

# Strongly Secure Certificateless Signature Scheme in the Standard Model With Resisting Malicious-But-Passive KGC Attack Ability

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Digital signature is an important cryptographic tool in the security and privacy of smart city. Certificateless signature has not only simplified certificate management of traditional public-key signature, but also solved the private key escrow problem of ID-based signature. Recently, Pang *et al.*'s proposed a certificateless signature scheme in the standard model. We find that their scheme is vulnerable to the attack of malicious-but-passive KGC adversary. From the analysis of Pang *et al.*'s secure proof, we give a suggestion for the proof of certificateless signature, *i.e.*, we cannot remain some possible trapdoor information for KGC. Then we propose a strongly secure certificateless signature scheme, and give the secure proof in standard model. Compared with Pang *et al.*'s and other certificateless signature scheme in standard model, our proposed scheme can resist attack of malicious-but-passive KGC adversary.

**Keywords:** certificateless signature, smart city, malicious-but-passive KGC attack, public key replacement attack, standard model

## 1. INTRODUCTION

Smart city [1, 2] will be the next generation of urbanization by using information and communication technologies (ICT) to make cities “smarter”, such as government services, smart grids, transport and traffic managements, water controlling, waste treatment and recycling, *etc.* While the smart city provides a novel way to improve the people's quality of life, it will also introduce new security issues in the smart city. Ijaz *et al.* [3] categorized the security of smart city into governance, socioeconomic, and technological factors, and RFID (Radio Frequency Identification System), WSN (wireless sensor networks), M2M (machine to machine) communication, smart grids, smartphones, biometrics are the important component of technological factor security in smart city. Digital signature is usually used in WSN [3], smart grids [4, 5], smartphones [6] *etc.* to ensure the security of smart city. Furthermore, as an IoT tested for a smart city in Santander, Spain, SmartSantander project [7] used digital signature to ensure the security of power grid. So, digital signature is an important cryptographic tool in the security and privacy

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of smart city.

The notion of certificateless public-key cryptography was first proposed by Al-Riyami and Paterson [8] in 2003. In certificateless public-key cryptography, the user's private key is composed by the user's secret key and partial secret key. The user's secret key is selected by the user behind closed doors, and the partial secret key is generated by key generation center (KGC) from the user's identity. The user generates and publishes his/her public key without certificate.

Certificateless public-key cryptography has not only simplified certificate management of traditional public-key cryptography, but also solved the private key escrow problem [9] of ID-based public-key cryptography [10]. So, many application scenarios such as searchable encryption [11, 12] were solved by certificateless public-key cryptography [13] instead of traditional public-key cryptography [14, 15] or ID-based public-key cryptography [16]. Therefore, a lot of certificateless schemes were proposed following Al-Riyami and Paterson's work, including many certificateless signature schemes.

Al-Riyami and Paterson [8] proposed the first certificateless signature scheme, and introduced two adversaries: public key replacement adversary  $A_1^*$  and honest-but-curious KGC adversary  $A_{II}^*$ . Unfortunately, Huang *et al.* [17] pointed out that Al-Riyami and Paterson's scheme cannot resist the attack of the public key replacement adversary  $A_1^*$ . Latter, Zhang *et al.* [18] and Gorantla and Saxena [19] proposed certificateless signatures scheme based on bilinear pairings respectively; Harn *et al.* [20] proposed certificateless signature scheme based on discrete logarithm problem. He *et al.* [21] constructed a high-efficiency certificateless signature scheme without bilinear pairings. However, Tian *et al.* [22], pointed out that He *et al.*'s scheme cannot resist the attack of honest-but-curious KGC adversary  $A_{II}^*$ . In 2014, Tsai *et al.* [23] and Gong and Li [24] proposed improved schemes respectively. In 2007, Au *et al.* [25] introduced malicious-but-passive KGC adversary  $A_1^+$ , and pointed out that the schemes [8, 17] cannot resist this attack. Based on the hard lattice problems, Kim and Jeong [26] proposed a new certificateless signature scheme against the attack of malicious-but-passive KGC adversary  $A_{II}^+$ .

However, all the certificateless signature schemes mentioned above are provable security in the random oracle. In fact, Canetti *etc.* [27] pointed out that random oracle model is not secure in actual applications. So, it is more practical to design cryptography schemes which are secure in the standard model [28]. Based upon Waters' signature [29], Liu *et al.* [30] proposed the first certificateless signature scheme in the standard model. However Xiong *et al.* [31] pointed out that Liu *et al.*'s scheme cannot resist the attack of malicious-but-passive KGC adversary  $A_{II}^+$ , and proposed a modified scheme. Huang *et al.* [32] proposed a generic certificateless signature scheme for resisting malicious-but-passive KGC adversary. Yuan *et al.* [33] also proposed a certificateless signature scheme in the standard model. Unfortunately, Xia *et al.* [34] pointed out that [31, 33] cannot resist the attack of Xia's public key replacement adversary  $A_1^+$ , *i.e.*, when adversary  $A_1^+$  obtains a signature on a message of a signer, he/she can forge valid signatures on the same message under the replaced public key. In 2012, Yu *et al.* [35] constructed a new certificateless signature scheme without random oracle model to avoid the attack of Xia's public key replacement adversary  $A_1^+$ , however, Cheng *et al.* [36] pointed out that Yu *et al.*'s scheme was vulnerable to the attacks of malicious-but-passive KGC adversary  $A_{II}^+$  and public key replacement adversaries  $A_1^+$  and  $A_{II}^+$ , and, Yuan *et al.* [37] proposed an improved scheme. In 2015, Pang *et al.* [38] proposed a new certificateless signature scheme

without random oracle model, however, we found that Pang *et al.*'s scheme cannot resist the attacks of malicious-but-passive KGC adversary  $A_{\Pi}^+$ . In other words, we have found that most of the existing schemes in the standard model are insecure. Therefore, to construct a secure certificateless signature scheme in the standard model is worthwhile.

Inspired by Waters' signature [29], Parampalli and Narayan's [39] identity based signature, and Pang *et al.*'s [38] certificateless signature scheme, we proposed a strongly secure certificateless signature scheme with resisting the attack of malicious-but-passive KGC adversary  $A_{\Pi}^+$  in the standard model. In the following, we summarize our contributions.

- We point out that Pang *et al.*'s [38] certificateless signature scheme is vulnerable to the attack of malicious-but-passive KGC adversary  $A_{\Pi}^+$ . The reason is partly because that the authors construct the hardness problem by using the value generated by KGC to complete their proof of resisting malicious-but-passive KGC attack.
- We proposed a strongly secure certificateless signature scheme with resisting the attack of malicious-but-passive KGC adversary  $A_{\Pi}^+$  in the standard model.
- We gives a suggestion for proof the certificateless signature resisting malicious-but-passive KGC attack, *i.e.*, we cannot remain some possible trapdoor information for KGC, especially cannot construct the hardness problem by using the value generated by KGC.

The rest of the paper is organized as follows. Section 2 gives the preliminaries including bilinear pairing, hard problems, and definition and security model of certificateless signature scheme. Section 3 reviews Pang *et al.*'s [38] certificateless signature scheme, and analyzes the flaws in their scheme. We propose our scheme in Section 4, and give the formal security proof in Section 5. In Section 6, we compare our scheme with others, and give the conclusion.

## 2. PRELIMINARIES

In this section, we introduce the conception of bilinear pairings and certificateless signature, and talk about the security model of certificateless signature, and hard problems in our scheme.

