(\rho)}(\eta), f_{F(\rho)}(\eta) : \rho \in E\ \eta \in X\}\ and\ F(\rho) = \phi$$
$$i.e.\ F(\rho) = \{t_{F(\rho)}(\eta) = 0, i_{F(\rho)}(\eta) = 0, f_{F(\rho)}(\eta) = 1 : \rho \in E - A\}$$

where $t_{F(\rho)}(\eta), i_{F(\rho)}(\eta)$ and $f_{F(\rho)}(\eta)$ represents the truth-membership degree, the indeterminacy -membership degree and the falsity -membership degree of an object η holds on parameter ρ respectively.Thus neutrosophic soft set is a parametrization tool.

Remark 2.2. *We write $NSS(X)$ for neutrosophic soft set.*

Example 2.2. *Consider the example 2.1. Suppose that*

$$F(e_1) = \{\langle .4, .8, .3\rangle, \langle .3, .7, .1\rangle\ \langle 0, 0, .1\rangle, \langle 0, 1, 0\rangle\}$$
$$F(e_2) = \{\langle 0, 0, 1\rangle, \langle 0, 0, 1\rangle\ \langle .8, .2, .1\rangle, \langle .0, .0, 1\rangle\}$$
$$F(e_3) = \{\langle 0, 1, 0\rangle, \langle 0, 0, 1\rangle\ \langle .6, .1, .2\rangle, \langle .1, .6, .3\rangle\}$$

Then $F_A(E)$ is a parametrized family of $NS(X)$.

Definition 2.5. *[12] If $X = \{\eta_1, \eta_2, ..., \eta_m\}$, $E = \{\rho_1, \rho_2, ..., \rho_n\}$ and $A \subseteq E$ then the neutrosophic soft set $\langle F,E\rangle$ is uniquely characterised by the neutrosophic soft matrix $[a_{ij}]_{m\times n}$ where $a_{ij} = (t_{F(\rho_j)}(\eta_i), i_{F(\rho_j)}(\eta_i), f_{F(\rho_j)}(\eta_i))$.*

Example 2.3. *In example 2.2, the neutrosophic set $\langle F,E\rangle$ is characterised by the following neutrosophic soft matrix $D = (\alpha_{ij})_{4\times 3} =$*

$$\begin{bmatrix} 0.4 & 0.8 & 0.3 & & 0.0 & 0.0 & 1.0 & & 0.0 & 0.0 & 1.0 \\ 0.3 & 0.7 & 0.1 & & 0.0 & 0.0 & 1.0 & & 0.0 & 0.0 & 1.0 \\ 0.0 & 0.1 & 0.0 & & 0.0 & 0.0 & 1.0 & & 0.6 & 0.1 & 0.2 \\ 0.0 & 1.0 & 0.0 & & 0.0 & 0.0 & 1.0 & & 0.1 & 0.6 & 0.3 \end{bmatrix}$$

Definition 2.6. *[8] $\langle F,A\rangle$ is said to be a neutrosophic soft subset of $\langle G,B\rangle$ if $A\subseteq B$ where $A,B\subseteq E$ and $t_{F_A(\rho)}(\eta)\leq t_{G_B(\rho)}(\eta)$,$i_{F_A(\rho)}(\eta)\leq i_{G_B(\rho)}(\eta)$ and $f_{F_A(\rho)}(\eta)\geq f_{G_B(\rho)}(\eta)$ $\forall\,\rho\in A,\eta\in X$.*

3. SIMILARITY MEASURE USING NORMALISED ORTHOGONAL DISTANCE

A similarity measure or similarity function is a real-valued function that quantifies the similarity between two objects. Similarity measure takes large values on similar objects and either zero or a negative value for very dissimilar objects. Similarity measures are inversely proportional to distance between the sets. In this section we defined the orthogonal distance between two neutrosophic soft sets and the similarity measure using the normalised orthogonal distance.