### 2.1 Bilinear Pairings

Let  $G_1$  and  $G_2$  be two multiplicative cyclic groups with the same prime order  $p$ , while  $g$  is a generator of  $G_1$ . The map  $e: G_1 \times G_1 \rightarrow G_2$  is called a bilinear map [38] if the following three properties are held:

- (1) Bilinear: For all  $a, b \in Z_p^*$ , the equation  $e(g^a, g^b) = e(g, g)^{ab}$  is held.
- (2) Non-degenerate:  $e(g, g) \neq 1$ .
- (3) Computable: For any  $g_1, g_2 \in G_1$ , there is an efficient algorithm to compute  $e(g_1, g_2)$ .

### 2.2 Certificateless Signature

Generally, a certificateless signature scheme [38] involves three entities: the key

generation center (KGC), the signer, and the verifier; and five algorithms: Setup, UserKeyGen, ExtractPartialKey, Sign, and Verify.

- (1) **Setup:** The KGC runs this algorithm. Given the security parameter  $k$ , KGC generates the system public parameters  $pp$ , and master secret key  $msk$ .
- (2) **UserKeyGen:** The signer runs this algorithm. Given the public parameters  $pp$ , the signer generates the public key  $pk$  and the secret value  $sk$ .
- (3) **ExtractPartialKey:** The KGC runs this algorithm. Given the signer's identity  $ID$  and public key  $pk$ , KGC uses the system public parameters  $pp$  and the master secret key  $msk$  to generate the signer's partial secret key  $psk$ .
- (4) **Sign:** The signer runs this algorithm. Given the message  $m$ , the signer use the system public parameters  $pp$ , his/her identity  $ID$ , public key  $pk$ , secret value  $sk$ , and partial secret key  $psk$  to generate the signature  $\sigma$ .
- (5) **Verify:** The verifier runs this algorithm. Given the message  $m$ , the signature  $\sigma$ , the system public parameters  $pp$ , the signer's identity  $ID$  and public key  $pk$ , the verifier checks the validity of signature  $\sigma$ .

### 2.3 Security Models

According to [8, 25, 34], There are two types of adversaries in certificateless signature.

One is the public key replacement adversary  $A_I$ . Al-Riyami and Paterson [8] defined that the public key replacement adversary  $A_I^*$  cannot have access to master secret key, but can select identity and replace the public key. Xia *et al.* [34] expended the public key replacement adversary  $A_I^*$  to  $A_I^+$ , *i.e.*, obtained a signature on a message of a signer,  $A_I^+$  can forge valid signatures on the same message under the replaced public key. In our manuscript, we denote the public key replacement adversaries  $A_I^*$  and  $A_I^+$  as  $A_I$ .

The other is malicious-but-passive KGC adversary  $A_{II}$ . Al-Riyami and Paterson [8] defined that the honest-but-curious KGC adversary  $A_{II}^*$  can have access to master secret key, but cannot replace the public key. Au *et al.* [25] expended the honest-but-curious KGC adversary  $A_{II}^*$  to malicious-but-passive KGC adversary  $A_{II}^+$ , *i.e.*,  $A_{II}^+$  is malicious and tries to impersonate the user. Obviously, if a scheme can resist the attack of  $A_{II}^+$ , it can resist the attack of  $A_{II}^*$  too. In our manuscript, we denote malicious-but-passive KGC adversary  $A_{II}^+$  as  $A_{II}$ .

### 2.4 Complexity Assumptions

**Computational Diffie-Hellman (CDH) problem** [40]: Let  $G_1$  be a multiplicative cyclic groups with the prime order  $p$ , while  $g$  is a generator of  $G_1$ , and  $g^\alpha, g^\beta$  be two random elements of  $G_1$  with  $\alpha, \beta$  unknown. The CDH problem is to output  $g^\gamma$  such that  $\gamma = \alpha \cdot \beta$ .

We say that the CDH assumption holds in a group  $G_1$  if no algorithm running in polynomial time can solve the CDH problem in  $G_1$  with an advantage of at least  $\varepsilon$ .

**Square computational Diffie-Hellman problem (SCDH)** [40]: the SCDH problem is a variation of CDH problem except that is to output  $g^\gamma$  such that  $\gamma = \beta^2$ . The SCDH assumption and CDH assumption are equivalent [40].

### 3. OVERVIEW AND SECURITY ANALYSIS THE PANG *ET AL.*'S SCHEME

In 2015, Pang *et al.* [38] proposed a certificateless signature scheme in the standard model. In this section, we review and analyze their scheme.

#### 3.1 Overview the Pang *et al.*'s Scheme

In Pang *et al.*'s scheme [38], there are five phases, *i.e.*, Setup, UserKeyGen, ExtractPartialKey, Sign, and Verify, which are described as below.

##### (A) Setup

Given a security parameter  $k$ , the KGC performs the following steps.

**Step 1:** Choose two multiplicative cyclic groups  $G_1$  and  $G_2$  with prime order  $p$ , the group  $G_1$ 's generator  $g$ , and a bilinear pairing  $e: G_1 \times G_1 \rightarrow G_2$ .

**Step 2:** Select four collision resistant hash functions  $H_1: G_1 \times G_1 \rightarrow G_1$ ,  $H_2: \{0, 1\}^* \times G_1^2 \rightarrow Z_p^*$ ,  $H_3: \{0, 1\}^* \times G_1^3 \rightarrow \{0, 1\}^n$ , and  $H_4: \{0, 1\}^* \times G_1^3 \rightarrow Z_p^*$ .

**Step 3:** Choose a random number  $x \in Z_p^*$ , and compute  $g_1 = g^x$ , and  $g_2 = H_0(g, g_1)$ .

**Step 4:** Choose  $n + 3$  random elements  $t_0, t_1, v_0, v_1, \dots, v_n \in G_1$ , and denote  $V = (v_0, v_1, \dots, v_n)$ .

**Step 5:** Define a function  $f(z) = (z_x + z_y) \bmod 2$ , where  $z \in G_1$ , and  $z_x, z_y$  denotes  $x$ -coordinate and  $y$ -coordinate of  $z \in G_1$  respectively.

**Step 6:** The public parameters are  $pp = \{G_1, G_2, e, p, g, g_1, g_2, V, f, H_1, H_2, H_3, H_4\}$ , and the master secret key is  $msk = g_2^x$ .

##### (B) UserKeyGen

Given the public parameter  $pp$ , the user with identity  $ID$  selects three random numbers  $s_1, s_2, s_3 \in Z_p^*$ , and computes  $pk_1 = g_1^{s_1}, pk_2 = g_1^{s_2}, pk_3 = s_2 + s_3 \cdot H_2(ID, pk_1, pk_2)$ , and  $pk_4 = g_1^{s_3}$ , then he/she sets his/her secret value  $sk = s_2$ , publishes his/her public key  $pk = (pk_1, pk_2, pk_3, pk_4)$ , and sends his/her identity  $ID$  to KGC via a public channel.

##### (C) ExtractPartialKey

With the user's identity  $ID$  and public key  $pk$ , the KGC selects a random number  $r \in Z_p^*$ , computes  $Q = v_0 \prod_{i=1}^n v_i^{(H_3(ID, pk))[i]}$  where  $(H_3(ID, pk))[i]$  denotes the  $i$ th bit of  $H_3(ID, pk)$ ,  $psk_1 = g_2^r Q^r$ , and  $psk_2 = g^r$ , then sends the partial secret key  $pk = (pk_1, pk_2)$  to the user via a secret channel.

##### (D) Sign

With the identity  $ID$ , public key  $pk$ , secret value  $sk$ , and partial secret key  $psk$ , the signer performs the following steps to sign a given a message  $m$ .

**Step 1:** Choose two random numbers  $k, k' \in Z_p$ .

**Step 2:** Compute  $\sigma_3 = g^k, h = H_2(m, ID, \sigma_3, pk_2), h' = H_4(m, ID, \sigma_3, pk_2, g_2), b = f(\sigma_3), Q = v_0 \prod_{i=1}^n v_i^{(H_3(ID, pk))[i]}, \sigma_1 = psk_1^{h \cdot sk} \cdot Q^{k'} \cdot t_b^k \cdot pk_1^{h \cdot k}, \sigma_2 = psk_2^{h \cdot sk} \cdot g^{k'}$ .

**Step 3:** Output the signature  $\sigma = (\sigma_1, \sigma_2, \sigma_3)$  on the message  $m$ .

##### (E) Verify

Given the public parameter  $pp$ , the user's identity  $ID$  and public key  $pk$ , the mes-

sage  $m$ , and the signature  $\sigma$ , the verifier performs the following steps.

**Step 1:** Check whether the equation  $pk_1^{pk_3} = pk_2 \cdot pk_4^{H_3(ID, pk_1, pk_2)}$  is held, if so, go to Step 2, else, reject it.

**Step 2:** Compute  $h = H_2(m, ID, \sigma_3, pk_2)$ ,  $h' = H_4(m, ID, \sigma_3, pk_2, g_2)$ ,  $b = f(\sigma_3)$ ,  $Q = v_0 \cdot \prod_{i=1}^n v_i^{(H_3(ID, pk))(i)}$ .

**Step 3:** Check whether the equation  $e(\sigma_1, g) = e(g_2^{h'}, pk_2) \cdot e(Q, \sigma_2) \cdot e(t_b \cdot pk_1^h, \sigma_3)$  is held, if so, the signature  $\sigma$  is valid, otherwise, the signature  $\sigma$  is invalid.

### 3.2 Security Analysis

In Pang *et al.*'s scheme [38], the authors insisted that their scheme can resist the malicious-but-passive KGC attack, and give the security proof. However, we find that their scheme is vulnerable to the malicious-but-passive KGC attack.

In their security proof against the malicious-but-passive KGC attack, given a NGB-Many-DH problem instance  $(g^\alpha, g^\beta, g^\gamma, g^{\alpha\beta}, g^{\alpha\gamma}) \in G_1^6$ , the challenger  $C$  tries to output  $(g^{\alpha\beta\gamma}, g^x) \in G_1^2$  by interacting with adversary  $A_{II}$ . However, the adversary  $A_{II}$  is the malicious-but-passive KGC which generates the value of  $g^\alpha = g_1$ . Therefore, the security proof is not perfect and probably results that their scheme is vulnerable to the malicious-but-passive KGC attack. So, we suggest that we cannot remain some possible trapdoor information for KGC, especially cannot construct the hardness problem by using the value generated by KGC because that the KGC is the adversary  $A_{II}$  in the security proof against the malicious-but-passive KGC attack. We describe the details of attack process as follows.

**Step 1:** In Setup phase,  $A_{II}$  chooses  $2n+4$  random elements  $\tau_0, \tau_1, \alpha_0, \alpha_1, \dots, \alpha_n, \beta_0, \beta_1, \dots, \beta_n \in Z_p^*$ , computes  $t'_0 = g^{\tau_0}$ ,  $t'_1 = g^{\tau_1}$ , and  $v'_i = g_2^{\alpha_i} \cdot g^{\beta_i}$  for  $i = 0, 1, \dots, n$ , and denotes  $V' = (v'_0, v'_1, \dots, v'_n)$ , then replaces the  $t_0, t_1, V$  by  $t'_0, t'_1, V$ .

**Step 2:** In ExtractPartialKey phase,  $A_{II}$  computes  $\lambda = H_3(ID, pk)$ ,  $Q = F(\lambda) = v'_0 \prod_{i=1}^n v_i^{\lambda[i]} = g_2^{\alpha_0} \cdot g^{\beta_0} \prod_{i=1}^n (g_2^{\alpha_i} \cdot g^{\beta_i})^{\lambda[i]} = g_2^{\alpha_0 + \sum_{i=1}^n \alpha_i \cdot \lambda[i]} \cdot g^{\beta_0 + \sum_{i=1}^n \beta_i \cdot \lambda[i]} = g_2^\alpha \cdot g^\beta$ , sets  $\alpha = \alpha_0 + \sum_{i=1}^n \alpha_i \cdot \lambda[i]$ ,  $\beta = \beta_0 + \sum_{i=1}^n \beta_i \cdot \lambda[i]$ ,  $r' = -x \cdot \alpha^{-1}$ , computes  $psk_1 = g_2^x \cdot Q' = g_2^x \cdot Q^{-x\alpha^{-1}} = g_2^x \cdot (g_2^\alpha \cdot g^\beta)^{-x\alpha^{-1}} = g^{-x\alpha^{-1}\beta}$ , and  $psk_2 = g^{r'} = g^{-x \cdot \alpha^{-1}}$ , then sends the partial secret key  $psk = (psk_1, psk_2)$  to user via a secret channel.

**Step 3:** In Sign phase,  $A_{II}$  computes  $psk_1^{sk} = (g^{-x \cdot \alpha^{-1}\beta})^{sk} = (g^{x \cdot sk})^{-\alpha^{-1}\beta} = (g^{x \cdot sk})^{-\alpha^{-1}\beta} = psk_2^{\alpha^{-1}\beta}$ ,  $psk_2^{sk} = (g^{-x \cdot \alpha^{-1}})^{sk} = (g^{x \cdot sk})^{-\alpha^{-1}} = (g^{x \cdot sk})^{-\alpha^{-1}} = psk_2^{-\alpha^{-1}}$ . So, for every message  $m$ ,  $A_{II}$  can compute the user's signature as follows.  $A_{II}$  chooses two random number  $k, k' \in Z_p$ , computes  $\sigma_3 = g^k$ ,  $h = H_2(m, ID, \sigma_3, pk_2)$ ,  $h' = H_4(m, ID, \sigma_3, pk_2, g_2)$ ,  $b = f(\sigma_3)$ ,  $Q = F(H_3(ID, pk))$ ,  $\sigma_1 = (psk_1^{sk})^{h'} \cdot Q^{k'} \cdot t_b^k \cdot psk_1^h$ ,  $\sigma_2 = (psk_2^{sk})^{h'} \cdot g^{k'}$ , outputs the signature  $\sigma = (\sigma_1, \sigma_2, \sigma_3)$  on the message  $m$ .

## 4. OUR SCHEME

Inspired by Waters' signature [29], Parampalli and Narayan's [39] ID based signature, and Pang *et al.*'s [38] certificateless signature scheme, we proposed a strongly se-

cure certificateless signature scheme. Our scheme has five phases, *i.e.*, Setup, UserKeyGen, ExtractPartialKey, Sign, and Verify, which are described as follows.

#### 4.1 Setup

Given a security parameter  $k$ , the KGC performs the following steps.

- Step 1:** Choose two multiplicative cyclic groups  $G_1$  and  $G_2$  with prime order  $p$ , the group  $G_1$ 's generator  $g$ , and a bilinear pairing  $e: G_1 \times G_1 \rightarrow G_2$ .
- Step 2:** Choose  $n_1 + 2$  random elements  $g_2, u_0, u_1, \dots, u_{n_1} \in G_1$ , and denote  $U = (u_0, u_1, \dots, u_{n_1})$ .
- Step 3:** Choose  $n_2 + 2$  random numbers  $x, x_0, x_1, \dots, x_{n_2} \in Z_p^*$ , and denote  $msk_2 = (x_0, x_1, \dots, x_{n_2})$ .
- Step 4:** Compute  $g_1 = g^x, msk_1 = g_2^x, v_0 = g^{y_0}, v_1 = g^{y_1}, \dots, v_{n_2} = g^{y_{n_2}}$ , and denote  $V = (v_0, v_1, \dots, v_{n_2})$ .
- Step 5:** The public parameters are  $pp = \{G_1, G_2, e, p, g, g_1, g_2, U, V\}$ , and the master secret key is  $msk = (msk_1, msk_2)$ .