Let $E=\{\rho_1,\rho_2,...,\rho_n\}$ be the set of parameters in relation to objects in $X=\{\eta_1,\eta_2,...,\eta_m\}$ and $A\subseteq E$. $\alpha=\langle F,A\rangle$ and $\beta=\langle G,A\rangle$ are two neutrosophic soft sets over X where each $\eta_i\in X$

$$F(\rho_j)=\{(\eta_i,t_{F(\rho_j)}(\eta_i),i_{F(\rho_j)}(\eta_i),f_{F(\rho_j)}(\eta_i)):\rho_j\in E\}$$
$$G(\rho_j)=\{(\eta_i,t_{G(\rho_j)}(\eta_i),i_{G(\rho_j)}(\eta_i),f_{G(\rho_j)}(\eta_i)):\rho_j\in E\}$$

in which $1\leq i\leq m, 1\leq j\leq n$ and the neutrosophic components are not a scalar multiple of each other.

Definition 3.1. *The normalized orthogonal distance between α and β can be denoted and defined as*

$$d^{\perp}(\alpha,\beta)=\sum_{i=1}^{i=m}\sum_{j=1}^{j=n}\frac{\sqrt{(T_{\alpha\beta}(\eta_i))^2+(I_{\alpha\beta}(\eta_i))^2+(\Gamma_{\alpha\beta}(\eta_i))^2}}{\max(|F(e_j)|,|G(e_j)|)} \text{ where}$$
$$T_{\alpha\beta}(\eta_i)=[t_{F(\rho_j)}(\eta_i)i_{G(\rho_j)}(\eta_i)-i_{F(\rho_j)}(\eta_i)t_{G(\rho_j)}(\eta_i)]$$
$$I_{\alpha\beta}(\eta_i)=[i_{F(\rho_j)}(\eta_i)f_{G(\rho_j)}(\eta_i)-f_{F(\rho_j)}(\eta_i)i_{G(\rho_j)}(\eta_i)],$$
$$\Gamma_{\alpha\beta}(\eta_i)=[f_{F(\rho_j)}(\eta_i)t_{G(\rho_j)}(\eta_i)-t_{F(\rho_j)}(\eta_i)f_{G(\rho_j)}(\eta_i)],$$
$$|F(\rho_j)|=\sqrt{(t_{F(\rho_j)}(\eta_i))^2+(i_{F(\rho_j)}(\eta_i))^2+(f_{F(\rho_j)}(\eta_i))^2},$$
$$|G(\rho_j)|=\sqrt{(t_{G(\rho_j)}(\eta_i))^2+(i_{G(\rho_j)}(\eta_i))^2+(f_{G(\rho_j)}(\eta_i))^2}.$$

Proposition 3.1. *$d^{\perp}(\alpha,\beta)$ where α and $\beta\in NSS(X)$ satisfies the following axioms*

1. $d^{\perp}(\alpha,\beta)\geq 0$
2. $\alpha=\beta\Rightarrow d^{\perp}(\alpha,\beta)=0$
3. $d^{\perp}(\alpha,\beta)=d^{\perp}(\beta,\alpha)$
4. $d^{\perp}(\alpha,\gamma)\leq d^{\perp}(\alpha,\beta)+d^{\perp}(\beta,\gamma)$ *where γ is any third neutrosophic soft set over X.*

Proof. It is obvious from Definition 3.1 □

Definition 3.2. *[9] A similarity measure between two neutrosophic sets A and B on X is a function defined as $S: X \times X \to [0,1]$ which satisfies the following properties*

1. $S(A,B) \in [0,1]$
2. $S(A,B) = 1 \Leftrightarrow A = B$
3. $S(A,B) = S(B,A)$
4. $A \subset B \subset C \Longrightarrow S(A,C) \leq S(A,B) \wedge S(B,C)$

Theorem 3.1. *A real valued function $S^{\perp}(\Upsilon,\Gamma): NSS(X) \times NSS(X) \to [0,1]$ where Υ and $\Gamma \in NSS(X)$ which is defined as $S^{\perp}(\Upsilon,\Gamma) = \frac{1}{1+d^{\perp}(\Upsilon,\Gamma)}$ is a similarity measure .*

Proof. To prove the function $S^{\perp}(\Upsilon,\Gamma)$ is a similarity measure, it is enough to prove that the function $S^{\perp}(\Upsilon,\Gamma)$ satisfies the properties in Definition 3.2: Proof of the property 1,2,3: It is clear from the given function
Proof of property 4: Given $\Upsilon \subseteq \Gamma \subseteq \Pi$ where $\forall\ \eta_i \in X$

$$\Upsilon = F(\rho_j) = \{(\eta_i, t_{F(\rho_j)}(\eta_i), i_{F(\rho_j)}(\eta_i), f_{F(\rho_j)}(\eta_i)) : \rho_j \in E\}$$

Now consider

$$\Gamma = G(\rho_j) = \{(\eta_i, t_{G(\rho_j)}(\eta_i), i_{G(\rho_j)}(\eta_i), f_{G(\rho_j)}(\eta_i)) : \rho_j \in E\}$$

$$\Pi = H(\rho_j) = \{(\eta_i, t_{H(\rho_j)}(\eta_i), i_{H(\rho_j)}(\eta_i), f_{H(\rho_j)}(\eta_i)) : \rho_j \in E\}$$

Then,

$$t_{F(\rho_j)}(\eta_i) \leq t_{G(\rho_j)}(\eta_i) \leq t_{H(\rho_j)}(\eta_i),\ i_{F(\rho_j)}(\eta_i) \leq i_{G(\rho_j)}(\eta_i \leq i_{H(\rho_j)}(\eta_i), \\ f_{F(\rho_j)}(\eta_i)) \geq f_{G(\rho_j)}(\eta_i)) \geq f_{H(\rho_j)}(\eta_i)$$

Consider

$$i_{H(\rho_j)}(\eta_i) \geq i_{G(\rho_j)}(\eta_i) \Rightarrow t_{F(\rho_j)}(\eta_i) i_{H(\rho_j)}(\eta_j) \geq t_{F(\rho_j)}(\eta) i_{G(\rho_j)}(\eta_i) \tag{1}$$

$$t_{H(\rho_j)}(\eta_i) \geq t_{G(\rho_j)}(\eta_i) \Rightarrow t_{H(\rho_j)}(\eta_i) i_{F(\rho_j)}(\eta_i) \geq t_{G(\rho_j)}(\eta_i) i_{F(\rho_j)}(\eta_i) \tag{2}$$

Eqs. (1)-(2),

$$t_{F(\rho_j)}(\eta_i) i_{H(\rho_j)}(\eta_i) - t_{H(\rho_j)}(\eta_i) i_{F(\rho_j)}(\eta_i) \geq t_{F(\rho_j)}(\eta_i) i_{G(\rho_j)}(\eta_i) - t_{G(\rho_j)}(\eta_j) i_{F(\rho_j)}(\eta_i)$$

$$\Rightarrow T_{\Upsilon\Pi} \geq T_{\Upsilon\Gamma} \text{ similarly } I_{\Upsilon\Pi} \geq I_{\Upsilon\Gamma}$$

Consider

$$t_{G(\rho_j)}(\eta_i) \leq t_{H(\rho_j)}(\eta_i) \Rightarrow f_{F(\rho_j)}(\eta_i) t_{G(\rho_j)}(\eta_i) \leq f_{F(\rho_j)}(\eta_i) t_{H(\rho_j)}(\eta_i) \tag{3}$$

$$f_{G(\rho_j)}(\eta_i) \geq f_{H(\rho_j)}(\eta_i) \Rightarrow f_{G(\rho_j)}(\eta_i) t_{F(\rho_j)}(\eta_i) \geq f_{H(\rho_j)}(\eta_i) t_{F(\rho_j)}(\eta_i) \tag{4}$$

$$\Rightarrow -f_{G(\rho_j)}(\eta_i) t_{F(\rho_j)}(\eta_i) \leq -f_{H(\rho_j)}(\eta_i) t_{F(\rho_j)}(\eta_i \tag{5}$$

Eqs. (3) and (5),

$$f_{F(\rho_j)}(\eta_i)t_{G(\rho_j)}(\eta_i) - f_{G(\rho_j)}(\eta_i)t_{F(\rho_j)}(\eta_i) \leq f_{F(\rho_j)}(\eta_i)t_{H(\rho_j)}(\eta_i) - f_{H(\rho_j)}(\eta_i)t_{F(\rho_j)}(\eta_i)$$

$\Rightarrow\ \mathit{\Gamma}_{\Upsilon\Gamma} \leq \mathit{\Gamma}_{\Upsilon\Pi} \Rightarrow \mathit{\Gamma}_{\Upsilon\Pi} \geq \mathit{\Gamma}_{\Upsilon\Gamma}$. Similarly, $T_{\Gamma\Pi} \leq T_{\Upsilon\Pi}$, $I_{\Gamma\gamma} \leq I_{\Upsilon\Pi}$ and $\mathit{\Gamma}_{\Gamma\Pi} \leq \mathit{\Gamma}_{\Upsilon\gamma}$