#### 4.2 UserKeyGen

Given the public parameter  $pp$ , the user with identity  $ID$  selects  $n_2 + 2$  numbers  $y, y_0, y_1, \dots, y_{n_2} \in Z_p^*$ , and computes  $pk_1 = g^y, w_0 = g^{y_0}, w_1 = g^{y_1}, \dots, w_{n_2} = g^{y_{n_2}} \in G_1$ , chooses a random element  $pk_2 \in G_1$ , and denotes  $pk_3 = (w_0, w_1, \dots, w_{n_2}), sk_2 = (y_0, y_1, \dots, y_{n_2})$ , then he/she sets his/her secret value  $sk = (sk_1, sk_2) = (pk_2, sk_2)$ , publishes his/her public key  $pk = (pk_1, pk_2, pk_3)$ .

#### 4.3 ExtractPartialKey

With the user's identity  $ID$  and public key  $pk$ , the KGC selects a random number  $r_1 \in Z_p^*$ , computes  $psk_1 = g_2^x(u_0 \prod_{i=1}^{n_1} u_i^{(ID||pk)i})^{r_1}$ , and  $psk_2 = g^{r_1}$ , then sends the partial secret key  $psk = (psk_1, psk_2)$  to user via a secret channel.

#### 4.4 Sign

With the identity  $ID$ , public key  $pk$ , secret value  $sk$ , and partial secret key  $psk$ , the signer performs the following steps to sign a given message  $m$ .

- Step 1:** Choose a random number  $r_2 \in Z_p^*$ .
- Step 2:** Compute  $d_0 = v_0^{y_0}, d_1 = v_1^{y_1}, \dots, d_{n_2} = v_{n_2}^{y_{n_2}}, \sigma_1 = psk_1 \cdot sk_1 \cdot (d_0 \cdot \prod_{i=1}^{n_2} d_i^{m[i]})^{r_2}, \sigma_2 = g^{r_2}, \sigma_3 = psk_2, \sigma_4 = (d_0, d_1, \dots, d_{n_2})$ .
- Step 3:** Output the signature  $\sigma = (\sigma_1, \sigma_2, \sigma_3, \sigma_4)$  on the message  $m$ .

#### 4.5 Verify

Given the public parameter  $pp$ , the user's identity  $ID$  and public key  $pk$ , the message  $m$ , and the signature  $\sigma$ , the verifier performs the following steps.

**Step 1:** For  $i = 0, 1, \dots, n_2$ , check whether the equations  $e(d_i, g) = e(v_i, w_i)$  are held and  $e(\sigma_1, g) = e(\sigma_2, (d_0 \cdot \prod_{i=1}^{n_2} d_i^{m[i]}) \cdot e(\sigma_3, u_0 \cdot \prod_{i=1}^{n_1} u_i^{(ID||pk)[i]}) \cdot e(g_1, g_2) \cdot e(pk_1, pk_2)$  is held, if so, the signature  $\sigma$  is valid, otherwise, the signature  $\sigma$  is invalid.

**Correctness:**

$$\begin{aligned} e(\sigma_1, g) &= e(psk_1 \cdot (sk_1 \cdot (d_0 \cdot \prod_{i=1}^{n_2} d_i^{m[i]})^{r_1}), g) \\ &= e((g_2^x \cdot (u_0 \cdot \prod_{i=1}^{n_1} u_i^{(ID||pk)[i]}))^{r_1} \cdot (g_4^y \cdot (d_0 \cdot \prod_{i=1}^{n_2} d_i^{m[i]})^{r_2}), g) \\ &= e((d_0 \cdot \prod_{i=1}^{n_2} d_i^{m[i]})^{r_2}, g) \cdot e((u_0 \cdot \prod_{i=1}^{n_1} u_i^{(ID||pk)[i]}))^{r_1}, g) \cdot e(g_2^x, g) \cdot e(pk_2^y, g) \\ &= e(d_0 \cdot \prod_{i=1}^{n_2} d_i^{m[i]}, g^{r_2}) \cdot e(u_0 \prod_{i=1}^{n_1} u_i^{(ID||pk)[i]}, g^{r_1}) \cdot e(g_2, g^x) \cdot e(pk_2, g^y) \\ &= e(\sigma_2, (d_0 \cdot \prod_{i=1}^{n_2} d_i^{m[i]})) \cdot e(\sigma_3, u_0 \cdot \prod_{i=1}^{n_1} u_i^{(ID||pk)[i]}) \cdot e(g_1, g_2) \cdot e(pk_1, pk_2) \end{aligned}$$

## 5. SECURITY PROOF

In this section, we give the security proof of our scheme. In order to resist the attack of Xia's public key replacement adversary  $A_1^+$ , we bind the signer's public key and his/her identity to his partial secret key, which is inherited the idea of Pang *et al.*'s scheme [38]. In order to resist the attack of malicious-but-passive KGC adversary  $A_{II}$ , we set some values generated by cooperation of the signer and the KGC to ensure that there is no trapdoor information for KGC. Next, we prove our scheme can resist the attack mentioned in subsection 2.3. The proof of resisting the attack of public key replacement adversary  $A_1$  is described in subsection 5.1, and the proof of resisting the attack of malicious-but-passive KGC adversary  $A_{II}$  is described in subsection 5.2.

### 5.1 Resisting the Attack of Public Key Replacement Adversary

**Theorem 1:** If there is public key replacement adversary  $A_1$ , who can break our proposed scheme in polynomial time with the success probability  $\varepsilon$ . Then there is an algorithm, which interacting with  $A_1$ , can solve the SCDH problem with the probability  $\varepsilon' \geq \frac{\varepsilon}{16 \cdot (q_{psk} + q_s) \cdot q_s \cdot (n_1 + 1) \cdot (n_2 + 1)}$ , where  $q_{psk}$  is the number of queries made to partial-secret-key-extract-queries, and  $q_s$  is the number of queries made to sign-queries respectively.

**Proof:** given a SCDH problem instance  $g^\beta$ , challenger  $C$  tries to output  $g^\beta$ , with the help of the adversary  $A_1$ . The challenger  $C$  does the following steps.

**Setup:**  $C$  sets  $G_1, G_2, e, p, g$  as same as subsection 4.1,  $l_1 = 2(q_{psk} + q_s)$ ,  $l_2 = 2q_s$ , and two random integers  $k_1$  and  $k_2$ , with  $0 \leq k_1 \leq n_1$ , and  $0 \leq k_2 \leq n_2$ .  $l_1 \cdot (n_1 + 1) < p$ , and  $l_2 \cdot (n_2 + 1) < p$  for the given values  $q_{pk}, q_s, n_1, n_2$ ; chooses randomly  $\alpha_0, \alpha_1, \dots, \alpha_{n_1} \in Z_{l_1}, \beta_0, \beta_1, \dots, \beta_{n_1} \in Z_p, \gamma_0, \gamma_1, \dots, \gamma_{n_2} \in Z_{l_2}, \omega_0, \omega_1, \dots, \omega_{n_2} \in Z_p$ ; constructs the functions  $F(\zeta) = \alpha_0 + \sum_{i=1}^{n_1} \alpha_i \cdot \zeta[i] - l_1 \cdot k_1, J(\zeta) = \beta_0 + \sum_{i=1}^{n_1} \beta_i \cdot \zeta[i], K(\xi, Y) = Y_0 \cdot \gamma_0 + \sum_{i=1}^{n_2} Y_i \cdot \gamma_i \cdot \xi[i] - Y_0 \cdot l_2 \cdot k_2, L(\xi, Y) = Y_0 \cdot \omega_0 + \sum_{i=1}^{n_2} Y_i \cdot \omega_i \cdot \xi[i]$ , where  $\omega_i = g^{\gamma_i}$ , for  $i = 0, 1, \dots, n_2$ ; computes the values:  $g_2 = g^\beta, g_1 = g_2^t = g^{t\beta} = g^\alpha, u_0 = g_2^{-l_1 k_1 + \alpha_0} \cdot g^{\beta_0}, u_i = g_2^{\alpha_i} \cdot g^{\beta_i}$ , for  $1 \leq i \leq n_1, v_0 = g_2^{-l_2 k_2 + \gamma_0} \cdot g^{\omega_0}, v_i = g_2^{\gamma_i} \cdot g^{\omega_i}$ , for  $1 \leq i \leq n_2$ ; and sends the public parameters  $pp = \{G_1, G_2, e, p, g, g_1, g_2, U,$