Thus,

$$d^{\perp}(\Upsilon,\Gamma) \leq d^{\perp}(\Upsilon,\Pi) \Rightarrow S^{\perp}(\Upsilon,\Gamma) \geq S^{\perp}(\Upsilon,\Pi)$$

$$d^{\perp}(\Gamma,\Pi) \leq d^{\perp}(\Upsilon,\Pi) \Rightarrow S^{\perp}(\Gamma,\Pi) \geq S^{\perp}(\Upsilon,\Pi)$$

$$\Rightarrow S^{\perp}(\Upsilon,\Pi) \leq S^{\perp}(\Upsilon,\Gamma) \wedge S^{\perp}(\Gamma,\Pi)$$

□

Definition 3.3. *The weighted similarity measure can be denoted and defined as follows*

$$WS^{\perp}(\Upsilon,\Gamma) = \frac{1}{1+\sum_{i=1}^{i=m}\sum_{j=1}^{j=n} w_i \frac{\sqrt{(T_{\Upsilon\Gamma}(\eta_i))^2+(I_{\Upsilon\Gamma}(\eta_i))^2+(\mathit{\Gamma}_{\Upsilon\Gamma}(\eta_i))^2}}{\max(|F_A(\rho_j)|,|G_A(\rho_j)|)}}$$

where $\Upsilon,\Gamma \in NSS(X)$ and $w_i \in [0,1]$ be the weight of each object η_i $(i = 1,2,...,m)$ with the property that sum of weights of each object is one.

4. DECISION MAKING USING WEIGHTED SIMILARITY MEASURE

In this section a retrospective decision-making model is proposed using weighted similarity measure of neutrosophic soft set through the analytic hierarchy process [16].

Definition 4.1. *[5] The Analytic Hierarchy Process (AHP) is a theory of measurement through pairwise comparisons and relies on the judgements of experts to derive priority scales. The comparisons are made using a scale of absolute judgements that represents, how much more, one element dominates another with respect to a given attribute.*

4.1 Methodology

A detailed overview of the steps taken to make the multi-criteria decision is provided. The following steps describe the category of retrospective decision-making and how to make a structured decision to create objectives in neutrosophic soft environment.

1. Develop a model for the decision making: It consists of building a hierarchy to analyze the decision.
   a. Choose the best alternative: The objects in the universal set $X = \{\eta_1,\eta_2,...,\eta_m\}$
   b. Choose the criteria: Let $E = \{\rho_1,\rho_2,...,\rho_n\}$ be the set of parameters or criteria in relation to objects in $X = \{\eta_1,\eta_2,...,\eta_m\}$ and $A \subseteq E$.

2. Derive weights (Priorities) and consistency ratio for the criteria .

3. Derive local preferences for the criteria: In this step we are first required to derive the pairwise comparison of each criterion with respect to each other using a numerical scale for comparison developed by Saaty [16] and prepare a comparison matrix of the criteria to perform the pairwise comparison of criteria.

4. Derive model synthesis: Let w_j be the weight of criterion ρ_j where $j = 1,2,...,n$ determined by the decision maker or team of experts. The evaluation of the alternatives η_i where $i = 1,2,...,m$ on criterion ρ_j *where* $1 \leq j \leq n$ is denoted by the following neutrosophic soft set form defined on ρ_j. $\langle F,A \rangle$ is defined as follows $\forall\ \eta_i \in X$.

$$F_A(\rho_j) = \{\eta_i, t_{F_A(\rho_j)}(\eta_i), i_{F_A(\rho_j)}(\eta_i), f_{F_A(\rho_j)}(\eta_i)\}$$

5. Formation of decision matrix: A decision matrix $D = [a_{ij}]_{m\times n}$ of $\langle F,A \rangle$ represents evaluation of each object η_i $(0 \leq i \leq m)$ in X on each parameter ρ_j $(0 \leq j \leq n)$ in E, constructed from the above equation. In multi-criteria decision making neutrosophic environment, the concept of ideal point has been used to identify the best attribute in the decision set. *i.e.* $F_A(\rho_j) = \{\eta_i, a_{ij}, b_{ij}, c_{ij}\}$.

   Definition 4.2. *[24] In the decision making procedure, the criteria are classified into two, according to their nature where* $i = 1,2,...,m$. *and* $j = 1,2,...,n$

   (a) Benefit criteria (α_j)*: Maximum operator is used for identifying ideal alternative in benefit criteria where* $\alpha_j = \langle \max_i a_{ij}, \max_i b_{ij}, \min_i c_{ij} \rangle = \langle a_j, b_j, c_j \rangle$

   (b) Cost criteria (α_j)*: Minimum operator is used for identifying ideal alternative in cost criteria where* $\alpha_j = \langle \min_{\mathbf{i}} a_{ij}, \max_{\mathbf{i}} b_{ij}, \max_{\mathbf{i}} c_{ij} \rangle = \langle a_j, b_j, c_j \rangle$

6. Perform sensitivity analysis: Calculate the weighted similarity measure between an alternative η_i and the ideal attribute α_j

7. Making a final decision based on model synthesis and sensitivity analysis. The ranking order of all attributes can be determined using the relation $\eta_i^* = \sum_{j=1}^{i=n} WS^{\perp}(\eta_i, \alpha_j)$.Then the best decision can be selected easily.

4.2 Algorithm for Decision Making in Neutrosophic Soft Environment

Step 1: Identify the alternatives and criteria as input data.

Step 2: Break down the model into hierarchy of goals,criteria and alternatives.

Step 3: Define the weight w_j of each criterion.

Step 4: Derive model synthesis *i.e.* construct a decision matrix $D = (\alpha_{ij})_{m\times n}$.

Step 5: Calculate the ideal attribute α_j using the evaluation of each η_i on each ρ_j.

Step 6: Calculate weighted similarity measure (WSM) $WS^{\perp}(\eta_i, \alpha_j)$.

Step 7: Determine ranking order of all alternatives using $\eta_i^* = \sum_{j=1}^{i=n} WS^{\perp}(A_i, \alpha_j)$.

Step 8: Pick the alternative relating to rank hierarchy as output.

5. EXPERIMENT-CLINICAL APPLICATION

5.1 Decision Making Problem and Objective

The experiment we consider in this section is clinical application of neutrosophic soft set. Let us consider a decision making problem of best radio therapy treatment in oncology. An external machine [Linear Accelerator] focuses the radiation beam on the treatment area using high-energy X-rays (which is known as external beam radiation). The following table gives the details of treatment procedure and concerns of external beam radiation.

The degree of success of radiation therapy of tumor cells in mobile organs such as the lungs or chest wall depends entirely on correct tumor location data and tumor position monitoring during treatment. Non regular motion is a constraint to achieve the accurate knowledge of a tumor location during treatment. For example, some typical tumors' location in a patient's lung region or pancreas or chest wall change due to breathing cycle abnormalities that lead to data ambiguity, imprecision and inconsistency. Therefore, with certain parameters, we find neutrosophic data as input data.

Table 1. External beam radiation treatment procedure and concerns.

Serial number	Treatment Procedure	Concerns
1	Simulation	Degree of success
2	Planning	Accuracy in tumor position
3	Treatment Delivery	Tracing of tumor motion

Objective: Identify the best available radiotherapy treatment by evaluating the alternatives and criteria.

5.2 Data Set

The first step in this framework is a set $X = \{\eta_1, \eta_2, \eta_3, \eta_4\}$ with four elements (alternatives) which are available treatment methods for monitoring and transmitting radiation to mobile organs. The oncologist must take a decision according to three criteria or parameters $E = \{\rho_1, \rho_2, \rho_3\}$. The table below gives a detailed overview of the four alternatives and three criteria we take for experimentation.

Table 2. Treatment methods and medical parameters.