$V\}$  to  $A_1$ , where  $U = (u_0, u_1, \dots, u_{n_1})$ ,  $V = (v_0, v_1, \dots, v_{n_2})$ .

$$\text{Note that } u_0 \prod_{i=1}^{n_1} u_i^{\xi[i]} = g_2^{F(\xi)} g^{J(\xi)}, v_0^{y_0} \prod_{i=1}^{n_2} v_i^{y_i \cdot \xi[i]} = g_2^{K(\xi, Y)} g^{L(\xi, Y)}.$$

Create-user-queries:  $C$  keeps a list  $L$  to store user's  $ID$ , public key and secret key. Received a query for a public key on an  $ID$ ,  $C$  searches  $L$  to find the corresponding  $pk$  and return to  $A_1$ . If the  $pk$  haven't in  $L$ ,  $C$  generates a public key by using UserKeyGen phase in subsection 4.2, and stores the value  $(ID, pk, sk)$  in the  $L$ , and sends  $pk$  to  $A_1$ .

Secret-key-extract-queries:  $C$  searches  $L$  to find the corresponding  $sk$ , otherwise,  $C$  generates a public key by using UserKeyGen phase in subsection 4.2, and stores the value  $(ID, pk, sk)$  in the  $L$ , and sends  $sk$  to  $A_1$ .

Partial-secret-key-extract-queries: Received an inquiry for the  $psk$  on an identity  $ID$  and public key  $pk$ ,  $C$  checks if the equation  $F(ID||pk) = 0 \bmod p$  is held. If so, aborts. Otherwise,  $C$  selects a random number  $r_1$ , and computes  $psk_1 = g_1^{-J(ID||pk)/F(ID||pk)} = (u_0 \prod_{i=1}^n u_i)^{r_1}$ ,  $psk_2 = g_1^{-1/F(ID||pk)} = g^{r_1}$ , and sends  $psk = (psk_1, psk_2)$  to  $A_1$ .

The correctness of  $psk$  is because that:

$$\begin{aligned} psk_1 &= g_1^{-J(ID||pk)/F(ID||pk)} (u_0 \prod_{i=1}^{n_1} u_i^{(ID||pk)[i]})^{r_1} = g_2^\alpha (g_2^{F(ID||pk)} \cdot g^{J(ID||pk)})^{-\alpha/F(ID||pk)} \cdot (g_2^{F(ID||pk)} \cdot g^{J(ID||pk)})^{r_1} \\ &= g_2^\alpha (g_2^{F(ID||pk)} \cdot g^{J(ID||pk)})^{r_1 - \alpha/F(ID||pk)} = g_2^\alpha (u_0 \prod_{i=1}^{n_1} u_i^{(ID||pk)[i]})^{\tilde{r}_1}, \\ psk_2 &= g_1^{-1/F(ID||pk)} g^{r_1} = g^{r_1 - \alpha/F(ID||pk)} = g^{\tilde{r}_1}, \text{ where } \tilde{r}_1 = r_1 - \alpha/F(ID||pk). \end{aligned}$$

Note that  $F(ID||pk) = 0 \bmod p$  implies that  $F(ID||pk) = 0 \bmod l_1$ , and  $F(ID||pk) \neq 0 \bmod l_1$  implies that  $F(ID||pk) \neq 0 \bmod p$ .

Public-key-replace-queries:  $C$  creates a list  $L'$  to for an public key on an  $ID$ .  $C$  searches  $L'$  to find the corresponding  $pk$ , otherwise,  $C$  generates a public key by using UserKeyGen phase in subsection 4.2, and stores the value  $(ID, pk, sk)$  in the  $L'$ .

Sign-queries: When  $C$  receives an inquiry for the signing on an identity  $ID$  and message  $m$ ,  $C$  searches  $L$  to check if there is a replaced public key. If so, aborts, otherwise,  $C$  searches  $L$  to find the corresponding  $pk$ , if the  $ID$  is not in the  $L$ ,  $C$  generates a public key by using UserKeyGen phase in subsection 4.2, and stores the value  $(ID, pk, sk)$  in the  $L$ .  $C$  checks if the inequation  $F(ID||pk) \neq 0 \bmod l_1$  is held. If so,  $C$  can generate the  $psk$  from partial-secret-key-extract-queries and the  $sk$  from  $L$  to sign  $m$ . Otherwise, if  $K(m, sk_2) = 0 \bmod l_1$ , aborts, if not, it implies that  $K(m, sk_2) \neq 0 \bmod p$ . Known the  $sk$  from  $L$ ,  $C$  picks two random numbers  $r_1, r_2 \in Z_p$ , computes  $\sigma_1 = (u_0 \prod_{i=1}^{n_1} u_i^{(ID||pk)[i]})^{r_1} \cdot sk_1 \cdot g_1^{-L(m, sk_2)/K(m, sk_2)} \cdot (v_0^{y_0} \cdot \prod_{i=1}^{n_2} v_i^{y_i \cdot m[i]})^{r_2}$ ,  $\sigma_2 = g_1^{-1/K(m, Y)} g^{r_2}$ ,  $\sigma_3 = g^{r_1}$ ,  $\sigma_4 = (d_0, d_1, \dots, d_{n_2})$ , and sends  $\sigma = (\sigma_1, \sigma_2, \sigma_3, \sigma_4)$  to  $A_1$ . The correctness of  $\sigma$  is because that:

$$\sigma_1 = (u_0 \prod_{i=1}^{n_1} u_i^{(ID||pk)[i]})^{r_1} \cdot sk_1 \cdot g_1^{-L(m, sk_2)/K(m, sk_2)} \cdot (v_0^{y_0} \cdot \prod_{i=1}^{n_2} v_i^{y_i \cdot m[i]})^{r_2}$$

$$\begin{aligned}
&= g_2^\alpha \cdot (u_0 \prod_{i=1}^{n_1} u_i^{(ID||pk)[i]})^{r_1} \cdot sk_1 \cdot (g_2^{K(m,sk_2)} \cdot g_1^{L(m,sk_2)/\alpha})^{-\alpha/K(m,sk_2)} \cdot (v_0^{y_0} \cdot \prod_{i=1}^{n_2} v_i^{y_i \cdot m[i]})^{r_2} \\
&= g_2^\alpha \cdot (u_0 \prod_{i=1}^{n_1} u_i^{(ID||pk)[i]})^{r_1} \cdot sk_1 \cdot (g_2^{K(m,sk_2)} \cdot g^{L(m,sk_2)})^{-\alpha/K(m,sk_2)} \cdot (v_0^{y_0} \cdot \prod_{i=1}^{n_2} v_i^{y_i \cdot m[i]})^{r_2} \\
&= g_2^\alpha \cdot (u_0 \prod_{i=1}^{n_1} u_i^{(ID||pk)[i]})^{r_1} \cdot sk_1 \cdot (v_0^{y_0} \cdot \prod_{i=1}^{n_2} v_i^{y_i \cdot m[i]})^{-\alpha/(K(m,sk_2))} \cdot (v_0^{y_0} \cdot \prod_{i=1}^{n_2} v_i^{y_i \cdot m[i]})^{r_2} \\
&= g_2^\alpha \cdot (u_0 \prod_{i=1}^{n_1} u_i^{(ID||pk)[i]})^{r_1} \cdot sk_1 \cdot (v_0^{y_0} \cdot \prod_{i=1}^{n_2} v_i^{y_i \cdot m[i]})^{r_2 - \alpha/(K(m,sk_2))} \\
&\sigma_2 = g_1^{-1/K(m,sk_2)} g^{r_2} = g^{r_2 - \alpha/K(m,sk_2)} = g^{\tilde{r}_2}, \text{ where } \tilde{r}_2 = r_2 - \alpha/K(m,sk_2).
\end{aligned}$$