Alternatives	Treatment Methods	Criteria	Medical Parameters
η_1	Target Tracking Cyberknife	ρ_1	Dosimetry (Benefit Criteria)
η_2	Automatic Breath Control Device	ρ_2	Prognosis (Benefit Criteria)
η_3	Cone Beam Computerized Tomography	ρ_3	Environmental Impact (Cost Criteria)
η_4	Fluro Portal Imaging		

5.3 Experiment Setup

The weight $w = (0.35, 0.25, 0.40)$ of the criteria is given by experts in decision making using analytic hierarchical process [16] . All alternatives are evaluated under the available criteria and the neutrosophic soft decision matrix $D = (a_{ij})_{4\times 3}$ is constructed as follows.

$$\begin{bmatrix} .45 & .25 & .35 & .50 & .20 & .30 & .80 & .25 & .45 \\ .65 & .15 & .25 & .65 & .15 & .25 & .45 & .40 & .45 \\ .45 & .25 & .35 & .55 & .25 & .35 & .45 & .30 & .80 \\ .75 & .5 & .15`qw & .65 & .15 & .20 & .65 & .35 & .85 \end{bmatrix}$$

From the above neutrosophic decision matrix D, the ideal attribute α_j $(j = 1, 2, 3)$ can be defined as follows

$$\alpha_j = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} = \begin{bmatrix} 0.75 & 0.05 & 0.15 \\ 0.65 & 0.15 & 0.20 \\ 0.45 & 0.40 & 0.85 \end{bmatrix}$$

5.4 Evaluation

The proposed experiment's objective is to identify the best treatment to mobile organs in radiotherapy. The parameter given in the neutrosophic soft environment is evaluated by the available treatment methods or alternatives. The weighted similarity measure between alternatives and each criteria is then calculated with the help of ideal attribute.

Table 3. Calculation of weighted similarity measure (WSM) using the criteria.

Attribute	WSM on ρ_1	WSM on ρ_2	WSM on ρ_3
η_1	0.84574	0.97953	0.82013
η_2	0.81107	0.95875	0.97004
η_3	0.86534	0.99960	0.73936
η_4	0.7471	0.80000	0.92489

From Table 3

$\eta_1^* = 2.6454,\ \eta_2^* = 2.7399,\ \eta_3^* = 2.6043,\ \eta_4^* = 2.4723$

where $\eta_i^* = \sum_{j=1}^{3} WS^{\perp}(\eta_i, \alpha_j)$ and $\eta_2^* > \eta_1^* > \eta_3^* > \eta_4^*$.

Also

$Rank\ 1 = \eta_2,\ Rank\ 2 = \eta_2,\ Rank\ 3 = \eta_3\ and\ Rank\ 4 = \eta_4$

5.5 Comparative Analysis

Comparative analysis plays a key role in the framework of decision-making. Experts are able to identify significant benefits and forecast future trends with consistency by evaluating comparative study. Here we consider three available weighted similarity measures (WSM) in literature [24] with proposed similarity measure for comparative study and and identify the significant benefits. The following methods represent weighted similarity measure (WSM) between of alternatives ρ_i (from Definition 2.5, the neutrosophic soft components of $\eta_i = (a_{ij}, b_{ij}, c_{ij})$) and ideal attribute α_j with neutrosophic components

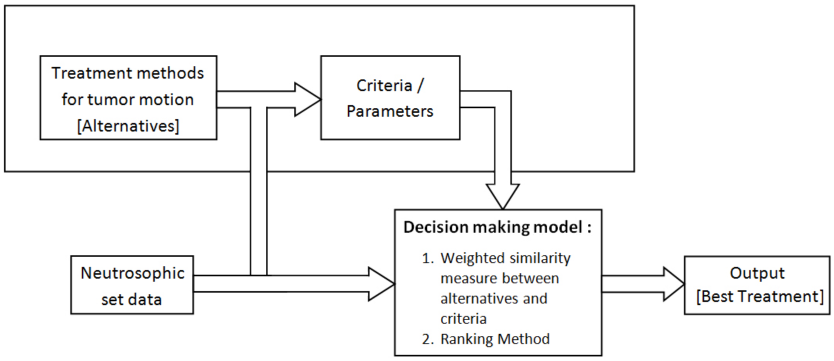

Fig. 1. Block diagram for proposed decision making model.