Solving the SCDH problem:

Finally,  $A_1$  outputs a forged signature  $\sigma^* = (\sigma_1^*, \sigma_2^*, \sigma_3^*, \sigma_4^*)$  with an identity  $ID^*$  on message  $m^*$ . If  $C$  does not terminate during the simulation, and satisfies the following conditions:

1.  $ID^*$  was never submitted to the partial-secret-key-extract-queries;
2. ①  $(ID^*, m^*)$  was never submitted to the sign-queries, or ②  $(ID^*, m^*)$  was submitted to the sign-queries, but the forged signature  $\sigma^*$  forged signature by the replaced  $pk^*$ .
3.  $\sigma^*$  is correct signature on message  $m^*$

Note that 2② is for Xia's public key replacement adversary  $A_1^+$ .

If the  $pk$  is not replaced,  $C$  knows the  $sk$ . If the public key is replaced, we discuss the methods for  $A_1$  to replace  $pk_3^*$  in forging signature first. Although we cannot imitate  $A_1$ 's process of forged signature, note that  $A_1$  cannot solve the CDH problem, we can explain why  $A_1$  can provide additional information for the following reasons. If  $A_1$  selects a random element  $w_i^*$  from  $G_1$  to construct  $pk_3^*$ , he/she must compute  $d_i^*$  for forging signature, i.e.,  $A_1$  can solve the CDH problem. Note the CDH problem is hard, so  $A_1$  must use the public parameters or the value from required the  $C$  to replace  $pk_3^*$ . Furthermore, he/she must compute  $pk_3^*$  by operations used in our scheme in section 5 such as multiplication, power, etc., otherwise, there is nothing but select random elements from  $G_1$ . Then he/she sends the computation methods to  $C$ , and  $C$  can obtain that  $w_i^* = g^{\theta_i} \cdot g_2^{\theta_i} = g^{\theta_i + \rho_i \beta}, 0 \leq i \leq n_2$ , with the  $\theta_i, \rho_i$  are known.

If there is a  $j$  such that  $\rho_j \neq 0$ , if  $j \neq 0$ ,  $C$  can compute  $g^{\beta^2} = (d_j^* \cdot (g^{\theta_j \cdot \alpha_j} \cdot g_2^{\theta_j \cdot \gamma_j + \alpha_j \cdot \rho_j})^{-1})^{(\rho_j \cdot \gamma_j)^{-1}}$ . It is because that  $e(v_j, w_i^*) = e(g^{\theta_j} \cdot g_2^{\gamma_j}, g^{\theta_j} \cdot g_2^{\rho_j}) = e(g^{\theta_j + \gamma_j \beta}, g^{\theta_j + \rho_j \beta}) = e(g^{(\theta_j + \rho_j \beta)(\alpha_j + \gamma_j \beta)}, g) = e(g^{\theta_j \cdot \alpha_j + (\theta_j \cdot \gamma_j + \alpha_j \cdot \rho_j) \beta + \rho_j \cdot \gamma_j \beta^2}, g)$ , and  $e(v_j, w_j^*) = e(d_j^*, g)$ .

If  $j = 0$ ,  $C$  can compute  $g^{\beta^2} = (d_0^* \cdot (g^{\theta_0 \cdot \alpha_0} \cdot g_2^{\theta_0(-l_2 k_2 + \gamma_0) + \alpha_0 \cdot \rho_0})^{-1})^{(\rho_0(-l_2 k_2 + \gamma_0))^{\pm 1}}$  similarly.

Otherwise, for  $i = 0$  to  $n_2$ ,  $\rho_i = 0$ , so  $w_i^* = g^{\theta_i}, 0 \leq i \leq n_2$ .

Furthermore,  $A_1$  uses the same method to replace  $pk_1^*, pk_2^*$ . With the same reason, at least one of  $pk_1^*, pk_2^*$  is equal to  $g^{\theta'}$ , where  $\theta'$  is known by  $C$ . Without loss of generality, we set  $pk_1^* = g^{\theta'}$ . Therefore, regardless of the public key replaced or not,  $C$  knows the  $\theta'$ ,  $\theta_i$  of  $pk_1^* = g^{\theta'}$ ,  $w_i^* = g^{\theta_i}, 0 \leq i \leq n_2$ . We denote  $Y^* = \{\theta_0, \theta_1, \dots, \theta_{n_2}\}$ .

If  $F(ID^*||pk^*) \neq 0 \pmod p$ , or  $K(m^*, Y^*) \neq 0 \pmod p$ , aborts. If  $F(ID^*||pk^*) = 0 \pmod p$ , and  $K(m^*, Y^*) = 0 \pmod p$ ,  $C$  can compute  $g^{\beta^2} = (\sigma_1^* \cdot ((\sigma_2^* \cdot L(m^*, Y^*) \cdot (\sigma_3^*)^{J(ID^*||pk^*)} \cdot (pk_2^*)^{\theta'})^{-1})^{r^t}$ . It is because that

$$\begin{aligned}
e(\sigma_1^*, g) &= e(\sigma_2^*, (d_0^* \cdot \prod_{i=1}^{n_2} (d_i^*)^{m^*[i]})) \cdot e(\sigma_3^*, u_0 \cdot \prod_{i=1}^{n_1} u_i^{(ID^*||pk^*)[i]}) \cdot e(g_1, g_2) \cdot e(pk_1^*, pk_2^*) \\
&= e(\sigma_2^*, g_2^{K(m^*, Y^*)} g^{L(m^*, Y^*)}) \cdot e(\sigma_3^*, g_2^{F(ID^*||pk^*)} g^{J(ID^*||pk^*)}) \cdot e(g^\alpha, g^\beta) \cdot e(g^\theta, pk_2^*)
\end{aligned}$$

$$\begin{aligned}
&= e(\sigma_2^*, g^{L(m^*, Y^*)}) \cdot e(\sigma_3^*, g^{J(ID^*||pk^*)}) \cdot e(g^{t\beta}, g^\beta) \cdot e(g^{\theta'}, pk_2^*) \\
&= e((\sigma_2^*)^{L(m^*, Y^*)}, g) \cdot e((\sigma_3^*)^{J(ID^*||pk^*)}, g) \cdot e(g^{t\beta^2}, g) \cdot e((pk_2^*)^{\theta'}, g) \\
&= e((\sigma_2^*)^{L(m^*, Y^*)} \cdot (\sigma_3^*)^{J(ID^*||pk^*)} \cdot g^{t\beta^2} \cdot (pk_2^*)^{\theta'}, g)
\end{aligned}$$

Now, we discuss the success probability of solving the SCDH problem.  $C$  will not abort during the proof if the following conditions satisfied simultaneously.