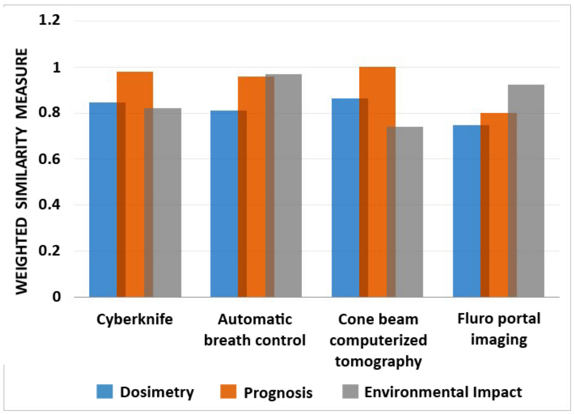

Fig. 2. Weighted similarity measure of criteria with each alternatives.

$\alpha_j = (a_j, b_j, c_j)$. Table 4 indicates that there is a difference in the order of the ranking. Not all strategies yielded the same results. To eliminate the variation and identify the best alternative, pair wise evaluation of alternatives, consistency ratio of criteria and clinical observations were applied by experts.

5.6 Clinical Observations from Experts

Experts deduce the following conclusions using statistical data and a direct interview on the above-mentioned decision-making clinical problem.

1. No restriction of automatic breath control device (ABC) tumor scale. The level of availability of this treatment strategy in common pool is high.

2. The degree of success in ABC technique is very high because it is possible to easily detect correct tumor location data and tumor movement during treatment.

3. Gross tumor volume coincides with planned and clinical target volume in ABC which prevents additional exposed dose to normal structures.

Table 4. Weighted similarity measures between the alternatives (η_i) and criteria (α_j).

Method	Evaluation Parameter	Measuring Value	Ranking order
Method 1	$WSM_1(\eta_1,\alpha_j)$	0.9768	$\eta_2 > \eta_1 > \eta_3 > \eta_4$
	$WSM_1(\eta_2,\alpha_j)$	0.9773	
	$WSM_1(\eta_3,\alpha_j)$	0.8547	
	$WSM_1(\eta_4,\alpha_j)$	0.7579	
Method 2	$WSM_2(\eta_1,\alpha_j)$	0.9880	$\eta_2 > \eta_1 > \eta_4 > \eta_3$
	$WSM_2(\eta_2,\alpha_j)$	0.9884	
	$WSM_2(\eta_3,\alpha_j)$	0.8594	
	$WSM_2(\eta_4,\alpha_j)$	0.9224	
Method 3	$WSM_3(\eta_1,\alpha_j)$	0.9896	$\eta_1 > \eta_2 > \eta_3 > \eta_4$
	$WSM_3(\eta_1,\alpha_j)$	0.9894	
	$WSM_3(\eta_2,\alpha_j)$	0.9276	
	$WSM_3(\eta_3,\alpha_j)$	0.8676	
Proposed Method	$\sum_{j=1}^{3} WS^{\perp}(\eta_1,\alpha_j)$	2.6454	$\eta_2 > \eta_1 > \eta_3 > \eta_4$
	$\sum_{j=1}^{3} WS^{\perp}(\eta_2,\alpha_j)$	2.7399	
	$\sum_{j=1}^{3} WS^{\perp}(\eta_3,\alpha_j)$	2.6043	
	$\sum_{j=1}^{3} WS^{\perp}(\eta_4,\alpha_j)$	2.4723	

4. The risk factor for cone beam computerized tomography and Fluro portal imaging is high because it affects the underlying anatomical structures (unsafe for patients due to additional exposure).

5.7 Inference and Results

To summarize the inference and discussions on the experiments, the following points are listed.

- The best choice of alternative is η_2 (Automatic breath control device), considering given parameters dosimetry, prognosis and environmental impact.
- The neutrosophic soft set is used to analyze the data in all possible forms.
- This approach reduces the data processing computational complexity.
- This method is very simple and effective to take an intelligent decision in neutrosophic soft set environment.

6. CONCLUSION

The proposed normalized orthogonal distance and weighted similarity measure in neutrosophic soft set one of the most generalized notions of classical theories to describe ambiguous or uncertain or indeterminate conditions. The decision making process using weighted similarity measure can be extended to different fields like engineering and medicine and other highly complex decision making situations. The procedure proposed in this paper for decision making is convenient and simple to adopt for practical purposes.The application of normalized similarity measure in multi attribute decision making in neutrosophic soft environment is illustrated through an example in medical field. We propose to extend the decision-making methodology for medical diagnosis, data mining and the theory of forecasting in our future research.

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