1.  $F(ID||pk) \neq 0$  in the partial-secret-key-queries.
2.  $F(ID||pk) \neq 0 \bmod l_1$  or  $K(m, sk_2) \neq 0 \bmod l_2$  in the sign-queries.
3.  $F(ID^*||pk^*) \neq 0 \bmod p$ , and  $K(m^*, Y^*) = 0 \bmod p$ , in the forged signature phase.

For simple, we ignore the condition that  $F(ID||pk) \equiv 0 \bmod l_1$  and  $K(m, sk_2) \neq 0 \bmod l_2$  in the sign-queries. Let  $ID_1, ID_2, \dots, ID_{q_1}$  be the identities appearing in either partial-secret-key-extract-queries or sign-queries not involving the challenge identity and  $m_1, m_2, \dots, m_{q_2}$  be the messages appearing in sign-queries involving the challenge identity  $ID^*$ . Obviously,  $q_1 \leq q_{psk} + q_s$  and  $q_2 \leq q_s$ . Define the events  $A_i, A^*, B_j, B^*$  as

$$\begin{aligned}
A_i: F(ID||pk_i) = 0 \bmod l_1, A^*: F(ID^*||pk^*) = 0 \bmod p, \\
B_j: F(m_j, sk_{2,j}) = 0 \bmod l_2, B^*: K(m^*, Y^*) = 0 \bmod p.
\end{aligned}$$

Then the probability of  $C$  not aborting is  $\Pr[\neg \text{abort}] \geq \Pr[\wedge_{i=1}^{q_1} \bar{A}_i \wedge A^* \wedge \wedge_{i=1}^{q_2} \bar{B}_i \wedge B^*]$ . Note that the events  $\wedge_{i=1}^{q_1} \bar{A}_i \wedge A^*$  and  $\wedge_{i=1}^{q_2} \bar{B}_i \wedge B^*$  are independent, and the assumption  $l_1 \cdot (n_1 + 1) < p$  leads to  $F(\zeta) = 0 \bmod p \Rightarrow F(\zeta) = 0 \bmod l_1$ , so  $\Pr[A^*] = \Pr[F(ID^*||pk^*) = 0 \bmod p \wedge F(ID^*||pk^*) = 0 \bmod l_1] = l_1^{-1} \cdot (n_1 + 1)^{-1}$ , and  $\Pr[\wedge_{i=1}^{q_1} \bar{A}_i] = 1 - \Pr[\vee_{i=1}^{q_1} A_i | A^*] \geq 1 - \sum_{i=1}^{q_1} \Pr[A_i | A^*] = 1 - \Pr[\vee_{i=1}^{q_1} A_i | A^*] \geq 1 - \sum_{i=1}^{q_1} \Pr[A_i | A^*] = 1 - q_1 \cdot l_1^{-1}$ . Therefore,  $\Pr[\wedge_{i=1}^{q_1} \bar{A}_i \wedge A^*] = \Pr[A^*] \cdot \Pr[\wedge_{i=1}^{q_1} \bar{A}_i \wedge A^*] = \Pr[A^*] \cdot \Pr[\wedge_{i=1}^{q_1} \bar{A}_i] \geq l_1^{-1} \cdot (n_1 + 1)^{-1} \cdot (1 - (q_{psk} + q_s) \cdot l_1^{-1})$ . Set  $q_1 = q_{psk} + q_s$ , then,  $\Pr[\wedge_{i=1}^{q_1} \bar{A}_i \wedge A^*] \geq (4 \cdot (q_{psk} + q_s))^{-1}$ , similarly,  $\Pr[\wedge_{i=1}^{q_2} \bar{B}_i \wedge B^*] \geq (4 \cdot q_s \cdot (n_2 + 1))^{-1}$ , therefore,  $\Pr[\neg \text{abort}] \geq (16 \cdot (q_{psk} + q_s) \cdot (n_1 + 1) \cdot (n_2 + 1))^{-1}$ .

## 5.2 Resisting the Attack of Malice-but-Passive KGC Adversary

**Theorem 2:** If there is malice-but-passive KGC adversary  $A_{II}$ , who can break our proposed scheme in polynomial time with the success probability  $\varepsilon$ . Then there is an algorithm, which interacting with  $A_{II}$ , can solve the CDH problem with the probability  $\varepsilon' \geq \frac{(q_{sk}-1) \cdot \varepsilon}{4 \cdot q_{sk} \cdot q_s \cdot (n_2 + 1)}$ , where  $q_{sk}$  is the number of queries made to secret-key-extract-queries, and  $q_s$  is the number of queries made to sign-queries respectively.

**Proof:** given a CDH problem instance  $(g^\alpha, g^\beta)$ , challenger  $C$  tries to output  $g^{\alpha\beta}$  with the help of the adversary  $A_{II}$ . The challenger  $C$  does the following steps.

**Setup:**  $C$  sets  $l_1 = 2(q_e + q_s)$ ,  $l_2 = 2q_s$ , and selects two random integers  $k_1$  and  $k_2$ , with  $0 \leq k_1 \leq n_1$ , and  $0 \leq k_1 \leq n_2$ .  $l_1(n_1 + 1) < p$ , and  $l_2(n_2 + 1) < p$  for the given values  $q_e, q_s, n_1, n_2$ . Then  $C$  uses the setup phase in subsection 4.1 to generate the public parameters and the master secret key except  $U$ , and chooses randomly  $\alpha_0, \alpha_1, \dots, \alpha_{n_1} \in Z_{l_1}$ , computes  $u_i = g^{\alpha_i}$ ,  $1 \leq i \leq n_1$ , sends the public parameters  $pp$  and the master secret key  $msk$  to  $A_{II}$ . Next,

$C$  chooses randomly  $\gamma_0, \gamma_1, \dots, \gamma_{n_2} \in Z_{l_2}$ ,  $\omega_0, \omega_1, \dots, \omega_{n_2} \in Z_p$ , sets two functions  $K(\xi) = x_0 \cdot \gamma_0 + \sum_{i=1}^{n_2} x_i \cdot \gamma_i \cdot \xi[i] - x_0 \cdot l_2 \cdot k_2$ ,  $L(\xi) = x_0 \cdot \omega_0 + \sum_{i=1}^{n_2} x_i \cdot \omega_i \cdot \xi[i]$ . Finally,  $C$  picks a target identity  $ID^*$ , and sets the public key  $pk_1 = g^\alpha$ ,  $pk_2 = g^\beta$ ,  $w_0 = pk_2^{-l_2 \cdot k_2 + \gamma_0} \cdot g^{\omega_0}$ ,  $w_i = pk_2^{x_i} \cdot g^{\omega_i}$ ,  $1 \leq i \leq n_2$ , and sends the user's identity  $ID^*$  and his/her public key to  $A_{II}$ .

Note that  $v_0^{Y_0} \prod_{i=1}^n v_i^{Y_i \cdot m[i]} = pk_2^{K(v, Y)} g^{L(v, Y)}$ . Furthermore, there is no additional trapdoor information for  $C$ . Although  $C$  knows the power of  $u_i$ ,  $1 \leq i \leq n_1$ , it cannot give  $C$  additional information because that adversary  $A_{II}$  has the ability of generation  $psk$ .

**Public-key-extract-queries:**  $C$  keeps a list  $L$  to stores user's  $ID$ , public key and secret key. Received a query for a public key on an  $ID$ ,  $C$  searches  $L$  to find the corresponding  $pk$ , otherwise,  $C$  generates a public key by using UserKeyGen phase in subsection 4.2, and stores the value  $(ID, pk, sk)$  in the  $L$ , and sends  $pk$  to  $A_{II}$ .

**Secret-key-extract-queries:** Received a query for a public key on an  $ID (\neq ID^*)$ ,  $C$  searches  $L$  to find the corresponding  $pk$ , otherwise,  $C$  generates a public key by using UserKeyGen phase in subsection 4.2 and stores the value  $(ID, pk, sk)$  in the  $L$ , and sends  $sk$  to  $A_{II}$ .

**Sign-queries:** If  $ID \neq ID^*$ ,  $C$  searches  $L$  to find the corresponding  $pk$ , if the  $ID$  is not in the  $L$ ,  $C$  generates a public key by using UserKeyGen phase in subsection 4.2, and stores the value  $(ID, pk, sk)$  in the  $L$ , so  $C$  knows the  $psk$ , and  $sk$ , and can sign any message  $m$ .

If  $ID = ID^*$ , if  $K(m, msk_2) = 0 \bmod l_2$  aborts. Otherwise, which implies that  $K(m, msk_2) \neq 0 \bmod p$ ,  $C$  knows the  $psk$ , and picks two random numbers  $r_1, r_2 \in Z_p$ , computes  $\sigma_1 = psk_1 \cdot pk_1^{-L(m)/K(m)} \cdot (w_0^{x_0} \cdot \prod_{i=1}^{n_2} w_i^{x_i \cdot m[i]})^{r_2}$ ,  $\sigma_2 = pk_1^{-1/K(m)} \cdot g^{r_2}$ ,  $\sigma_3 = psk_2$ ,  $\sigma_4 = (d_0, d_1, \dots, d_{n_2})$ , and sends  $\sigma = (\sigma_1, \sigma_2, \sigma_3, \sigma_4)$  to  $A_{II}$ . The correctness of  $\sigma$  is because that:

$$\begin{aligned} \sigma_1 &= psk_1 \cdot pk_1^{-L(m)/K(m)} \cdot (w_0^{x_0} \cdot \prod_{i=1}^{n_2} w_i^{x_i \cdot m[i]})^{r_2} \\ &= psk_1 \cdot pk_2^\alpha \cdot (pk_2^{K(m)} \cdot pk_1^{L(m)/\alpha})^{-\alpha/K(m)} \cdot (w_0^{x_0} \cdot \prod_{i=1}^{n_2} w_i^{x_i \cdot m[i]})^{r_2} \\ &= psk_1 \cdot pk_2^\alpha \cdot (pk_2^{K(m)} \cdot g^{L(m)})^{-\alpha/K(m)} \cdot (w_0^{x_0} \cdot \prod_{i=1}^{n_2} w_i^{x_i \cdot m[i]})^{r_2} \\ &= psk_1 \cdot pk_2^\alpha \cdot (w_0^{x_0} \cdot \prod_{i=1}^{n_2} w_i^{x_i \cdot m[i]})^{-\alpha/K(m)} \cdot (w_0^{x_0} \cdot \prod_{i=1}^{n_2} w_i^{x_i \cdot m[i]})^{r_2} \\ &= psk_1 \cdot sk_1 \cdot (w_0^{x_0} \cdot \prod_{i=1}^{n_2} w_i^{x_i \cdot m[i]})^{r_2 - \alpha/K(m)} \\ &= psk_1 \cdot sk_1 \cdot (w_0^{x_0} \cdot \prod_{i=1}^{n_2} w_i^{x_i \cdot m[i]})^{\tilde{r}_2} \\ \sigma_2 &= pk_1^{-1/K(m)} \cdot g^{r_2} = g^{r_2 - \alpha/K(m)} = g^{\tilde{r}_2}, \text{ where } \tilde{r}_2 = r_2 - \alpha/K(m). \end{aligned}$$

Solving the CDH problem:

If  $A_{II}$  outputs a forged signature  $\sigma^* = (\sigma_1^*, \sigma_2^*, \sigma_3^*, \sigma_4^*)$  with an identity  $ID^*$  on message  $m^*$ , if  $C$  does not terminate during the simulation, and satisfies the follows conditions:

1.  $(ID^*, m^*)$  is never submitted to the signing queries;
2.  $\sigma^*$  is correct signature on message  $m^*$ .

If  $K(m^*, msk_2) \neq 0 \bmod p$ , aborts. Otherwise  $K(m^*) = 0 \bmod p$ ,  $C$  can compute  $g^{\alpha/\beta} = \sigma_1^* \cdot ((\sigma_2^*)^{L(m^*)} \cdot (\sigma_3^*)^{\alpha_0 + \sum_{i=1}^{n_2} \alpha_i (ID^* || pk^*)[i]} \cdot g^x)^{-1}$ . It is because that:

$$e(\sigma_1, g) = e(\sigma_2^*, (d_0^* \cdot \prod_{i=1}^{n_2} (d_i^*)^{m^*[i]})) \cdot e(\sigma_3^*, u_0 \cdot \prod_{i=1}^{n_2} u_i^{(ID^* || pk^*)[i]}) \cdot e(g_1, g_2) \cdot e(pk_1^*, pk_2^*)$$

$$\begin{aligned}
&= e(\sigma_2^*, g_2^{K(m^*)} g^{L(m^*)}) \cdot e(\sigma_3^*, g^{\alpha_0} \cdot \prod_{i=1}^{n_1} (g^{\alpha_i})^{(ID^* \| pk^*)[i]}) \cdot e(g^x, g_2) \cdot e(pk_1^*, pk_2^*) \\
&= e(\sigma_2^*, g^{L(m^*)}) \cdot e(\sigma_3^*, g^{\alpha_0 + \sum_{i=1}^{n_1} \alpha_i \cdot (ID^* \| pk^*)[i]}) \cdot e(g, g_2^x) \cdot e(g^\alpha, g^\beta) \\
&= e((\sigma_2^*)^{L(m^*)}, g) \cdot e((\sigma_3^*)^{\alpha_0 + \sum_{i=1}^{n_1} \alpha_i \cdot (ID^* \| pk^*)[i]}, g) \cdot e(g_2^x, g) \cdot e(g^{\alpha, \beta}, g) \\
&= e((\sigma_2^*)^{L(m^*)} \cdot (\sigma_3^*)^{\alpha_0 + \sum_{i=1}^{n_1} \alpha_i \cdot (ID^* \| pk^*)[i]}, g_2^x \cdot g^{\alpha, \beta}, g)
\end{aligned}$$

Now, we discuss the success probability of solving the CDH problem.  $C$  Will not abort during the proof if the following conditions satisfied simultaneously.

1.  $ID \neq ID^*$  in the secret-key-extract-queries;
2.  $ID = ID^*$  and  $K(m) \neq 0 \pmod{l_2}$  or  $ID \neq ID^*$  in the sign-queries;
3.  $K(m^*) = 0 \pmod{p}$ , in the forged signature phase.

The probability that  $ID \neq ID^*$  in the secret-key-extract-queries is  $(q_{sk} - 1)/q_{sk}$ . Similar to the probability analysis in subsection 5.1, the probability of  $C$  not abort is at least  $(q_{sk} - 1) \cdot (4 \cdot q_{sk} \cdot q_s \cdot (n_2 + 1))^{-1}$ .

## 6. CONCLUSIONS

In smart city, digital signature is an important cryptographic tool which is used to protect the security and privacy. In this paper, we have reviewed the Pang *et al.*'s certificateless signature scheme, and point out their scheme is vulnerable to the attack of malicious-but-passive KGC adversary. Then we propose a strongly secure certificateless signature scheme, and give the secure proof in standard model. Compared Pang *et al.*'s [38] and other certificateless signature scheme [30, 33, 35] in standard model, Although the computational cost of our proposed scheme is slightly larger, our proposed scheme can resist attack of malicious-but-passive KGC adversary. Furthermore, we give a suggestion for proof the certificateless signature resisting malicious-but-passive KGC attack, *i.e.*, we cannot remain some possible trapdoor information for KGC.

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